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Some Statistical Properties of LF and VLF Propagation

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Summary

A statistical analysis has been conducted on the day-time phase fluctuations of the standard frequency and time Radio Stations WWVB (60 kHz) and WWVL (20 kHz) as received at Palo Alto, California, and of WWVB as received at the National Research Council, Ottawa, Canada. The analysis technique allows a meaningful determination of the low frequency spectral density, of the variance of the phase and frequency fluctuations, and of some cross-correlations. The analysis techniques used are appropriate for commonly encountered non-stationary as well as stationary noise processes.

The results of the analysis yielded a spectral density of the time fluctuations proportional to the reciprocal spectral frequency ($S_t(\omega) = \frac{h}{|\omega|}$, flicker noise) for the propagation noise on both WWVL and WWVB. The value of h was equal to 7.9 x 10^{-14} s^2 for WWVL and 2.2 x 10^{-14} s^2 for WWVB for the Palo Alto path, and h was 4.4 x 10^{-14} s^2 for WWVB over the Ottawa path. For flicker noise phase modulation a good model of the standard deviation of the fractional frequency fluctuations is: $\sigma = k|\tau|^{-1}$, where τ is the sample time in days. The values of k were 2.4 x 10^{-11} days for WWVL and 1.2 x 10^{-11} days for WWVB over the Palo Alto path and 1.8 x 10^{-11} days for WWVB over the Ottawa path.

A cross-correlation coefficient of -0.6 was found between WWVL and WWVB for the Palo Alto path. A linear combination of the two transmissions improved the flicker noise level by a factor of 11.5 over WWVL and by a factor of 2.7 over WWVB, allowing a precision of frequency measurement of 1 x 10^{-12} for a nine day average any time of the year and of 1 x 10^{-12} for a five day average over the summer months.

Introduction

The phase fluctuations introduced along a propagation path may be classified into two types. First there are the deterministic or predictable types of fluctuation such as the diurnal, seasonal, and tidal variations. Second there are the random and non-predictable fluctuations that remain after all known perturbations are removed. The deterministic type of fluctuations has been studied by others and much understanding has been given. The authors will treat some aspects of the non-deterministic propagation fluctuations with the intent of reducing the effect of the propagation phase fluctuations to a minimum.

The second section will review some useful statistical techniques. Application of these techniques in section three yields the spectral density of the phase and freq-

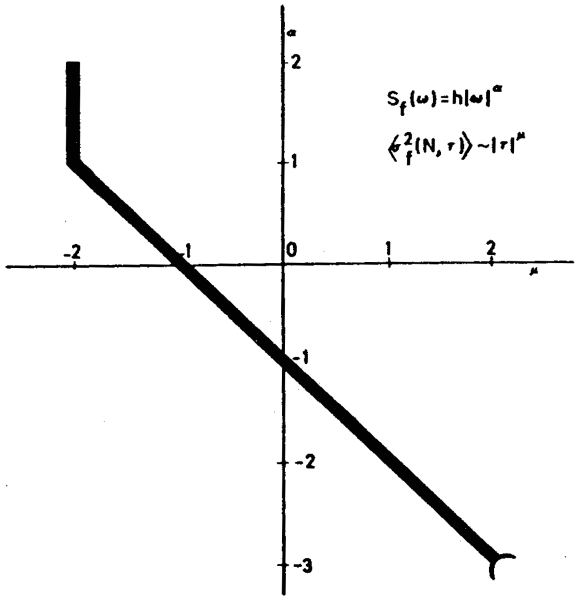

Fig. 15-1 Relationships between μ and α for different kinds of noise. $S_\varphi(\omega) = h\,|\omega|^{\alpha}$ and $\langle\sigma^2(N,\tau)\rangle = a(h,N)\,|\tau|^{\mu}$, where $\omega \gg 1/\tau$

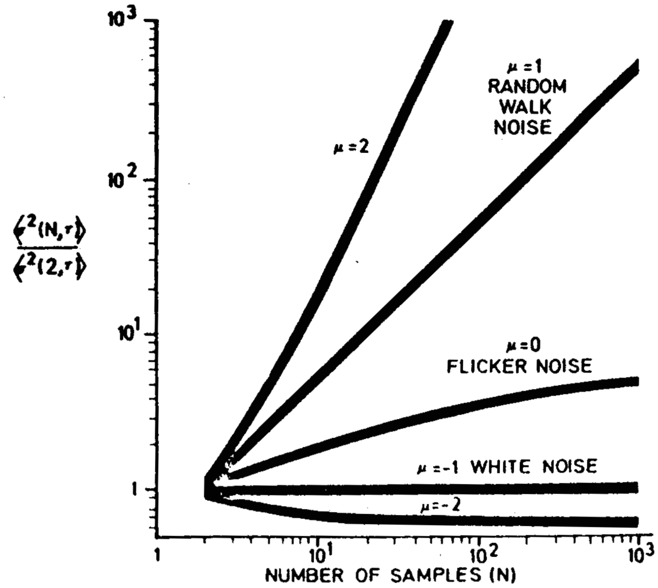

Fig. 15-2 The dependence of the ratio of the variance for N samples to 2 samples for various kinds of noise and as a function of N

uency fluctuations for the random propagation fluctuations. Also, the cross-correlation coefficient for two different frequencies along the same propagation path is calculated. Because the cross-correlation coefficient is negative, a combined statistical weighting of the observations at the two transmission frequencies gives a significant reduction of the effect of the random perturbing effects along the propagation path.

Statistical Analysis Techniques

Consider the phase fluctuations along a propagation path, assuming all the deterministic variations have been removed. These random fluctuations have a spectral density, $S_\varphi(\omega)$, where ω is the Fourier frequency. The authors have found that a good model for many long term random processes is:

$$S_\varphi(\omega) = (2\pi f)^2 h |\omega|^\alpha, \qquad \text{(Eq. 15-1)}$$

where f is the carrier frequency. If $\alpha \geq -1$, then the total power is infinite unless there is an upper frequency cutoff, ω_U. Every electronic system has a cutoff frequency either by limitation or by design; for example, the bandwidth of a VLF receiver. So if the spectral density lies in the above range, no divergence difficulties arise from the high frequency end of the spectrum. This is the case for white noise phase modulation, $S_\varphi(\omega) = (2\pi f)^2 h |\omega|^0$. If, however, $\alpha \leq -1$, the total power is infinite unless there is a low frequency cutoff, ω_L. A very common noise in the laboratory is flicker noise with $\alpha = -1$. Flicker noise has been observed on the outputs of a multitude of devices (1) but the low frequency cutoff has seldom, if ever, been observed. Data have been obtained with a flicker noise spectrum having frequency components as low as one cycle per century (2) and continuing. Obviously, any measurement parameter depending on ω_L would be essentially useless.

Any random process with $\alpha \leq -1$ is also non-stationary. The process has no time-average mean value, and the variance of the fluctuations over the whole data length tends to infinity as the data length increases without limit (3). If, for example, the phase deviations over some transmitter's propagation path had a spectral density given by:

$$S_\varphi(\omega) = (2\pi f)^2 h |\omega|^{-1}, \qquad \text{(Eq. 15-2)}$$

then the phase deviations would be unbounded and would not average around a mean value as time continued indefinitely.

Typically it is more convenient to analyze the phase fluctuations in the time domain as this is how the data are taken. A useful measure in the time domain has been shown to be the expectation value of the variance for a finite number, N, of data samples (4). In general the variance depends upon four variables: N, T, τ, and ω_U, where T is the time between the start of each sample and τ is the sample duration and does not depend upon ω_L. If the ratio T/τ is held constant along with N and ω_U, and the variance may be written:

$$\langle \sigma^2 (N, T, \tau, \omega_U) \rangle = a |\tau|^\mu, \qquad \text{(Eq. 15-3)}$$

then the spectral density is given by Eq.15.1 with the mapping of μ into α shown in figure 15-1. Also $|\tau\omega_U|$ is assumed to be much greater than unity.

The dependence of $\langle \sigma^2(N, T, \tau, \omega_U) \rangle$ on N is a useful measure of the noise type (4). The ratio of $\langle \sigma^2(N, T, \tau, \omega_U) \rangle$ to $\langle \sigma^2(2, T, \tau, \omega_U) \rangle$ is shown in figure 15-2 as a function of the number of samples, N, and for various pertinent values of μ. For $\mu \geq 0$ ($\alpha \leq -1$) this ratio tends to infinity as N approaches infinity, showing again the non-stationary behavior of these types of noise. A relationship exists between the coefficients 'a' and 'h' so that the level as well as the type of noise may be determined (4) (5).

Statistical Properties of LF and VLF Propagation

In considering the fluctuations on a transmitted signal induced by the ionosphere or some other mechanism in the propagation media, it is convenient to divide these fluctuations into their deterministic and random components. However, as will be shown later in the text, the random fluctuations may contain deterministic components that perhaps are resolvable only with careful analysis. The diurnal fluctuations on the phase of a radio signal are a very obvious deterministic process as shown in figure 15-3. Each section of the figure is the average of the daily fluctuations, taken over a month's time (6). One observes that there are also seasonal variations, which in detail exhibit an annual and semi-annual periodicity (6). Lunar tidal variations in the ionosphere have also been observed to affect the phase of VLF signals (7). Very little is known about the longer term phase stability and its correlation with other geophysical phenomena because of insufficient data.

Spectral Density of LF and VLF Propagation Fluctuations

Two propagation paths are considered as shown in figure 15-4. The perturbing effect due to the diurnal fluctuations was significantly reduced by sampling the phase for a few minutes at the same time each day. The phases of the WWVL and WWVB transmitters are synchronized with the time scale at the National Bureau of Standards in Boulder, Colorado. This time scale is derived from a cesium beam frequency standard. Mr. Phil Hand of the Hewlett-Packard Company took the phase data at Palo Alto for both WWVL and WWVB with reference to a cesium beam frequency standard at their laboratory. The data analyzed covered the period from 16 June 1966 to 28 February 1967, 258 days. Dr. Allan Mungall of the National Research Council at Ottawa, Canada, provided the frequency data analyzed for the Ottawa path for WWVB as it was taken with respect to their cesium beam frequency standard. These data covered the period from 21 October 1965 to 31 March 1967, 518 days.

WWVL is a dual frequency transmitter operating at 20 kHz and 19.9 kHz; only the 20 kHz signal was analyzed. WWVB transmits a 60 kHz frequency. The distances from Fort Collins, Colorado, to Palo Alto, California, and to Ottawa, Canada, are 960 miles (1540 km) and 1500 miles (2430 km), respectively.

Define φ as the fractional phase fluctuation of a signal. The phase fluctuations analyzed are composed of the following components:

$$\begin{aligned}\varphi\ (\text{measured}) = {}& \varphi\ (\text{propagation}) + \varphi\ (\text{cesium}) \\ & + \varphi\ (\text{transmitter}) + \varphi\ (\text{receiver}) \\ & + \varphi\ (\text{residual})\end{aligned} \qquad \text{(Eq.15-4)}$$

Assuming that there is no correlation between any of the components, the variance of the fractional phase fluctuations may be written:

$$\begin{aligned}\langle\sigma_{\varphi_m}{}^2(N,\tau)\rangle = {}& \langle\sigma_{\varphi_p}{}^2(N,\tau)\rangle + \langle\sigma_{\varphi_c}{}^2(N,\tau)\rangle + \langle\sigma_{\varphi_t}{}^2(N,\tau)\rangle \\ & + \langle\sigma_{\varphi_{rc}}{}^2(N,\tau)\rangle + \langle\sigma_{\varphi_{rs}}{}^2(N,\tau)\rangle .\end{aligned} \qquad \text{(Eq. 15-5)}$$

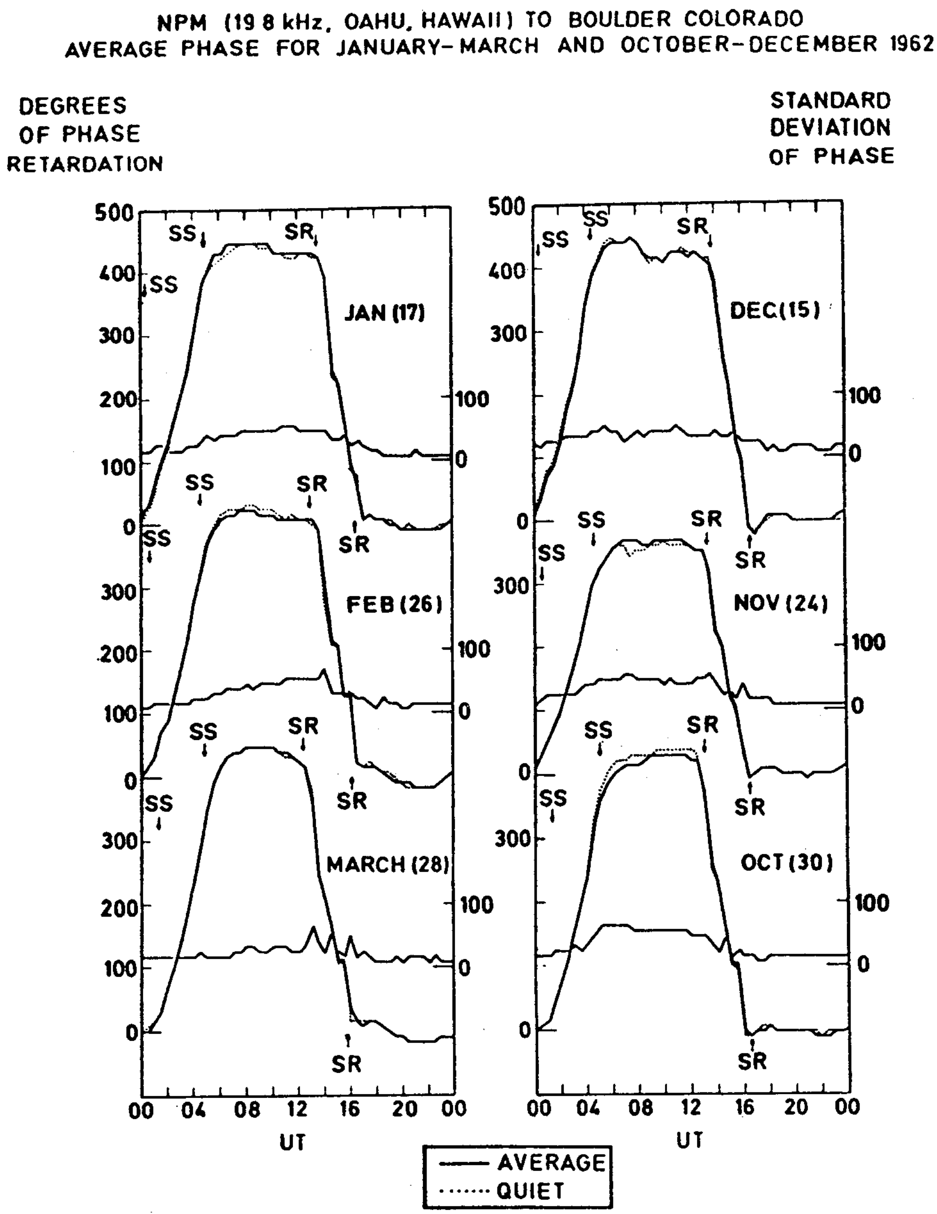

Fig. 15-3 Examples of the diurnal phase fluctuations. Adapted from A. H. Brady (see reference 4)

The variance in equation 15-5 is written with the assumption that ω_u, of the system bandwidth, is constant and the dead time between measurements is proportional to τ, i.e., T/τ is a constant. For N = 2 and τ = 1 day the square root of the variance for cesium and the transmitters is 0.1 microseconds or less. The phase noise in the receivers varies from one unit to the next, but this contribution along with the residual phase noise is assumed to be negligible with respect to the propagation fluctuations on a day to day basis. Hence, one may write the following equation:

$$\langle \sigma_m{}^2(2, \tau)\rangle \simeq \langle \sigma_p{}^2(2, \tau)\rangle, \qquad \text{(Eq.15-6)}$$

where τ is a few days.

A plot of the square root of the variance of the fractional phase fluctuations for N = 2 is shown in figure 15-5. WWVL exhibits flicker of phase noise ($\mu = 0$, $\alpha = -1$) for sample times ranging from 1 day to 64 days and at a level of about 1 microsecond over the Palo Alto path. WWVB exhibits flicker of phase noise on both the Palo Alto and Ottawa paths and at a level of about 0.5 microseconds and 0.9 microseconds, respectively. A random walk of phase noise ($\mu = 1$, $\alpha = -2$) predominates for the Ottawa path for sample times in excess of four days. This is the same type of noise one sees on the phase fluctuations of a cesium beam frequency standard, but for the cesium beam the level is typically less than one-tenth of that observed. This type of noise may be caused by making daily frequency measurements with a random uncertainty of 5×10^{-12}. If this were the case, the effect could be eliminated by making phase measurements. The Palo Alto path for WWVB also seems to exhibit random walk of phase noise for sample times longer than eight days, but the confidence limits on the data points preclude a definite determination of this effect.

If the spectral density of the phase fluctuations is flicker noise as given by equation 15-2, then the variance of the frequency fluctuations may be written (4):

$$\frac{\langle \sigma_f^2(N, \tau)\rangle}{(2\pi f)^2} = \frac{4h(N+1)}{N\tau^2}\left[2 + \ln(\tau\omega_u) - \frac{\ln N}{N^2-1}\right] \qquad \text{(Eq. 15-7)}$$

Using equation 15-7 gives the following values of h:

$h = 4.36 \times 10^{-14}\ s^2$, WWVB to Ottawa;

$h = 2.25 \times 10^{-14}\ s^2$, WWVB to Palo Alto;

and $h = 7.92 \times 10^{-14}\ s^2$, WWVL to Palo Alto.

For flicker noise (equation 15-2) the variance of the phase fluctuations may be written:

$$\frac{\langle \sigma_\varphi{}^2(N, \tau)\rangle}{(2\pi f)^2} = \frac{2hN}{N-1}\ \ln N, \text{ if } T = \tau. \qquad \text{(Eq. 15-8)}$$

Obviously this diverges with N increasing, but the divergence is mild being logarithmic. If for example the square root of the variance were 1 microsecond for N = 2, and τ = 1 day, then for N = 365 ($N\tau$ = 1 year), the expectation value of the square root of the variance would be only about 3 microseconds. Even though the effect of flicker noise precludes maintaining perfect time synchronization by tracking the phase of an LF or VLF signal, the rate of divergence may be negligible for most applications. Using portable clocks does not entirely solve the problem of synchronizing a remote clock to a master clock because it has been shown that the spectral

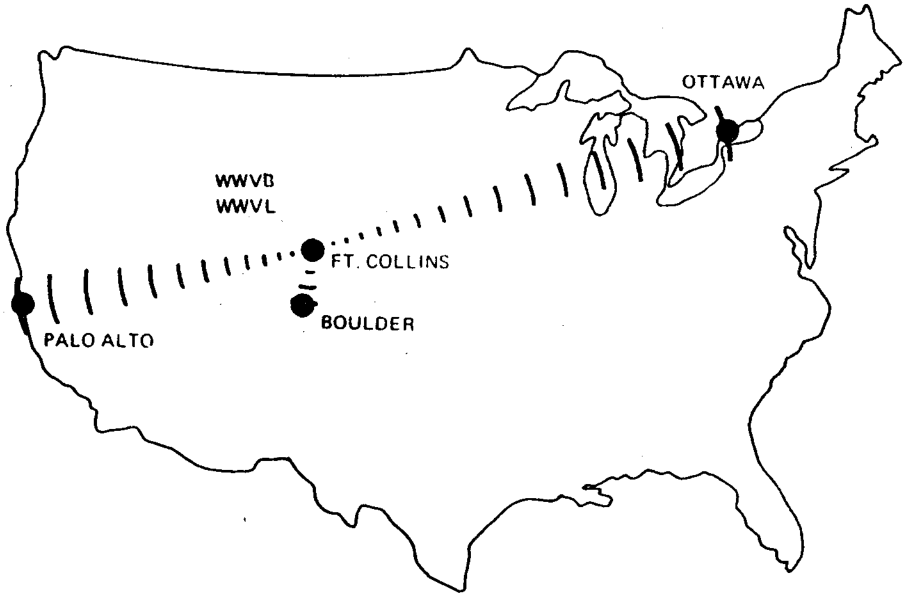

Fig. 15-4 A map of the propagation paths studied

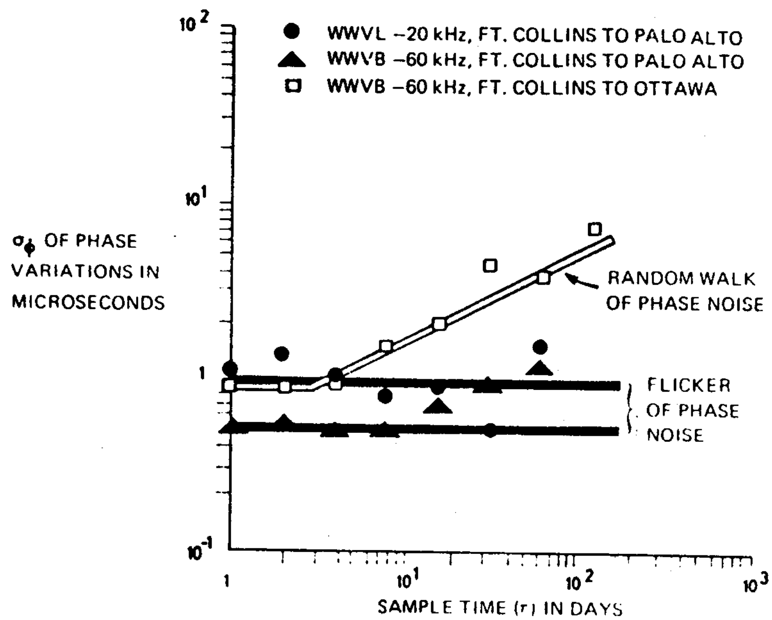

Fig. 15-5 The square root of the variance of the phase fluctuations for WWVL and WWVB over the Palo Alto and Ottawa paths

density of phase fluctuations for most frequency standards involved (see Appendix) is proportional to $|\omega|^{-3}$ (flicker of frequency noise). Though this noise is more divergent the level is sufficiently low to provide better synchronization with portable clocks for times less than a few months.

Cross-correlation of LF and VLF Propagation Fluctuations

Since both WWVL and WWVB were received over the Palo Alto path it was possible to use the following equation:

$$\varphi_m(60\text{ kHz}) - \varphi_m(20\text{ kHz}) \simeq \varphi_p(60\text{ kHz}) - \varphi_p(20\text{ kHz}), \qquad \text{(Eq. 15-9)}$$

neglecting transmitter, receiver, and residual noise. Define the phases of the two transmitter frequencies as x and y for the 20 kHz and 60 kHz respectively. Taking the variance of the measured phase deviations gives:

$$<\sigma^2{}_{(x-y)_m}(N,\tau)> \simeq <\sigma^2{}_{x_p}(N,\tau)> + <\sigma^2{}_{y_p}(N,\tau)> - 2<\sigma^2{}_{(xy)_p}(N,\tau)>. \qquad \text{(Eq. 15-10)}$$

Define the cross-correlation coefficient as:

$$C_{xy}(N,\tau) \equiv \frac{<\sigma^2{}_{(xy)_p}(N,\tau)>}{\left[<\sigma^2{}_{x_p}(N,\tau)>\cdot<\sigma^2{}_{y_p}(N,\tau)>\right]^{\frac{1}{2}}} \qquad \text{(Eq. 15-11)}$$

Combining equations 15-10 and 11 allows a determination of $C_{xy}(N,\tau)$. For $N = 2$ and $\tau = 1$ day, C_{xy} was equal to -0.58 ± 0.2. The implication from this very significant cross-correlation coefficient is that an apparent perturbing mechanism in the propagation medium affects both transmission frequencies, but the phase deviations are in opposite directions. One may assume therefore that an appropriate linear combination of the two received signals will significantly reduce the effect of this perturbing mechanism.

Define θ as true phase, and ϵ and δ as the fluctuations so that one may write:

$$x = \theta + \epsilon$$

and

$$y = \theta + \delta \qquad \text{(Eq. 15-12)}$$

Let W be the linear combination of x and y such that:

$$W = \lambda x + (1 - \lambda)y. \qquad \text{(Eq. 15-13)}$$

Taking the variance of W and minimizing it with respect to λ and the use of equations 15-10 and 12 yields an optimum value of λ given by:

$$\lambda = \frac{<\sigma^2{}_{x-y}(N,\tau)>+<\sigma^2{}_y(N,\tau)> - <\sigma^2{}_x(N,\tau)>}{2<\sigma^2{}_{x-y}(N,\tau)>} \qquad \text{(Eq. 15-14)}$$

The following is a list of lambdas for N = 2 and various values of τ:

τ (days)	λ
1	0.28
2	0.31
4	0.25
8	0.23
16	0.36

Figure 15-6 shows a plot of the square root of the variance of the fractional frequency fluctuations as derived from the phase for the 20 kHz signal, the 60 kHz signal and for a weighted combination ($\lambda = 0.3$). Figure 15-7 is a plot of the frequency fluctuations. The weighted combination is 1.6 times better than WWVB and 3.4 times better than WWVL. The square root of the variance for the weighted combination tends toward zero slope at about 4×10^{-13} - the flicker noise frequency modulation level of the cesium beam frequency standards involved in the experiment.

The phase fluctuations are noticeably smaller prior to November 1966. The data were divided into two groups, one prior to 1 November 1966 and one after. The first group gave C_{xy} (2, 1 day) = +0.20 ± 0.2 and the second, -0.65 ± 0.2. The weighting factors, λ, were 0.27 and 0.28, respectively. During the first part of the data the weighted combination was 10 percent better than the 60 kHz and 44 percent better during the last part. For the first part of the data the weighted combination gave a precision of fractional frequency determination of 1×10^{-12} for a five day sample time.

Conclusion

For two LF transmission paths and one VLF transmission path on the North American continent the spectral density of the phase fluctuations was determined to be proportional to the inverse spectral frequency (flicker noise, $S_t = h/|\omega|$). The spectral range covered was from one cycle per day to one cycle per two months. The value of h was about 4 times larger for the 20 kHz signal than the 60 kHz signal over the Fort Collins to Palo Alto path. Because the propagation fluctuations have a flicker noise spectrum, the square root of the variance of the time fluctuations may be written:

$$\langle \sigma_t^2(N, \tau)\rangle^{\frac{1}{2}} = b(h, N)\,|\tau|^{0}. \qquad \text{(Eq.15-15)}$$

The value of b(h, 2) for WWVB over a 1540 km path and a 2430 km path was 0.5 microsecond and 0.9 microsecond respectively and the value for WWVL over a 1540 km path was 1.0 microsecond.

A non-zero cross-correlation coefficient was determined between WWVL and WWVB. Using a weighted combination of the two transmissions improved the apparent level of flicker of phase noise by a factor of 2.7 over WWVB and 11.4 over WWVL. The coefficient changed from positive to negative from summer to winter implying the introduction of another perturbation mechanism such as a change in the charge density in the ground-ionosphere propagation waveguide, or perhaps an actual change in the waveguide propagation mode of at least one of the transmission frequencies.

Though the data analyzed are only one sample the confidence in the non-zero cross-correlation coefficient is based on the following: first, the value of the coefficient was the same over the spectral frequencies considered: second, personnel monitoring these two transmissions have observed visually a negative cross-correlation for other years; third, Dr. Sigfrido Leschiutta of the National Electrotechnical Institute at Torino, Italy has reported visual observation of a negative cross-correlation coefficient between GBZ (19.6 kHz) and MSF (60 kHz); and fourth, an increase in the phase fluctuations has occurred for every winter that WWVL and WWVB have been operational (11).

The introduction of flicker noise on the phase of a transmission along its propagation path causes a mild time divergence between the transmitter and the receiver. This divergence is of the order of 3 microseconds after a year for the Palo Alto path on the 20 kHz signal.

Appendix

Flicker noise frequency modulation has been measured by the authors on the frequency fluctuations of quartz crystal oscillators, rubidium gas cell frequency standards, cesium beam frequency standards, and on both hydrogen and ammonia maser frequency standards. The respective levels of the square root of the variance of the fractional frequency fluctuations for $N = 2$ are 5×10^{-13}, 7×10^{-13}, 3×10^{-13}, 1×10^{-14}, and 1×10^{-12} for some of the best standards measured. The range of the spectral frequency extends from 30 Hz to frequencies as low as one cycle per year for quartz crystal oscillators with intermediate ranges for the other frequency standards (4), (8), (9), (10). The rms time divergence of an oscillator clock system having a flicker noise frequency modulation spectral density is proportional to τ. A typical cesium beam, for example, would have an expected time divergence of the order of about 5 microseconds after one year (3), (4).

Acknowledgement

Most of the data for this work were provided by the laboratories of the Hewlett-Packard Company and of the Conseil National des Recherches. The authors extend sincere appreciation to the personnel of these laboratories for their very generous cooperation and in particular to Mr. Phil Hand and Dr. Allan Mungall. Appreciation is also expressed to Dr. Donald Halford, Mr. Douglass Crombie, and Mr. Lowell Fey for their cooperation and criticisms of the text. Mr. Howard Machlan was very helpful in the data analysis.

References

1. **McWhorter, A.L.**, '1/f noise and related surface effects in germanium', MIT Lincoln Laboratory Technical Report No.80, Lexington, Massachusetts: MIT, 1955.

2. **Mandelbrot, B.**, private communication.

3. **Barnes, J.A.**, Proc. IEEE **54**, 207, 1966.

4. **Allan, D.W.**, Proc. IEEE **54**, 221, 1966.

5. **Lighthill, M.J.**, Fourier Analysis and Generalised Functions, Cambridge, The University Press, 1964.

6. **Brady, A.H.**, Radio Sci. J. Res. NBS **68D**, 283, 1964.

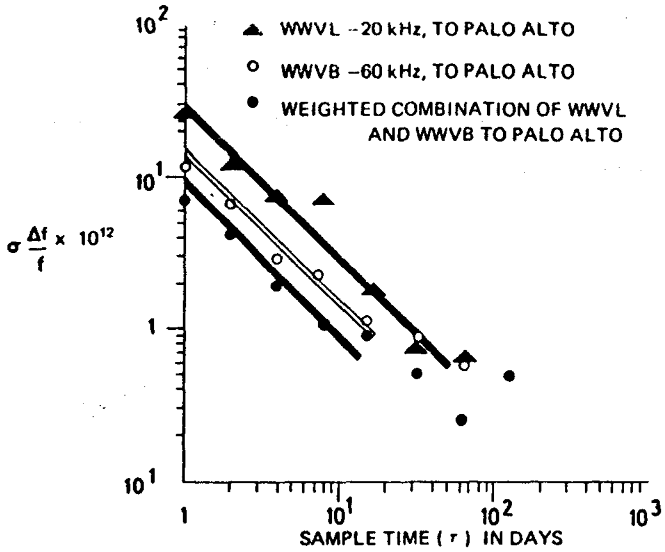

Fig. 15-6 The square root of the variance of the frequency fluctuations for WWVB, WWVL, and the weighted combination over the Palo Alto path

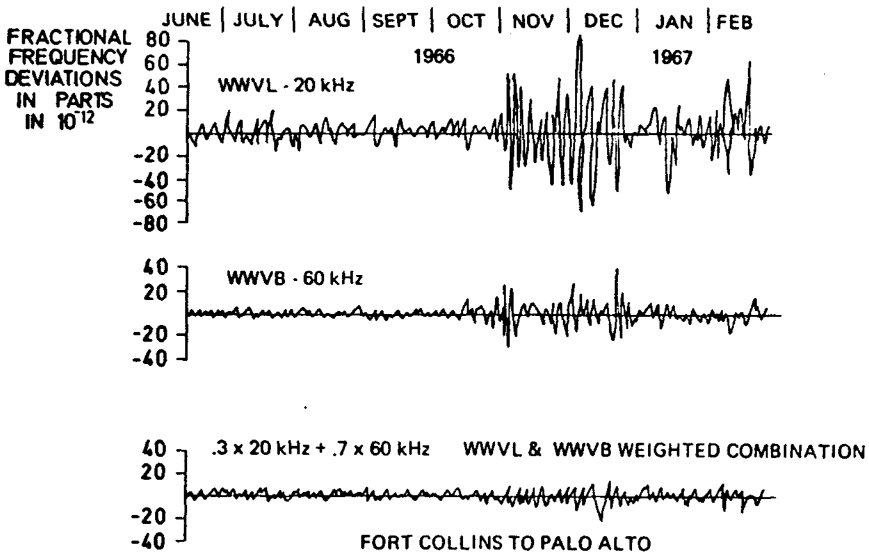

Fig. 15-7 A plot of the fractional frequency fluctuations for WWVL, WWVB, and the weighted combination over the Palo Alto path

7. **Brady, A.H., Crombie, D.D.,** J. Geophys. Res. 68, 5437, 1963.

8. **Vessott, R., Peters, H., Vanier, J., Beehler, R., Halford, D., Harrach, R., Allan, D., Glaze, D., Snider, C., Barnes, J., Cutler, L., Bodily, L.,** IEEE Trans. Instr. Measr. IM-15, 165, 1966.

9. **Atkinson, W.R., Fey, L., Newman, J.,** Proc. IEEE 51, 379, 1963.

10. Unpublished work at the National Bureau of Standards, Boulder, Colorado.

11. **Leschiutta, S.,** Nat. Electrotechnical Inst. Torino, Italy; **Hand, P.,** Hewlett-Packard Co., Palo alto, California; **Fey, L.,** Nat. Bureau of Standards, Boulder, Colorado; private communication.