RADIO SCIENCE, Vol. 1 (New Series), No. 12, December 1966

New Measurements of Phase Velocity at VLF

G. Kamas, A. H. Morgan, and J. L. Jespersen

Radio Standards Laboratory, National Bureau of Standards, Boulder Laboratories, Boulder, Colo. 80302, U.S.A.

(Received June 6, 1966; revised July 13, 1966)

The purpose of this note is to describe some new phase velocity measurements using signals from the VLF stations NPG (18.6 kc/s) near Seattle, Wash., and WWVL (20.0 kc/s) near Fort Collins, Colo. Previous experimental results have been described by Wait [1961].

The new measurements consist of determining the accumulated phase over a known distance along the surface of the earth. These measurements are related to the phase velocity by the following expressions. The definition of the phase velocity at a point R is given by

$$V_p(R) = \frac{2\pi f}{d\phi/dR} \tag{1}$$

where f is the radio frequency and ϕ is the phase. Therefore, the total phase accumulated over a distance D is

$$\phi_T = 2\pi f \int_0^D \frac{dR}{V_p(R)}.$$

If V_p is a constant, then

$$V_p = \frac{2\pi f D}{\phi_T}; \tag{2}$$

otherwise, (2) gives some average effective velocity.

The measurements were accomplished using phase-locked VLF receivers and very stable atomic reference frequency standards mounted in a mobile laboratory which traversed a roughly north-south path from Boulder, Colorado, to Austin, Texas. This method is the opposite of that used in VLF navigation, where the velocity of the wave is assumed known and position is then determined [Stanbrough and Keily, 1964; Stanbrough, 1965] from the measurements of the phase.

The results in terms of the quantity, $\Delta\phi$, are shown in figure 1. The term $\Delta\phi$ is the measured accumulated phase in microseconds minus the phase that would have accumulated if the phase velocity of the wave equaled that of light, c, in a vacuum. It is plotted as a function of radial distance R from the transmitter. The phase in microseconds is related to the phase in radians by the expression, ϕ microseconds $= \phi$ radians$/2\pi f$, where f is the radio frequency. Each value of $\Delta\phi$ at the two frequencies, 18.6 and 20.0 kc/s is an average of six measurements obtained on different days within 4 hours of local noon at the midpoint of the path. There was no unusual solar activity during any of the measurements. Because the measurements did not start at the transmitter, the accumulated phase between the transmitter and the starting point is unknown. Therefore, we have arbitrarily set $\Delta\phi$ equal to zero for the point closest to the transmitter at each of the two frequencies. The error due to uncertainties in the great-circle-path length, the reference-oscillator frequency, and the phase-measurement equipment is less than 0.5 μs.

Although it is not our intention in this paper to discuss these results in any detail, there are two features of the data which deserve comment. First, the standard deviation of the measurements is considerably more than one would expect from the measurement errors alone. This spread in the data is probably due, primarily, to varying local anomalies and to differences in the ionosphere between measurements. Second, the phase in general does not accumulate uniformly with distance at either frequency, but rather appears to have an oscillatory pattern whose amplitude decreases with distance from the transmitter. This is to be expected, since ionospheric waveguide mode theory shows [Wait, 1962] that the phase pattern across the ground depends upon the interaction of several ionospheric waveguide modes. Near the transmitter several modes are important; however, the higher-order modes are attenuated rapidly with distance, so that at great distances the phase pattern is primarily due to mode 1. All of the measurements shown in figure 1 appear to be in an oscillatory pattern except, perhaps, for the four most distant measurements at 18.6 kc/s whose average values lie on a straight line. Assuming that these four values are produced primarily by mode 1, we have calculated the phase velocity by fitting a line, using the least-squares method, to the phase versus distance measurements. The slope of this line, by (1), is inversely proportional to the phase velocity. Although there is no guarantee that more distant measurements would not have shown further oscillations, it is interesting to note that the result obtained was

$$V_p/c - 1 = -0.0026 \pm 0.0007. \tag{3}$$

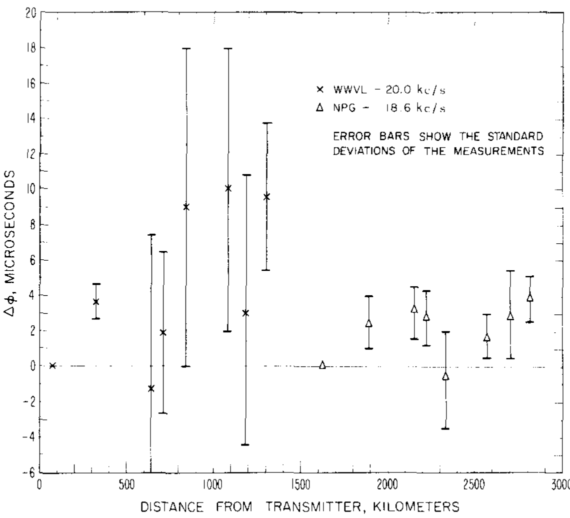

FIGURE 1. *Measured phase minus calculated accumulated phase, in microseconds, at discrete distances from the transmitter.*

This value, within the precision of the measurement, agrees with a previous experimental result, using a different method, obtained by Steele and Chilton [1964] where they interpret their measurement as applying to mode 1. The value is also in agreement with a theoretical mode 1 value obtained by Wait and Spies [1964] for an exponential model of the ionosphere with a reference height of about 80 km (case with no earth magnetic field). In this model the conducting parameter ω_r varies with height, z, as $e^{+\beta z}$, where $\beta=0.3$ km^{-1} and where $\omega_r=2.5\times10^5$ sec^{-1} at the reference height. The ground conductivity is assumed infinite.

As a statistical check, the phase velocity, using the same method, was obtained from the rest of the 18.6-kc/s measurements with the result that $V_p/c-1=-0.0003\pm0.0007$. It may be shown that this value differs from the previous value at the 5-percent significance level, thus indicating that no meaningful phase velocity may be obtained by fitting a straight line to all of the 18.6-kc/s data.

Since figure 1 shows rapid fluctuations in $\Delta\phi$ with distance at 20.0 kc/s, no attempt was made to apply (1) to this data. To determine the phase velocity as a function of distance, where more than one mode is significant, measurements must be made sufficiently close together along the path to accurately sample the variation. A later paper will report on the results of such closely spaced measurements and on the extension of the measurements reported above.

References

Stanbrough, J. H., Jr. (1965), A VLF radio relative navigation system, Navigation **11**, 4, 417–428.

Stanbrough, J. H., Jr., and D. P. Keily (1964), Long range navigation by means of VLF transmissions, Deep-Sea Res. **11**, No. 2, 249–255.

Steele, F. K., and C. J. Chilton (1964), Measurement of the phase velocity of VLF propagation in the earth-ionosphere waveguide, Radio Sci. J. Res. NBS **68D**, No. 12, 1269–1273.

Wait, J. R. (1961), A comparison between theoretical and experimental data on phase velocity of VLF radio waves, Proc. IRE **49**, No. 6 1089–1090.

Wait, J. R. (1962), Electromagnetic Waves in Stratified Media (Pergamon Press, New York, N.Y.).

Wait, J. R., and K. Spies (Dec. 30, 1964), Characteristics of the earth-ionosphere waveguide for VLF radio waves, NBS Tech. Note No. 300.

(Paper 1–12–168)