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SOME METHODS OF MAINTAINING AND/OR GENERATING TIME AND FREQUENCY AT ARBITRARY POINTS ON SURFACE* OF THE EARTH

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THE USEFULNESS of time and frequency standards within such fields as navigation and communication is assumed to be understood within the context of this paper. This paper will discuss some of the ways and techniques of maintaining, setting up, generating, and comparing time and frequency (T/F) standards at remote locations on the surface of the earth. The paper is not intended to be exhaustive, but rather to give some guidelines and nominal values of performance, both for clocks and for comparison systems, and it will compliment Roger Beehler's paper (Beehler 1981), which outlines in more detail the practical methods of T/F dissemination.

CONCEPTS

Three useful concepts related to T/F and within the context of this paper are: first, the concept of time interval which we may denote as τ or Δt. The reciprocal of this quantity is frequency; in other words, the second is identically equal to the reciprocal of the Hz. Currently, the standards employed for generating "the second" are frequency standards utilizing a resonant frequency in the caesium atom: "the second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom."

The second concept is that of date often called epoch, clock time, or clock reading. Typically the date or the time, t, of a clock is recorded coincident with some event. Every clock reads a different time, if one observes in fine enough detail; it is only a matter of degree. Operationally, time becomes a definition; we may pick a particular clock or a set of clocks to define time for a system whether it be for the laboratory setting or for the International Atomic Time scale (TAI).

The third concept is that of synchronization, or closely associated with it, simultaneity. In some reference frame, we may talk of two events occurring simultaneously without having a frequency standard or a clock. Similarly ,we may keep the ticks of two clocks occurring simultaneously, which of course, causes them to be synchronized. In contrast, we may synchronize two clocks at one point in time and that may be the only point in time over a long period during which they are synchronized.

One may describe the reasons for two clocks not remaining synchronized in two categories. First, there are systematic difficulties within the clocks and/or within the comparison systems, and secondly, there are random fluctuations of either the clocks and/or the comparison systems.

The typical arrangement is to have a frequency standard which determines time interval, *i.e.*, "the second", and then to have a frequency divider or counter attached to that clock standard to generate clock time. Alternatively, one may have an ensemble of clocks that is calibrated by the frequency standard, and the clocks in the ensemble are combined in some manner. One may choose to compare the time so generated with the time generated at another location either within a laboratory or at a remote site by some comparison system, *e.g.*, in the case of TAI, it is typically Loran-*C*. It is often the case, at least in the short term, that the method of comparison limits the ability to ascertain the performance of clocks, to maintain synchronization, or to compare the frequency standards at the two locations.

To illustrate the above, Fig. 1 shows a plot of the best time scales in the world. The times and rates have been changed in the software so that the time scales were constrained to be synchronized at the origin and syntonized (same frequency) when averaged over the first year. After that, these time scales were allowed to free run. Clearly, regardless of having the best clocks available, one still has significant time dispersion. In the short term (Times less than about 2 months), the fluctuations are most probably due to the comparison method, *i.e.*, Loran-*C*.

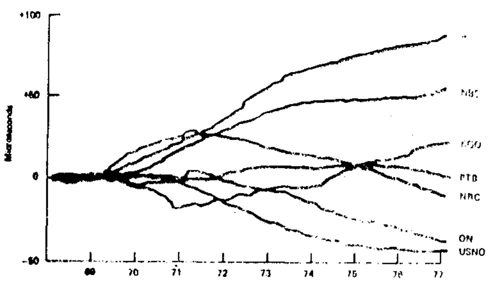

Fig. 1. *Data with corrections removed : TAI—TA (i).*

CLOCKS AND THEIR CHARACTERISTICS

To describe clocks, let us define the output frequency of an oscillator as $\nu(t)$ and the normalized fractional frequency as $y(t) = [\nu(t) - \nu_0]/\nu_0$ where ν_0 is the nominal frequency for the oscillator. Further let us define $x(t)$ to be the integral from 0 to t of $y(t')$, t' being the integration variable.

Figures 2 and 3 show the performance of a commercial caesium and a commercial rubidium; respectively. These

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are plots of $y(t)$ versus t. Figure 4 is $x(t)$; the time error for the commercial rubidium—the integral of the data shown in Fig. 3. One can clearly see the presence of both systematics and random fluctuations in these data. We will discuss later some ways of ascertaining the systematics in clock data.

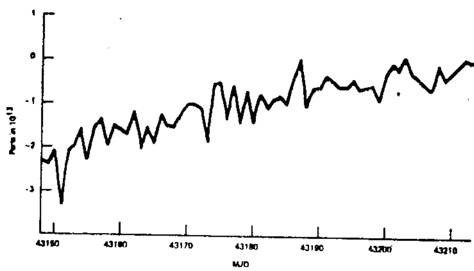

Fig. 2 *Frequency vs. time of commercial caesium.*

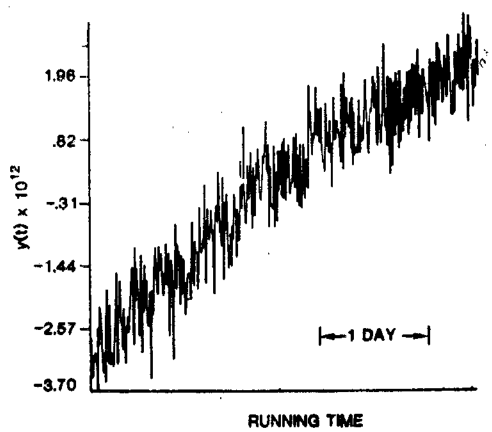

Fig. 3. *Rb gas cell frequency standard.*

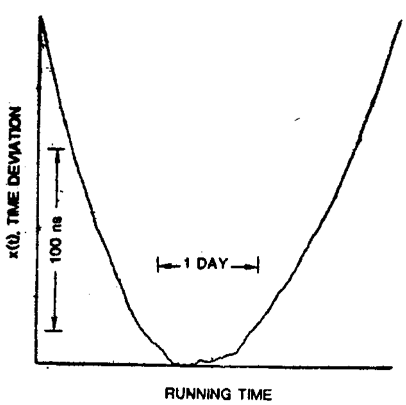

Fig 4.. *Rb gas cell frequency standard.*

For characterizing the random fluctuations in the time-domain, it has become customary to use what has been named the two-sample Allan variance—denoted $\sigma_y^2(\tau)$ (Barnes *et al.* 1971).

$$\sigma_y^2(\tau) = \tfrac{1}{2} \langle [(\bar{y}_{k+1} - \bar{y}_k)]^2 \rangle$$

where the brackets "$\langle\ \rangle$" denote infinite time average and τ is taken as the measurement or sample time for adjacent frequency averages, $\bar{y}_k$, from the data. The operational realization of eqn. 1 for digitized data is given in eqn. 2:

$$\sigma_y(\tau) = \left[\{2(M-1)\}^{-1} \sum_{K=1}^{M-1} (\bar{y}_{k+1} - \bar{y}_k)^2 \right]^{\frac{1}{2}}$$

where, M is the number of adjacent samples of fractional frequency; each sample is over a time interval τ. As has been shown in several other papers (Barnes 1971, Allan 1966, Allan *et al.* 1974), the dependence of $\sigma_y(\tau)$ on τ allows one, in most cases, to ascertain a model for the type and level of random fluctuations present in the data. Figures 5 and 6 are some of the best $\sigma_y(\tau)$ versus τ performances which we have seen for commercial caesiums and for a laboratory passive hydrogen maser respectively. Figure 7 is a plot of

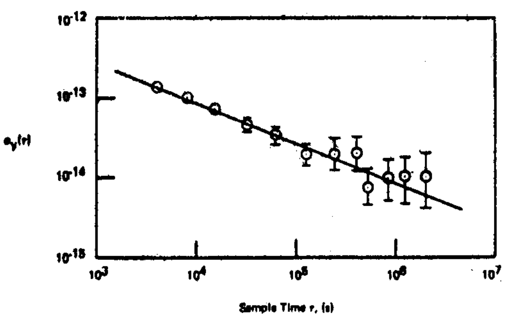

Fig. 5. *Frequency stability of two high performance commercial caesium beam standards.*

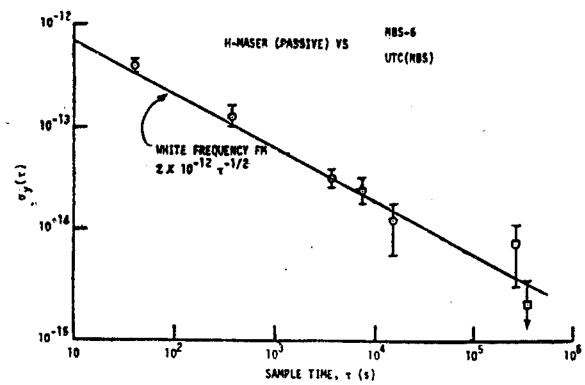

Fig. 6. *Frequency stability.*

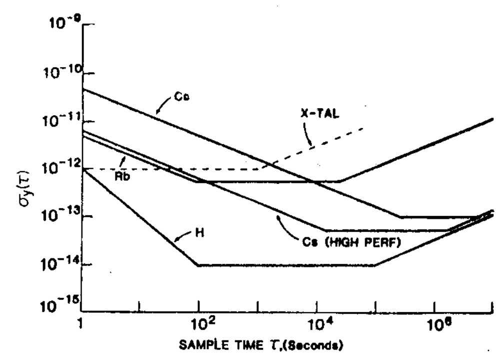

Fig. 7. *Normal performanc·.*

the performance of common kinds of frequency standards. The stability values for quartz crystal oscillator standards (X-TAL) are approximateld by the dashed line as actual performance will vary markedly above and below these values, depending upon the manufacturer and on each unit.

It is very important to realize that the performance of a clock depends not only upon its internal resonator and associated electronics, but also on the environment. All oscillators are affected by their environment to some extent, whether it be caused by temperature, humidity, pressure, shock, vibration, or *g* forces. Reference 5 gives some of the nominal co-efficients for the sensitivity of some oscillators of interest. The performance shown in figures 5 and 6 were taken with near ideal environments for these particular frequency standards.

GENERATING A TIME SCALE

The accuracy of a time and frequency standard can be significantly improved if one knows and utilizes the characteristics of the standard. A reasonable model for most precision clocks and oscillators, which are used for time and frequency standards, is given in eqn. (3):

$$x(t) = x(o) + y(o)\cdot t + \tfrac{1}{2} D \cdot t^2 + \varepsilon(t),$$

where the first three terms are representative of the systematics (synchronization error, syntonization error, and frequency drift, respectively) and the last term is representative of the random fluctuations. It is not so important that the co-efficients in the systematic terms be zero, as it is that they be known with small uncertainties in their estimates. The third term may or may not be important in atomic frequency standards and clocks. Caution should be used in assuming that a measured frequency drift will persist and judgement should be made based on the individual clock's performance, rather than categorically; however, rubidium standards tend to have more frequency drift by about an order of magnitude than do caesium frequency standards. The kinds of noise processes involved in the fourth term, $\varepsilon(t)$, can easily be characterized using a $\sigma_y(\tau)$ vs. τ diagram without subtracting the systematics unless there is significant frequency drift in the data. If there is significant frequency drift, then this drift should be subtracted from the data before the $\sigma_y(\tau)$ vs. τ analysis. This can be easily done by removing a linear-least-squares fit to the frequency or a quadratic fit to the time deviations of the data. For current commercial caesium and rubidium standards, the former is usually more nearly optimum. If one has time readings, rather than frequency readings, then eqn. (4) can be used instead of eqn. (2) to estimate $\sigma_y(\tau)$:

$$\sigma_y(\tau) = [\{2(M-1)\tau^2\}^{-1} \sum_{t=1}^{M-1} (x_{k+2} - 2x_{k+1} + x_k)^2]^{\frac{1}{2}}$$

where, there are $M+1$ time measurements spaced τ apart. Once the random fluctuations are characterized, then appropriate filters can be designed allowing one to do optimal time prediction from the data. The final uncertainties in the time prediction of a clock will be a composite of the uncertainties in ascertaining its systematic parameters plus the uncertainties in the prediction technique employed. There are several different kinds of prediction techniques being used in the *T/F* field. At NBS, we typically have employed, and plan to employ, Kalman-type filters after the data have been properly characterized.

At the Bureau International de l'Heure (BIH), the following approach is taken in generating TAI (ref. 6). The correction to a given clock reading at a time, *t*, is as calculated from the first two terms in eqn. (3). Once each clock produces an estimate of its time versus TAI, then all of the clocks are combined in a weighted estimate to produce an official TAI value. The weights are chosen in proportion to the inverse variance for that particular clock. The algorithm so employed by the BIH is called ALGOS; the time so produced has been a very accurate time scale over several years.

Similar approaches have been taken by other laboratories and observatories with similar and different weighting factors for the members of the clock ensemble. The USNO has chosen to use equal weights for their clocks and not to use those clocks which have poor performance. NBS, has used a similar approach to the BIH—but also applying a simplified Kalman filter with weighting factors proportional to an inverse variance. An example of the software package is listed by Allan *et al.* (1974) Chapter 9, with the availability of faster and better computation power, NBS has been working on more sophisticated and elaborate Kalman-filter approaches for its future time scale. In addition to an ensemble of clocks, the NBS time scale utilizes the associated primary frequency standards. Current state-of-the-art in time scales is such that dispersions of the order of about 1 ns in one day are possible and of a few μs in 1 year.

ROLE OF FREQUENCY STANDARDS AND TAI

The inaccuracies and long-term instabilities of commercial frequency standards are typically an order magnitude worse than can be achieved with primary frequency standards of a laboratory type. Currently, the absolute accuracies of the primary frequency standards, NBS-6, Cs V at NRC in Ottawa, Canada, and Cs 1 at the PTB in Braunsweig, West Germany are 8 parts in 10^{14}, 5 parts in 10^{14}, and 6 parts in 10^{15}, respectively. With these levels of accuracy, it is clearly possible to improve the long-term stability of a clock ensemble by periodic calibrations. It is essential to have such primary frequency standards for assuring that sets of commercial clocks do not depart too far in frequency from the definition, and it is also important that we make every effort to realize the definition as best as possible for congruity in international *T/F* metrology. These three primary standards are the standards used in determining the rate of TAI. Figure 8 is a plot of the frequency of some of these primary frequency standards, with each point being a two-month average. Also shown are three new

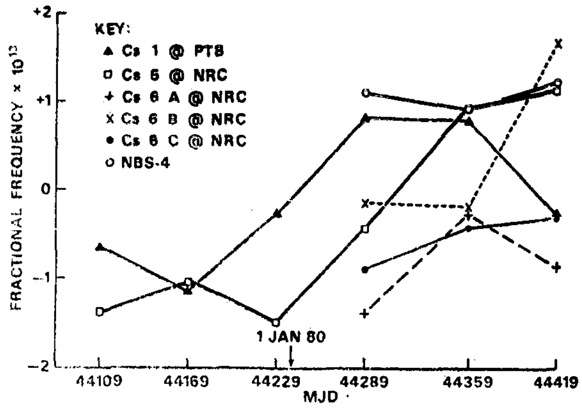

Fig. 8. *Bi-monthly rates of TAI minus primary clocks.*

standards being developed by NRC, Caesium VI *A*, *B* and *C*. One gains a feeling for the internal consistency of the standards involved from this figure, as well as the possibility of trends in TAI, or the possibility of an annual term. Some of the fluctuations in the data could also be the comparison link, Loran-*C*. Figure 9 shows an interesting stability diagram, which not only includes the stability of anticipated state-of-the-art type frequency standards, but also the impact of having a good comparison link whose time-transfer accuracy is of the order of a nanosecond. The stability of Loran-*C* is also shown for comparison purposes.

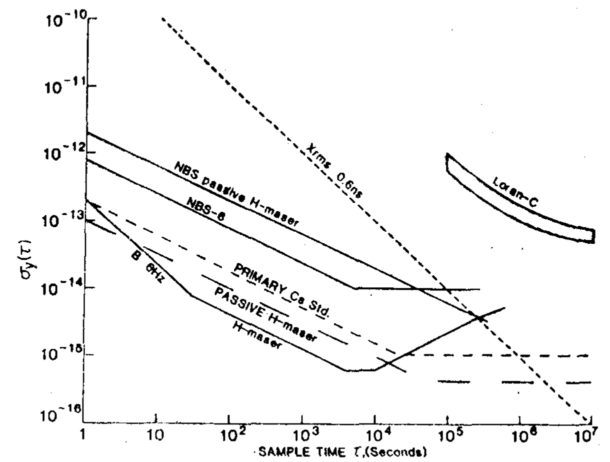

Fig. 9. *State-of-the-art and anticipated frequency stabilities; long-dashed lines are 1984 projections.*

METHODS OF REMOTE SYNCHRONIZATION AND SYNTONIZATION

Clearly, for now and into the future, one should research alternatives to Loran-*C* in order to compare clocks at remote sites on the surface of the earth. Most of the current and future techniques being investigated involve satellites. The two-way satellite technique has some very interesting attractions, namely that the propagation path can be calibrated assuming reciprocity. An interesting experiment was done between Boulder, Colorado through NRC Canada, to Pleumeur—Bodou, France, via the CTS or Hermes communications satellites and via a Symphonie satellite (Costain *et al.* 1979). The Hermes and Symphonie data stream were tied together at the NRC and the stability for the two-hop two-way signal was only 6 ns between Boulder, and Pleumeur—Bodou, France. In one-hop two-way situations, time stabilities of less than 1 ns have been achieved taken over several minutes of data. The time-transfer accuracy is still being researched and investigated, but it appears that 50 ns or better is achievable.

Currently, there are 6 Navstar GPS satellites orbiting the earth with 12 hours periods and with atomic clocks and quartz oscillators on board, one of which is transmitting signals from a caesium standard; the others employ rubidium gas-cell frequency standards, except for one which has a quartz crystal standard. There are four basic approaches of using signals from the GPS satellites. We have been assured that the C/A signal will be available during peacetime. The first three approaches utilize the C/A signal. GPS time will eventually be synchronized to UTC (USNO) to within 100 ns and if one knows his own co-ordinates accurately enough, this approach should have an accuracy internationally of about 100 ns. The second approach, shown in the upper right, is what we might call the clock-flyover mode, one uses the same clock at both sites. Any two sites on the surface of the earth will be able to observe the same clock within a 12-hour interval. It is projected that accuracies of the order of 50 ns can be achieved in this mode by comparing the received GPS times of the two ground-station clocks. The third approach, shown in the lower left, is called the common-view approach; *i.e.*, both ground station clocks observe the same satellite simultaneously (Allan & Weissly 1980). In this case, the clock time error of the satellite clock has no effect on the time-transfer error, and the ephemeris error is reduced by approximately an order of magnitude, depending upon geometries and propagation circumstances. Accuracies of about 10 ns are believed achievable using this approach. NBS is pursuing this approach and is near completion of a prototype receiver. For short baseline, this approach should yield accuracies approaching 1 ns as the common-mode cancellation of errors improves for both the satellite ephemeris error and the ionosphere propagation errors. The last approach, shown in the lower right-hand corner, is being developed by Jet Propulsion Laboratories for geodesy purposes and is the same idea as that employed in very long baseline interferometry (VLBI) (MacDoran 1979). In this case, however, the baselines are very short and one simultaneously looks at 4 satellites from two ground stations using wideband receivers tuned to the full GPS signal. The potential accuracies are subnanosecond, but the equipment employed is quite a bit more complex, as a computer cross-correlator and radiometers similar to VLBI are needed. We project that out of the common-view aproach, which NBS is pursuing, industry may make available a receiver which can be used in any of the first three modes. The price range should be compatible to that of a commercial caesium standard, and with an overall performance which exceeds that of a commercial standard in long-term because synchronization can be maintained. The prototype tested at NBS using a simulated satellite signal showed a receiver stability of the order of 1 ns. We have yet to determine experimentally the instabilities introduced by the receiver front-end, by the propagation fluctuations, and by the effect of the satellite ephemeris. Figure 10 shows a calculated time-transfer error due to the satellite ephemeris between the BIH and NBS, with a baseline of 74.4^0; and one has time transfer errors of about 4 ns from a 25 ns RMS ephemeris uncertainty when the satellite is nominally midway between those two sites.

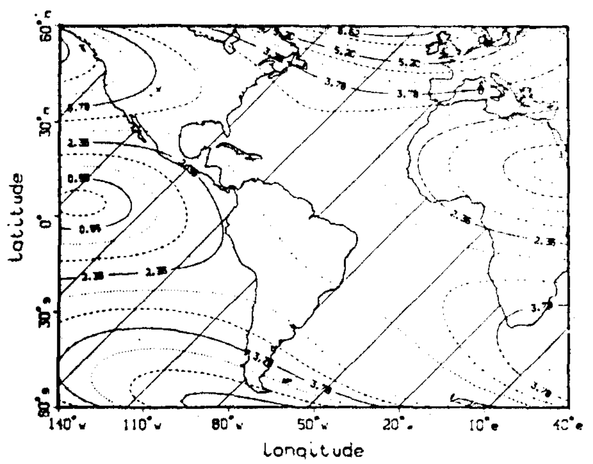

Fig. 10. *NBS—BIH time transfer error from rms ephemeris error =25.46 ns, units=nonoseconds, direction=south.*

Another experiment using the Space Shuttle is anticipated for the 1985 time frame (ref. 10). The Shuttle will have a hydrogen maser clock on board. The *T/F* comparisons to the ground will feature both a three-way microwave Doppler cancellation ground link and a two-way laser link with the laser on the ground and a retro-reflector on the Shuttle. These systems have all been separately tested in other experiments. The coverage should be worldwide for most of the populated regions of the earth. A time-transfer accuracy of 1 ns or better is anticipated, as well as worldwide frequency comparisons of 10^{-14} or better. Table 1 is a comparison of the time and frequency transfer capabilities of several satellite techniques. Also for comparison purposes, Loran-*C* is shown.

The Laser Synchronization by Stationary Orbit (LASSO) experiment is due to take place in the Fall, 1981. It has the accuracy potential of 1 ns for sites in common-view of the geostationary satellite involved. This experiment is being supported by the European Space Agency. It features laser stations on the ground with a retro-reflector on-board the Serio-2 satellite at 25°W longitude. Each laser-to-satellite path can be calibrated by the reflected signal. An event timer with 0.1 ns accuracy on-board the satellite will measure the arrival times of the laser pulses. One ns time-transfer accuracy is possible.

In all of these high-accuracy time-transfer techniques, account must be taken of the effect of the spinning earth as a non-inertial frame—the Sagnac correction (Ashby & Allan 1979). This amounts to 207.4 ns at the equator at sea level for a slow clock circumnavigating the earth. The correction is positive eastward; *i.e.*, one has to add time to the clock readings, and negative westward. The size of this effect for satellite experiments can be much greater than for a terrestrial clock and has to be taken into account to be consistent with the TAI co-ordinate time scale. When transferring time from one point to the other by means of

Table 1. *INTERNATIONAL TIME AND FREQUENCY COMPARISON* ($\leqslant 1\ \mu s$)

	Method	*Inaccuracy*	*Stability*	†*Cost-Effectiveness* (*M$.ns*)	*24-Hour Frequency Accuracy*	*Coverage*	*When Available*
(1)	GPS (Common-view)	10 ns	1 ns	0.25	$\lesssim 10^{-13}$	Global	1981
(2)	Shuttle (STIFT)	1 ns	0.001 ns	0.25	$\lesssim 10^{-14}$	To ±57° Latitude	1985
(3)	TDRSS	10 ns	1 ns	1.0	$< 10^{-13}$	All but India Longitudes	1983
(4)	LASSO	1 ns	0.1 ns	1.0	$\sim 10^{-14}$	SERIO-2 at 25° W. Longitude	1981
(5)	GPS	50 ns*	10 ns	2.0	$\sim 3 \times 10^{-13}$	Global	1981
(6)	2-Way (Communication Satellite)	40 ns	$\lesssim$ 1 ns	5.0**	$\sim 10^{-13}$	All but near the poles	1981
(7)	Portable Clock	100 ns	N/A	6.0	$\lesssim 10^{-12}$	Global (Best accuracy within reasonable driving vicinity of Air Ports)	1981
(8)	Loran-C	500 ns	$\lesssim$ 40 ns	3.0	$\lesssim 10^{-12}$	Excludes Most of Asia and Southern Hemisphere	1981

*This inaccuracy may increase if the GPS C/A code is deteriorated for strategic reasons.

**Cost includes estimate of annual rental.

†Estimate of user receiver cost produced with the time transfer accuracy in Megadollar. nanoseconds.

an electromagnetic signal, the co-ordinate time correction that one needs to apply is $\Delta\tau = 2\omega^2 A_E/c^2$ where ω is the angular velocity of the earth, c is the speed of light, and A_E is the area circumscribed by the equatorial projection of the triangle whose vertices are at the centre of the earth at the point of transmission of the signal and at the point of reception of the signal. The area A_E, is positive if the signal path has an eastward component; *i.e.*, this correction has to be added to that calculated assuming the earth was an inertial frame ($2\omega/c^2 = 1.6227.10^{-6}$ ns/km^2). The correction can amount to several hundred nanoseconds for practical values of A_E.

ACCURACY, COST, COMPLEXITY, AND AVAILABILITY OF VARIOUS TIMING SYSTEMS

It is very important that someone coming into the field of time and frequency, who does not have a large experimental background or who lacks experience in how to use some of the *T/F* systems, be able to purchase an operating system that would allow him to get the time and/or frequency needed in his country or at that site without great difficulty. All of the methods described in the previous Section are experimental and should be pursued only by those who have the scientists who wish to be involved in such experiments. The operational aspects, as indicated, are some years in the future. Also, these methods were meant to complement some of the several other methods described by Roger Beehler in his paper. There are only two operational time and frequency transfer techniques available from satellites (Beehler *et al.* 1979). These are GOES and TRANSIT with accuracies of 50μs and 10μs respectively, and with receiver costs of $2,000 to $4,000, and $12,000, respectively. The two GOES geostationary satellites are located at 75°W and 135° W longitude, and they are described in much more detail in Roger Beehler's paper. TRANSIT has worldwide coverage. Both systems should be operational upon receipt of the receivers conjoint with the ground clocks and measurement equipment. Others of these experimental systems mentioned in the previous section are stated to become operational sometime during the 1980s. GPS is in some sense operational now; however, the receiver cost is in excess of $50,000 and in addition requires a fairly sophisticated computer system and time interval counter to support it. That may improve significantly in the next few years.

CONCLUSION

We are now in a transition period for methods of comparison of *T/F* at remote sites. The past has featured terrestrial electromagnetic signals; *e.g.*, WWV and Loran-*C*. The current state-of-the-art methods feature satellite techniques, some of which will become operationally available during this decade with synchronization and syntonization accuracies approaching 1 ns and parts in 10^{-14}, respectively.

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