PERFORMANCE OF AN AUTOMATED HIGH ACCURACY PHASE MEASUREMENT SYSTEM

S. Stein, D. Glaze, J. Lavine, J. Gray, D. Hilliard, D. Howe
Time and Frequency Division
National Bureau of Standards
Boulder, Colorado 80303

and

L. Erb
ERBTC Engineering Inc.
Boulder, Colorado

Summary

A fully automated measurement system has been developed that combines many properties previously realized with separate techniques. This system is an extension of the dual mixer time difference technique, and maintains its important features: zero dead time, absolute phase difference measurement, very high precision, the ability to measure oscillators of equal frequency and the ability to make measurements at the time of the operator’s choice. For one set of design parameters, the theoretical resolution is 0.2 ps, the measurement noise is 2 ps rms and measurements may be made within 0.1 s of any selected time. The dual mixer technique has been extended by adding scalers which remove the cycle ambiguity experienced in previous realizations. In this respect, the system functions like a divider plus clock, storing the epoch of each device under test in hardware.

The automation is based on the ANSI/IEEE-583 (CAMAC) interface standard. Each measurement channel consists of a mixer, zero-crossing detector, scaler and time interval counter. Four channels fit in a double-width CAMAC module which in turn is installed in a standard CAMAC crate. Controllers are available to interface with a variety of computers as well as any IEEE-488 compatible device. Two systems have been in operation for several months. One operates 24 hours a day, taking data from 15 clocks for the NBS time scale, and the other is used for short duration laboratory experiments.

Review of the Dual Mixer Time Difference Technique

It is advantageous to measure time directly rather than time fluctuations, frequency or frequency fluctuations. These measurements constitute a hierarchy in which the subsequently listed quantities may always be calculated from the previous ones. However, the reverse is not true when there are gaps in the measurements. In the past, frequency was usually not derived from time measurements for short sample times because time interval measurements could not be performed with adequate precision. The dual mixer technique, illustrated in Figure 1, made it possible to realize the precision of the beat frequency technique in time interval measurements.

The signals from two oscillators (clocks) are applied to two ports of a pair of double balanced mixers. Another signal synthesized from one of the oscillators is applied to the remaining two ports of the mixer pair. The input signals may be represented in the usual fashion

\[ V_1(t) = V_{10} \sin [2\pi f_0 t + \phi_1(t)] \]
\[ V_2(t) = V_{20} \sin [2\pi f_0 t + \phi_2(t)] \]
\[ V_s(t) = V_{s0} \cos [2\pi f_s t + \phi_s(t)] \]

where \( f_s = f_0 (l-1/R) \) and \( R \) is a constant usually called the heterodyne factor.

The low passed outputs of the two mixers are

\[ V_{1B} = V_{10} \sin [\phi_1(t) - \phi_2(t)] \]
\[ V_{2B} = V_{20} \sin [\phi_2(t) - \phi_s(t)] \]

where

\[ \phi(t) = 2\pi f_s t + \phi(t) \]

The time interval counter starts at time \( t_M \) when \( V_{2B} \) crosses zero in the positive direction and stops at time \( t_N \), the time of the very next positive zero crossing of \( V_{2B} \). Thus

\[ \phi_s(t_M) - \phi_s(t_N) = 2\pi M \]

\[ \phi_s(t_N) - \phi_s(t_M) = 2\pi N \]

where

\( N \) and \( M \) are integers.

Subtracting the two equations in order to compare the phases of oscillators 1 and 2, one obtains

\[ \phi_2(t_N) - \phi_2(t_M) = \phi_1(t_N) - \phi_1(t_M) + 2(2N-M)\pi \]

The phase of an oscillator at time \( t_M \) may be written in terms of its phase at \( t_H \) and its
average frequency over the interval \( t_M < t_N \):

\[
\phi(t_N) = \phi(t_M) + 2n[V(t_M;t_N)](t_N-t_M)
\]

when we apply this equation to both \( \phi_2 \) and \( \phi_5 \) we find

\[
\phi_2(t_M)-\phi_5(t_M) = 2(N+M)n
\]

\[
-2n[V_{B2}(t_M;t_N)](t_N-t_M)
\]

where \( v_{B2} = v_2 - v_5 \).

Since \( M \) and \( N \) are not measurable with the equipment in Figure 1, the dual mixer technique has heretofore only been used to measure the phase difference between two oscillators modulo \( 2\pi \). We denote the period of the time interval counter time base by \( t_c \) and the number of counts recorded in a measurement by \( P \). Then the phase difference between the two oscillators is given by

\[
[\phi_2(t_M)-\phi_5(t_M)] \mod 2\pi = 2n[V_{B2}(t_M;t_N)]t_c^P
\]

Figure 2 illustrates the output of the measurement system over a period of time. If a measurement begins and ends without the time interval counter making a transition between zero and its maximum value, e.g., \( t_c < t_M < t_N < t_c \), then the phase difference can be calculated from the data. If \( t_M < t_c < t_N < t_c \), then the data must be corrected by \( 2\pi \) to calculate the phase difference. Experience has shown that there are many measurement situations for which the number of transitions of the time interval counter which occur between \( t_M \) and \( t_N \) cannot be known. For this reason, a modification has been developed which removes the ambiguity by measuring \( M \) and \( N \).

**Extended Dual Mixer Time Difference Measurement Technique**

In order to configure the system to acquire complete phase information, two scalers are added to count the zero crossings of each mixer. Figure 3 is the block diagram of a two channel system. It is constructed from identical circuit modules and therefore contains an unused time interval counter. However, this design permits very straightforward, and inexpensive extension to the comparison of an arbitrarily large number of oscillators with no need for switching any signals.

The counter outputs are combined to form the phase difference between oscillators.

\[
\phi_2(t_M)-\phi_5(t_M) = 2(N-M)n + 2(N-M)m
\]

\[
-2n[V_{B2}(t_M;t_N)]t_c^P
\]

The first term is a constant which represents the choice of the time origin and can be ignored. The last two terms and their sum are plotted in Figure 4.

The average beat frequency \( \bar{v}_{B2}(t_M;t_N) \) cannot be known exactly. However, \( \bar{v}_{B2} \) may be estimated with sufficient precision from the previous pair of measurements designated \( M \) and \( N \). The average frequency is approximately

\[
\bar{v}_{B2}(t_M;t_N) \approx \frac{(N-N')}{(R(M-M'))/v_{10} + t_c(P-P')} \]

provided that it changes sufficiently slowly compared to the interval \( t_M,t_N \). A typical value for this error will be given in the following section.

**Hardware Implementation**

All measurement channels consist of a mixer, zero-crossing detector, scaler and time interval counter. Four such circuits can be built in a double width CAMAC module. The system is easily expanded to compare many oscillators and a complete system for making phase comparisons among four clocks is shown in Figure 5. We have chosen parameters which are reasonable for comparing state-of-the-art atomic standards. Thus, the synthesizer is offset 10Hz below oscillator \#1 and \( R = 5 \times 10^4 \). The outputs from both mixers are approximately 10Hz. The noise bandwidth is 100 Hz. The time interval counter is twice the frequency of oscillator \#1 or approximately 50 MHz. The quantization error is \( 1/2R = 10^8 \) cycle or 0.2ps which is a factor of ten smaller than the measurement noise. As stated earlier, an error will result from frequency changes which violate the constancy assumption used to estimate \( v_{B2} \). A change in \( v_{B2} \) by \( 10^5 \) during the interval between two measurements will result in a time deviation error of 10ps. Thus, one must make more closely spaced measurements for oscillators which have large dynamic frequency changes than for more stable devices. Two other sources of inaccuracy are the sensitivities to the amplitude and phase of the common oscillator. Figure 6 shows the measured value of \( \chi = 6/2\pi \) as a function of the amplitude of the input signal and the phase of the synthesizer.

The new measurement system has many desirable features and properties:

1. It has very high resolution, limited by the internal counters to 0.2 ps and by noise to approximately 2 ps.
2. It has much lower noise than divider based measurement systems. However, compromises made to achieve low cost, low power, small size and automatic operation degrade the performance compared to state-of-the-art systems for comparing 2 oscillators.
3. The operation is fully automatic.
4. NBS has developed a detailed operating manual for the equipment and software.
(5) All oscillators in the range of 5 MHz ± 5 Hz may be compared. Other carrier frequencies such as 1 MHz, 5.115 MHz, 10 MHz and 10.23 MHz are also usable. However, different carrier frequencies may not be mixed on the same system. The system has been successfully tested with an oscillator offset 4.6 Hz from nominal 5 MHz. Measurements were made at intervals of 2 hours between which the system had to accumulate approximately 2 x 10^11. The system has also been tested with an oscillator offset 4 x 10^11, and no errors were detected during a period of 40 days.

(6) All sampling times in the range of 1 second to 16 days with a resolution of 0.1 second are possible. Measurements may be made on command or in a preprogrammed sequence.

(7) Measurements are synchronized precisely, i.e. at the picosecond level, with the reference clock. They may therefore be synchronized with important user system events, such as the switching times of a FSK or PSK system.

(8) All oscillators are compared synchronously and all measurements are performed within a maximum interval of 0.1 second. As a result, the phase of any oscillator needs to be interpolated to the chosen measurement time for an interval of 0.1 second maximum. This capability, which is not present in either single heterodyne measurement systems or switched measurement systems, eliminates a source of "measurement" error which is generally much larger than the noise induced errors. For example, interpolation of the phase of a high performance Cs clock (σ = 10^{-12}/√T) over a period of 3 hours would produce approximately 1.5 ns phase uncertainty. To maintain 4 ps accuracy requires measurements simultaneous to 0.1s.

(9) There are no phase errors due to the switching of rf signals since there is no switching anywhere in the analog measurement system.

(10) No appreciable phase errors are introduced when it is necessary to change the reference clock since, as shown in Figure 6, the peak error due to changes in synthesizer phase is 20 ps.

(11) The measurement system is capable of measuring its own phase noise when the same signal is applied to two input ports. Figure 7 shows the phase deviations between two such channels over a period of 75,000 seconds and Figure 8 is the corresponding Allan variance plot. Figure 9 shows the phase deviations between 2 input channels over a period of 40 days.

(12) Since the IEEE-583 (CAMAC) interface standard has been followed for all the custom hardware, the system may be easily interfaced to almost any instrument controller. NBS has already tested the system using a large minicomputer, a small minicomputer and a desk top calculator. Interfaces between IEEE-583 and IEEE-488 controllers are available and have been used successfully.

(13) The system is capable of comparing a very large number of oscillators at a reasonable cost per device.

There are also disadvantages to this measurement system. The most important are:

(1) The complexity of the hardware is greater than for some systems. It is possible that this will reduce reliability.

(2) A high level of redundancy is difficult to achieve. The system design stresses size, power, convenience and cost, resulting in an increase in the number of possible single point failure mechanisms compared to some other techniques. For example, a CAMAC power supply failure will result in a loss of data for all devices being measured.

(3) A substantial commitment is required in both specialized hardware and software.

(4) If an oscillator under test experiences a phase jump which exceeds 1 cycle, the measurement system records a jump with incorrect absolute magnitude. As a result, it may not be applicable to signals which are frequency modulated with discontinuous phase steps larger than 2π.

Conclusions

We have demonstrated a new phase measurement system with very desirable properties: All oscillators in the range of 5 MHz ± 5 Hz may be measured directly. The sampling times are only restricted by the requirement that they exceed one second. The noise floor is σ(2,τ) = 3 x 10^{-12}/√T in short term and the time deviations are less than 100 ps. All circuitry is designed as modules which allows expansion at modest cost. Compatibility with a variety of computers is insured through the use of the IEEE-583 interface and adapters are available to permit use with an IEEE-488 controller. The system makes it feasible to make completely automated phase measurements at predetermined times on large numbers of atomic clocks. It's own noise is one-hundred times less than the state-of-the-art in clock performance. It will be used in the near future to make all measurement needed to compute NBS atomic time, but it will also be very valuable for any laboratory which uses three or more atomic clocks.

References

1. D. W. Allan. "The measurement of frequency

Figure 4. Extended Dual Mixer Data

Figure 5. System Block Diagram

Figure 6. Measured Time Difference vs. Input Amplitude and Synthesizer Phase
Figure 7. Raw Phase Data for Two Channels Driven from the Same Source

Figure 8. Noise Floor of Measurement System
Figure 9. Raw Phase Data for Two Channels Driven from the Same Source