Offprint from "Metrologia"
Journal International de Métrologie scientifique — International Journal of Scientific Metrology
Internationale Zeitschrift für wissenschaftliche Metrologie. Vol. 8, No. 2, April 1972, Pages 64—72

Precision and Accuracy of Remote Synchronization via Network Television Broadcasts, Loran-C, and Portable Clocks * ** ***

David W. Allan, B. E. Blair, D. D. Davis, and H. E. Machlan

Time and Frequency Division, National Bureau of Standards, Boulder, Colorado 80302, U.S.A.

Received: December 29, 1971

Abstract

A comparison among three precise timing centers in the United States has been conducted for more than 1 year using three different synchronization methods. The timing centers involved were the United States Naval Observatory (USNO) in Washington, D. C., Newark Air Force Station (NAFS) in Newark, Ohio, and the National Bureau of Standards (NBS) in Boulder, Colorado. The three methods were cesium beam portable clocks; Loran-C transmissions from Cape Fear, North Carolina, and Dana, Indiana; and ABC, CBS, and NBC network television broadcasts commonly received by the three timing centers.

Cesium beam portable clocks have the capability of accurately and precisely synchronizing remote clocks to within 0.1 µs. The Loran-C data involved a 3500 km (2180 miles) ground wave path — the longest Loran-C ground wave path that has been studied with the precision and accuracy reported herein. The long-term precision achieved was about 1 µs over 1 year. The accuracy is limited on occasion by inability to resolve the 10 µs ambiguity of the 100-kHz pulse train. The precision capability of maintaining remote clock synchronization within the majority of the continental United States using network television broadcasts was inferred to be about $5\ \mathrm{ns} \cdot \tau^{1/3}\ \mathrm{s}^{-1/3}$ over the range of τ from 86400 s (1 day) to about 10^7 s (324 days) but with definite accuracy limitations caused by such factors as occasional network re-routing of the television signals. Some estimates of the long-term frequency stabilities among the references used at the three timing centers were measured or inferred.

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** Presented at the 25th Annual Symposium on Frequency Control, Atlantic City, N.J., 26—28 April 1971.

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Introduction

Time and frequency dissemination via television has received much attention during the past few years. Even though some very impressive results have already been obtained using television [1—6], it seems still to be a pioneer field. In this paper we compare television with two other state-of-the-art methods of time and frequency dissemination — Loran-C and portable clocks — and evaluate some precision and accuracy capabilities of each.

The television method is readily available, very inexpensive, and within the majority of the continental United States it is a common source for many users. Loran-C, being well established and well known for its precision and accuracy [7—8], has been chosen to compare with the television system. Portable clocks are used as a reference because of their precision and accuracy for remote synchronization — one of the best techniques yet available [9]. The portable clocks referred to in this paper are those of the United States Naval Observatory (USNO). We will exclude many other dissemination techniques such as VLF and satellite, as these have already been covered in some detail [10—12].

There are three fundamental aspects of time which can be disseminated via these methods. The first is time interval which can be related to frequency — frequency being the inverse period of an oscillation. The second aspect is that of date or clock reading which has often been called epoch. We prefer the use of the word date because epoch has alternate meanings that could lead to confusion. Often we have a master clock, and we wish to communicate its date or time by some technique to a slave station located elsewhere. The third aspect is simultaneity — the practical application of which is clock synchronization, i.e., two clocks have the same reading in some frame of reference.

In principle, if we had perfect clocks, we could synchronize them once and they would remain synchronized forever. There are two basic reasons in practice why the synchronization does not persist. First of all, systematic (non-random) effects such as frequency drift, frequency offset, and environmental effects on equipment often cause time dispersion. These must be analyzed and solved at each particular location. Secondly, there are different kinds of random noise, or what we might call non-deterministic kinds of processes, that affect these time and frequency centers and/or the time communication systems. These latter processes can typically be classified statistically. We will discuss and apply some useful statistical measures for the time and frequency dispersion of the dissemination systems in question.

Most of the data analyzed in this paper were taken by other people, and we wish to acknowledge the fine work and careful data taking and reporting of the personnel at the U.S. Naval Observatory (USNO) and the Newark Air Force Station (NAFS), as well as of the personnel in addition to the authors at the National Bureau of Standards (NBS).

In discussing the precision capabilities of the above three methods of time dissemination, it is of interest to have a statistical measure of the time and/or frequency dispersion characteristics of each method. The non-random aspects of the data, i.e., that due to the frequency and time differences between the three timing centers under consideration, are generally of primary interest. However, the precision with which these may be determined can best be evaluated by a study of the random processes that perturb a particular method of time dissemination. In general, if one knows the spectral density (or the autocorrelation) of a particular random process, and if the distribution is normal, one knows all there is to know about the statistical properties of the process. Since the distributions of the processes being studied in this paper are probably normal, it will be our primary concern to discuss and employ some statistical measures of time and/or frequency dispersion involving autocorrelation yielding spectral density estimates of the perturbing noise processes, and which also often yield information about the underlying perturbation mechanisms.

It is often the case that data are taken at a constant repetition rate with a period of sampling, T; and each data point is an average over a time τ called the sample time. Let the total number of data points taken in a continual data set be M. Further, for every measurement system there is a high frequency cutoff usually called the measurement system bandwidth, f_B, such that noise at frequencies greater than f_B will be attenuated and non-relevant. In the past it has been common practice to compute the standard deviation as a statistical measure of such a data set:

$$\sigma_{\text{std. dev.}} \equiv \left\{\frac{1}{M-1}\left[\sum_{i=1}^{M}(z_i-\bar{z})^2\right]\right\}^{1/2}, \qquad (1)$$

where z_i denotes the i^{th} data point and $\bar{z}$ denotes the average of all M of the z_i. For most of the noise processes that are pertinent in time and frequency metrology, it has been shown that $\sigma_{\text{std. dev.}}$ depends upon M, T, τ and f_B [13—15], and all of these parameters should be noted for each experiment. Furthermore, $\sigma_{\text{std. dev.}}$ is a measure of the distribution width only and gives no information that would allow a spectrum estimation. We have found it convenient to use the Allan variance [13—15]:

$$\sigma^2 \equiv \langle\sigma^2(N, T, \tau, f_B)\rangle\,, \qquad (2)$$

where the angle brackets denote the expectation value, and N is the number of data points for each estimate of $\sigma^2(N, T, \tau, f_B)$.

For many pertinent noise processes, we have found that a power law spectral density is a good model, i.e.,

$$S_y(f) = A f^{\alpha}\,, \qquad (3)$$

where f is the Fourier frequency and A is the intensity of the noise process. Throughout this paper y denotes the fractional frequency deviations, x denotes the time deviations, and y is thus proportional to the derivative of x. Further, it has been shown that if N, f_B, and the ratio $r = T/\tau$ are held constant, then σ_y^2 is equal to $a\tau^{\mu}$ with μ related to α as follows:

$$\mu = \begin{cases} -\alpha - 1 & -3 < \alpha \leq 1 \\ -2 & \alpha > 1\,, \end{cases} \qquad (4)$$

and with the constraint that $|\tau f_B|^{|\alpha - 1|} \gg 1$ [10, 13]. Thus, by finding the dependence of σ_y^2 as a function of τ, the spectral density may be inferred. For a more detailed explanation of the N, T, and τ dependence, the relationship between μ and α, and the relationship between the frequency domain and time domain coefficients, A and a, see Ref. [14] and [15].

There is a particular Allan variance which has been recommended by the IEEE subcommittee on frequency stability [14]. It is defined as follows:

$$\sigma_y^2(\tau) \equiv \langle\sigma_y^2(N = 2, T = \tau, \tau, f_B)\rangle\,. \qquad (5)$$

This measure has some very convenient experimental and theoretical characteristics. For example, with M values of $\bar{y}_i$ an estimate of $\sigma_y(\tau)$ is:

$$\sigma_y(\tau) \simeq \left[\frac{1}{2(M-1)}\sum_{i=1}^{M-1}(\bar{y}_{i+1} - \bar{y}_i)^2\right]^{1/2}, \qquad (6)$$

where

$$\bar{y}_i = \frac{x_{i+1} - x_i}{\tau} \qquad (7)$$

and the interval between each discrete time measurement x_i is τ. Also once $\sigma_y(\tau)$ has been calculated, the time dispersion may be *estimated* by simply calculating the product $\sigma_x(\tau) \equiv \tau\cdot\sigma_y(\tau)$ [14]. This estimate is good for white noise frequency modulation but is approximately a factor of 1.3 too optimistic for flicker noise frequency modulation and its approximately a factor of 1.7 too pessimistic for flicker noise phase modulation with respect to an optimum prediction routine [16].

The above statistical measures will be used to calculate the precision for the data used in this paper. From these analyses we will draw some conclusions regarding the relative precision of the three time and frequency dissemination techniques herein discussed, i.e., network television, Loran-C, and portable clocks. Accuracy, on the other hand, may be defined as follows: for frequency it is the confidence with which a frequency is known with respect to the currently defined resonance in cesium 133; and for time it is the confidence with which a date is known with respect to a reference time scale such as UTC (NBS).

Time and Frequency Stability of Network TV

Description of TV Line-10 Timing System

After the work of Tolman and others [1—5], the TV line-10 system was developed by NBS as a passive means of comparing precision clocks, geographically separated but periodically compared via commonly received network broadcasts [6]. An overview of the system is shown in Fig. 1. The broadcasts utilized in this paper originate from the New York City studios of three commercial TV networks (ABC, CBS, and NBC). These originating networks incorporate independent atomic frequency standards (rubidium gas cell) to stabilize their transmissions. The New York signals, broadcast without auxiliary time coding, traverse varied and long paths at microwave frequencies. This relay system is a chain of broadband radio links encompassing the continental United States at line-of-sight distances of some 40 to 60 km between repeaters. The microwave relay system carrying over 95% inter-city television programs is known as the TD-2 system [17]. At a terminating station, such as an affiliate local transmitter, the microwave signal from the applicable repeater station is converted to a video signal and retransmitted by VHF or UHF (commercial TV) to a local service area. Reception points of such broadcasts for our data were the U.S. Naval Observatory (USNO), Washington, D. C.; Newark Air Force Station (NAFS) Newark, Ohio; and the National Bureau of Standards (NBS), Boulder, Colorado.

This version of TV timing uses the pulse identifying line-10 of the odd field in the 525-line system M as a passive transfer pulse [3], [6]. This pulse occurs during the blanking re-trace interval between successive fields; the line-10 pulse was chosen for timing as it is the first horizontal synchronization (sync) pulse following the equalizing and vertical sync pulses and therefore is easy to identify with simple logic circuits. Fig. 2 shows a typical equipment configuration for line-10 synchronization. Almost any type of television receiver, black and white or color, is suitable for reception of the signals used for timing. Various combinations of auxiliary equipment can be used with the receiver to synchronize the clocks. A typical combination is a line-10 synchronized pulse generator (available at a cost of about $165), a digital counter-printer (about 0.1 μs resolution), and a clock whose frequency is known to an accuracy of a few parts in 10^9 and which has an output of 1 pulse per second (1 pps),

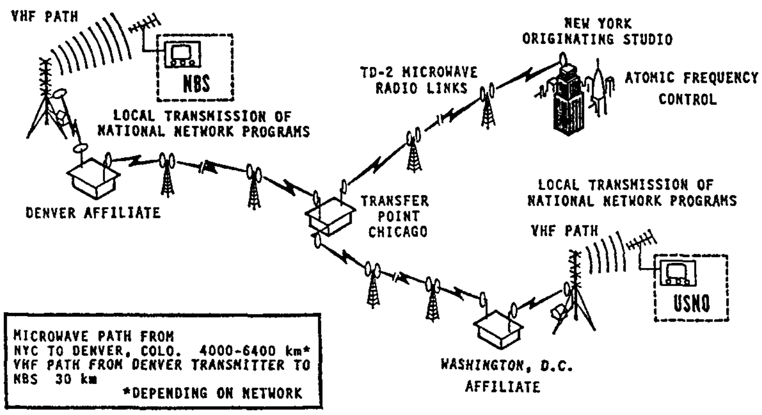

Fig. 1. Typical routing of TV signals from New York City's originating studios to distant receivers

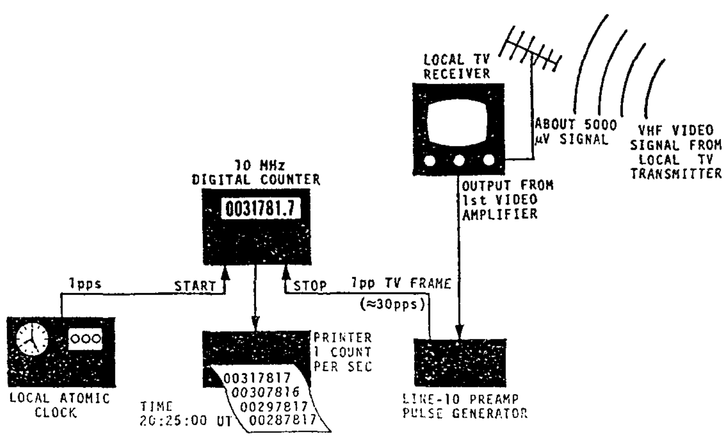

Fig. 2. Overview of receiving laboratory instrumentation for TV line-10 clock synchronization

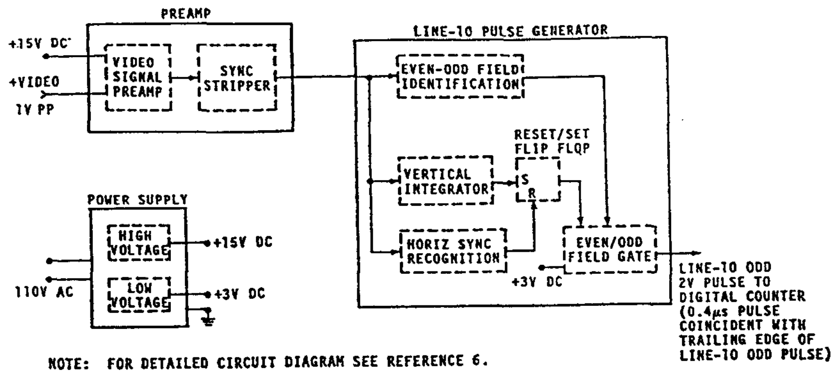

Fig. 3. Functional block diagram of TV line-10 identification circuitry

whose date should be known to about 16 μs. A functional block diagram of the line-10 identification circuitry employed is shown in Fig. 3.

The timing system employed to collect the data used in this paper works as follows: At the same date, to an accuracy of a few microseconds, which is much better than is needed, counters are started at all laboratories with a 1 pps tick from their local reference atomic clocks. Near this time a line-10 horizontal sync pulse is broadcast from one of the New York City originating TV transmitters. After diverse delays through both common and separate microwave links, the sync pulse — received by the laboratories at different times due to the delay and clock differences — stops the corresponding counters. The difference, then between each pair of counter readings at any two receiving laboratories, remains constant except for instabilities in the propagation delay and/or instability or relative frequency offsets among the reference atomic clocks.

Likewise, any other clock can be compared with the UTC (USNO) and UTC (NBS) scales through a similar reception-recording system. If the system is to be used for accurate setting of the clock's date the delay of the propagation paths involved must be calculated or calibrated, e.g., with a portable clock. Since the period of one TV frame is about 33 ms, it is also necessary to resolve this ambiguity at the receiving site to within about 16 ms. NBS distributes these line-10 daily measurements in terms of UTC (NBS) in the monthly NBS Time Service Bulletin [18]. The USNO distributes line-10 data in terms of UTC (USNO-MC) in the weekly Series 4 Time Services Bulletin [19]. These publications will publish future changes, modifications to the TV system, or other factors affecting a user in the field.

Advantages of line-10 timing include 1) simplicity and minimum cost of comparison equipment; 2) low cost of maintaining synchronization with long range and long term precision of better than 10 μs; 3) no effect on regular TV networks and with no external cost to user; 4) three TV networks with atomic clock references provide redundancy and backup data in case one TV channel shows a microwave reroute; and 5) a method for simultaneous maintainance of sub-microsecond synchronization of several clocks diversely located within the service area of a common transmitter, without regard to national programming. These advantages must be tempered by such factors as 1) microwave paths can be interrupted without notice; 2) there is limited simultaneous viewing time of nationwide network programs; 3) present network distribution does not allow common programming with West Coast transmission lines, although local synchronization from a common transmitter can be effected; 4) system is not compatible with tape delayed programs; and 5) the system ambiguity is 33 ms. (Note that time-of-day is "ambiguous" to 1 day.)

Stability of the USNO, NBS TV Path

The TV paths being considered are from two to four thousand kilometers in length. The particular problem mentioned above of an occasional TV network re-route will cause an effective change in the delay. So it is highly advantageous to use all three networks so that such a change can readily be identified. Conveniently, we do have three networks so that outliers and delay changes are easily recognized. During the analysis period studied, which was from 25 June 1969 to 30 December 1970, there were only about two network delay changes per year per network, so it is not a serious inconvenience. The television time measurements using the line-10 method were made at 2025 UT, 2026 UT, and 2027 UT (1 h earlier during Daylight Savings Time) on NBC, CBS, and ABC, respectively and nominally every work day at each of the three laboratories involved.

Fig. 4 is a plot of the fractional frequency stability, $\sigma_y(\tau)$ versus the sample time τ in days for the TV paths between Washington, D.C., and Boulder, Colorado, for each of the three networks (this assumes that the instabilities of the reference time scales are negligible). An ensemble of commercial cesium beam frequency standards and dividers to generate atomic time (AT) was used at each location as the 1 pps time reference, i.e., AT (USNO)[1] and AT (NBS) [20—23]. The dashed line in Fig. 4 corresponds to a noise process with α greater than or equal to 1. By analyzing the time fluctuations directly (rather than the frequency fluctuations), we determined the TV

[1] We have chosen the designation AT (USNO) to parallel our designation AT (NBS) and for consistency with the New Delhi CCIR Recommendation 458 of Study Group VII for Standard Frequency and Time Signals. USNO (Mean) or A.1 (Mean) in Ref. [22] is identical to AT (USNO) as used herein.

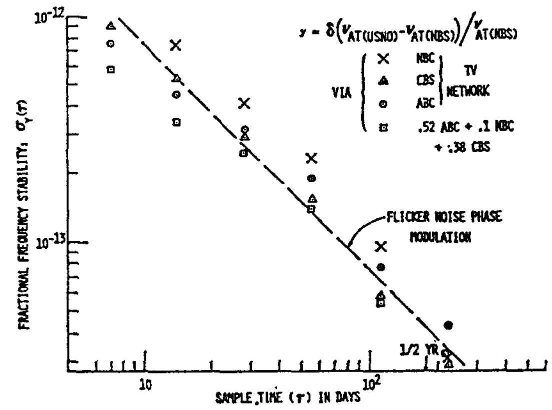

Fig. 4. Fractional frequency stability, $\sigma_y(\tau)$, of the AT(USNO) — AT (NBS) time scales compared by the 3-network TV line-10 technique

noise was reasonably modeled with an $\alpha = 1$ process, i.e., flicker noise phase modulation for this path. Calculating the time dispersion, $\sigma_x(\tau)$ for the dotted line gives the following equation [14]:

$$\sigma_x(\tau) = 62 \text{ ns } [6 + 3 \log_e(2\pi\,\tau\, f_B) - \log_e 2]^{1/2}, \quad (8)$$

where τ is in seconds; f_B for a color TV receiver is about 3.2 MHz. As can be seen from Fig. 4, Eq. (8) is apparently valid to within about 30% for values of τ in the range 7 days $\leq \tau \lesssim$ 200 days, and $\sigma_x(\tau)$ has a value of about 0.6 µs over this range.

The fractional frequency difference between AT (USNO) and AT (NBS) calculated over the period of analysis was:

$$\frac{\nu_{AT(USNO)} - \nu_{AT(NBS)}}{\nu_{AT(NBS)}} = \begin{cases} 4.48 \times 10^{-13} \text{ via ABC} \\ 4.43 \times 10^{-13} \text{ via CBS} \\ 4.56 \times 10^{-13} \text{ via NBC}. \end{cases} \quad (9)$$

The precision of these measurements is about $\pm 3 \times 10^{-14}$ as may be seen from the stability measured at $\tau =$ 224 days. An additional inference from Fig. 4 is that the combined instabilities of AT (USNO) and AT (NBS) were not worse than 3×10^{-14} over the period being considered and for $\tau = 224$ days. The confidence, however, on this single sample stability estimate is undoubtedly poor.

Since there are three essentially independent networks, they can be combined optimally by weighting each one inversely proportional to its variance. We chose the variance at $\tau = 7$ days, since it has the best confidence, to calculate the weights of 0.52, 0.38, and 0.10 for ABC, CBS, and NBC, respectively. This is optimum in the sense of giving a minimum variance, and it should be noted that these coefficients need to be calculated for each pair of receiving laboratories or effectively for each TV path. The squares in Fig. 4 represent the stability using this optimum weighting procedure. The squares in Fig. 5 show the time fluctuations of the weighted three network TV data between USNO and NBS as measured each Wednesday. Each of the points used in the computation was an average of 40 measurements taken at one-second intervals. A one-second measurement gives almost as good precision, but the averaging allows one to recognize outliers. The interval between the measured line-10 horizontal sync pulses is 1.001 s and hence there is a walk between a standard 1 pps and the line-10 pulse of 1 ms per second (modulo 33.366... ms).

TV and Cesium Beam Stability at NAFS

The same line-10 TV network method was employed as outlined above at Newark Air Force Station (NAFS) in Newark, Ohio. Their time reference was a commercial cesium beam frequency standard and clock. Both USNO and NBS data were used to study the path stability between Washington, D.C., and Newark, Ohio, and between Boulder, Colorado, and Newark, Ohio, over the periods from 24 September 1969 to 16 December 1970 and from 17 September 1969 to 30 December 1970 respectively.

Fig. 6 shows the fractional frequency stability $\sigma_y(\tau)$ for both paths using all three TV networks optimally weighted. The squares show the stability for the USNO, NAFS TV path with weights of 0.36, 0.57, and 0.07 for ABC, CBS, and NBC respectively. The circles show the stability for the NBS, NAFS TV path with weights of 0.29, 0.36, and 0.35 for ABC, CBS, and NBC respectively.

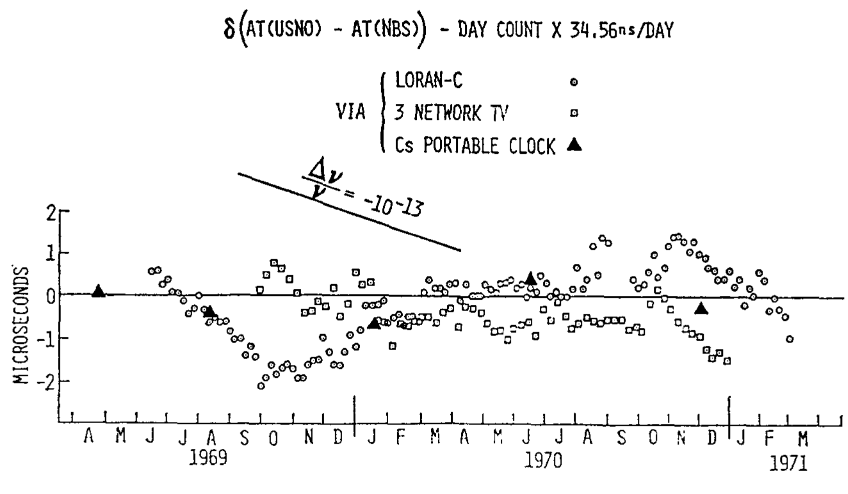

Fig. 5. Relative time differences of the AT (NBS) — AT (USNO) time scales compared by the Loran-C, 3-network TV line-10, and cesium portable clock techniques plus arbitrary constants

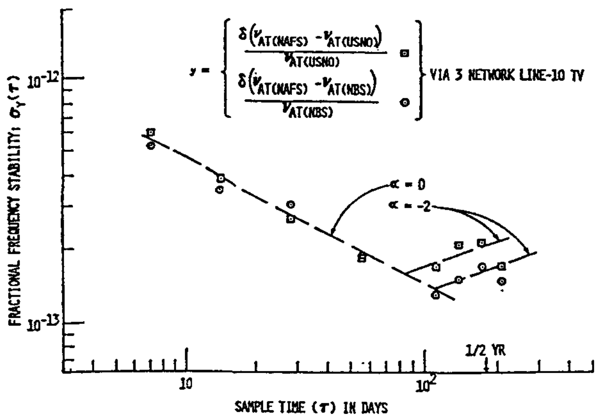

Fig. 6. Fractional frequency stability, $\sigma_y(\tau)$, of the Boulder-Newark and the Washington, D.C.-Newark paths via the 3-network TV line-10 technique

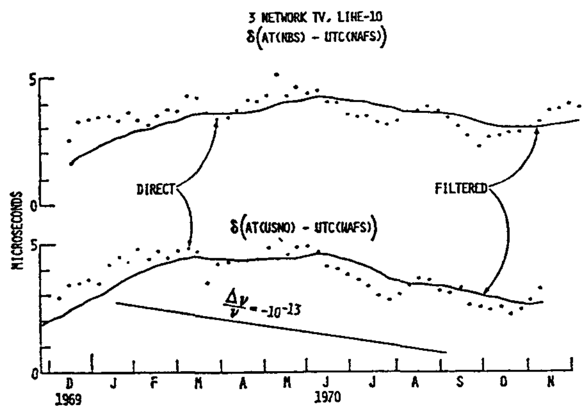

Fig. 7. Time differences (direct and filtered) of the Boulder-Newark and the Washington, D.C.-Newark paths via the 3-network TV line-10 technique plus an arbitrary constant

The slope indicated by $\alpha = 0$ is probably the noise of the TV line-10 time transfer system, but the type of noise is unexpected, i.e., white noise frequency modulation or random walk of phase noise. Calculating $\sigma_x(\tau) = \tau\sigma_y(\tau)$ gives:

$$\sigma_x(\tau) = (0.13\ \mu\text{s day}^{-1/2})\ \tau^{1/2} \tag{10}$$

where τ is in days, and is in the range 7 days $\leq \tau \leq$ 100 days. An explanation of this random walk of phase noise could be some step changes in the delay for which there was inadequate accounting during the data reduction.

Note that the stability gets worse for τ larger than 100 days with a maximum at τ equal to about $\frac{1}{2}$ year for both the squares (USNO vs NAFS) andthe circles (NBS vs NAFS). This part of the stability plot is probably due to a seasonal or annual effect on the time reference cesium standard at NAFS.

Wiener Filtering of TV Data

In Fig. 6 the dashed line representative of $\alpha = 0$ (white noise FM) appears to be a good model for the instabilities in the TV data over the two paths mentioned above. If one can assume that the dashed line representative of $\alpha = -2$ (random walk of frequency noise) is a good noise model for the cesium beam reference standard, then it has been shown that a Wiener filter may be applied to the data [11, 24]. Assuming that the $\alpha = 0$ process is noise and the $\alpha = -2$ process is signal, i.e., we wish to have a best estimate of the behavior of the cesium reference as observed through the noise of the TV line-10 time transfer system, then the filter takes on a very simple form, i.e., an exponential. We mean by best estimate a minimum mean squared error for $\langle[\hat{s}(t) - s(t)]^2\rangle$ where $s(t)$ is the true behavior of the cesium beam reference standard and $\hat{s}(t)$ is the Wiener filtered estimate.

The models for the noise and signal are:

$$S_{y(\text{noise})}(f) = A\, f^0\ , \tag{11}$$

and

$$S_{y(\text{signal})}(f) = B\, f^{-2} \tag{12}$$

respectively, and the form of the Wiener filter for the discrete case is as follows:

$$\hat{x}_i = \frac{\sum\limits_{j=-\infty}^{i} x_j\, e^{-\frac{i-j}{\xi}}}{\sum\limits_{j=-\infty}^{i} e^{-\frac{i-j}{\xi}}}\ , \tag{13}$$

where

$$\xi = \frac{1}{2\pi}\sqrt{\frac{A}{B}}\ . \tag{14}$$

and ξ is normalized to the same units as i and j, i.e., days, weeks, etc. A convenient recursive filter that approximates Eq. (13) is:

$$\hat{x}_i = \frac{1}{\xi + 1}\,[x_i + \xi\, \hat{x}_{i-1}]\ . \tag{15}$$

For the signal and noise processes given by Eqs. (11) and (12), using Ref. [14], ξ takes on the value

$$\xi = \frac{1}{\sqrt{3}}\,\tau_I\ , \tag{16}$$

where τ_I is the value of τ corresponding to the intercept of the dashed lines in Fig. 6. For these particular data sets, ξ had values of 7 weeks and 9-1/4 weeks for the Washington, D.C., to Newark, Ohio, and the Boulder, Colorado, to Newark, Ohio, paths respectively.

The dots in Fig. 7 show the weighted three network values measured each Wednesday for each of the above two paths. The solid lines show the result of an application on these data of the Wiener filter given by Eq. (13) with the sum being taken over about 1-1/2 time constants, i.e., the past 10 and 14 values (weeks) respectively for the above two paths. Note the slope of one part in 10^{13} and the strong correlation between the two paths both before and after filtering. It is apparent from Fig. 7 that the time constant, ξ, is some too large, which implies that the noise model, $\alpha = -2$, for the instabilities in the cesium beam frequency standard is too low-frequency (Fourier frequency) divergent. Probably $\alpha = -1$ (flicker noise frequency modulation) would be a better model, but the optimum filter for an $\alpha = 0$ noise and $\alpha = -1$ signal has not been worked out to the best of our knowledge.

Residual Time Dispersion of Optimum Processed TV Data

Taking the difference between the two solid lines in Fig. 7 gives us a filtered estimate of time fluctuations between AT (USNO) and AT (NBS). If we now apply a Wiener filter to the TV line-10 data plotted in Fig. 5, we have a direct path filtered estimate of the same fluctuations. Taking the difference between these two estimates leaves as a residual the noise or perturbations introduced during the data processing, i.e., filtering, improper adjustment for TV network delay changes, etc. This residual is plotted in Fig. 8. Note, the data fall within a vertical range of $\pm 1\ \mu s$ over about a 1-year period. If the data in Fig. 8 are analyzed statistically, the intensity of this residual noise is found to be only about 40% of that given in Eq. (10). Hence, we have an experimental consistency check that the errors introduced during data processing are significantly below the inherent noise of the TV line-10 timing system.

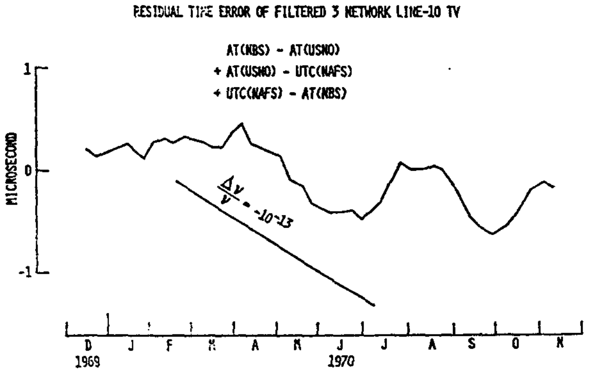

Fig. 8. Residual time differences of the 3-network TV line-10 technique between Boulder, Newark, and Washington, D.C.

Time and Frequency Stability of Loran-C

Stability of USNO, NBS Loran-C Path

The circles in Fig. 5 represent the time difference AT (NBS) — AT (USNO) plus an arbitrary constant via a 3,500 km Loran-C path going from Cape Fear, North Carolina, to USNO and to Dana, Indiana, and from Dana, Indiana, to Boulder, Colorado. Dana, Indiana, is phase controlled to within $0.2\ \mu s$ with respect to Cape Fear using as the place of phase reference Warner Robins Air Force Base in Georgia [25]. The circles represent the Loran-C measurements as made every fifth day. The triangles are the USNO portable clock trips between USNO and NBS with a reported accuracy of date transferral of $0.1\ \mu s$ [19].

The circles in Fig. 9 show the fractional frequency stability, $\sigma_y(\tau)$ for the Loran-C data plotted in Fig. 5.

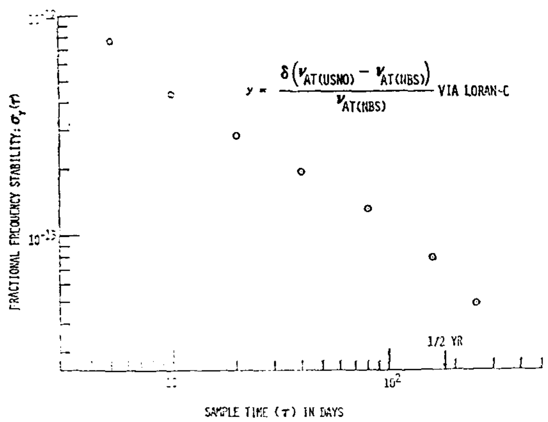

Fig. 9. Apparent Loran-C fractional frequency stability, $\sigma_y(\tau)$, over a continental USA path

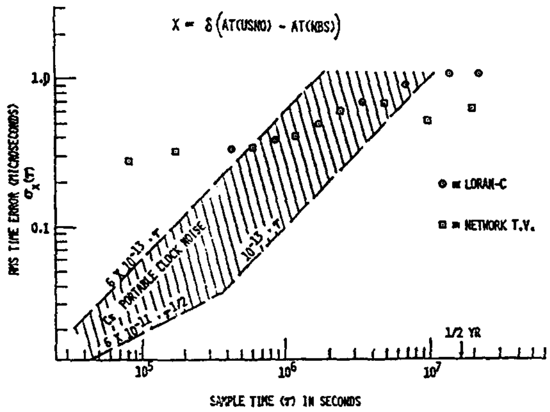

Fig. 10. Estimation of the rms time dispersion versus sample time for Loran-C, 3-network TV line-10, and cesium portable clock techniques with USNO and NBS as the time references

The circles in Fig. 10 show an estimate of the time dispersion, $\sigma_x(\tau) \equiv \tau\sigma_y(\tau)$ for the same data. The squares in Fig. 10 show the same estimate of time dispersion for the TV line-10 data shown in Fig. 5.

Time Accuracy of TV Line-10, Loran-C, and Cesium Portable Clocks

The ordinate shown in Fig. 5 for the TV line-10 system, for the Loran-C, and for the portable clocks has been chosen to provide a convenient display of the data and has no meaning for comparing the different time dissemination systems on an absolute basis since over these paths only the portable clocks have accurate date transferral capabilities at the levels plotted. For the paths being considered, both the TV line-10 system and Loran-C need a path delay calibration in order to establish sub-microsecond synchronization. Loran-C delays can often be calculated to better than $1\ \mu s$ for areas within good ground-wave coverage. For a TV line-10 system the continental path delay would be extremely difficult to calculate; however, the path delay may be readily calculated to within about $1\ \mu s$ when both receiving points are line-of-sight to the same TV transmitter [1]. On the other hand, the cycle ambiguity for Loran-C is $10\ \mu s$ within the pulse train whereas it is 33 ms for the TV line-10 system. This means that the cycle ambiguity could be resolved very easily on the TV line-10 system using the transmissions from WWV whereas the $10\ \mu s$ ambiguity of Loran-C requires a greater accuracy to adequately resolve.

Fig. 10 also shows an estimate of the time dispersion characteristics for cesium beam portable clocks presently available. Note that the dispersion degrades to be that of either TV or Loran-C for τ values of a few weeks. For portable clocks τ may be interpreted as the time since the last calibration date. It is obvious for this and other reasons that portable clock trips should be made quickly for the best accuracy in date communication.

Short-Term Frequency Stability of TV, Loran-C, and Portable Clocks

It is often desirable to have available a reference standard frequency for calibrating the frequency (time interval) of a clock, or of a frequency counter, etc. In color television broadcasting, a color "subcarrier" of 63/88 of 5 MHz (3.57... MHz) is transmitted on the VHF or UHF signal. It is used as a reference signal in the color television receiver to

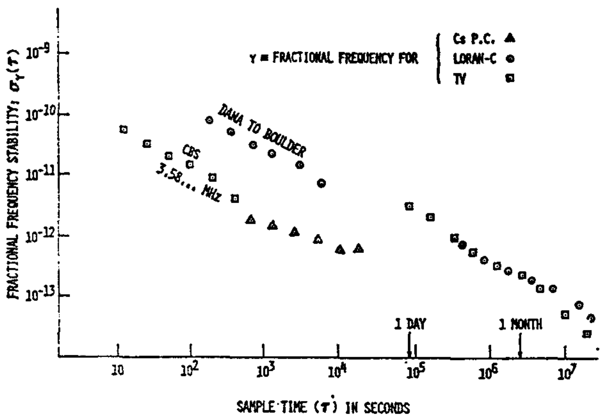

Fig. 11. Relative fractional frequency stability, $\sigma_y(\tau)$, versus sample time for Loran-C, 3-network TV line-10, CBS TV color subcarrier, and cesium portable clock

demodulate the chrominance sidebands. Since the major U.S. networks generate the color subcarrier with rubidium frequency standards, this color subcarrier may be used as a reference standard frequency. Frequency stability measurements of the color subcarriers of all three major U.S. networks (originating in New York) have been made at the NBS laboratories in Boulder [26]. NBS designed instrumentation both to synthesize the output of a 1- or 5-MHz local frequency standard to 3.57... MHz and to compare phases of the local synthesized signals to the received subcarrier frequency. A plot of the stability of some of the best data received in Boulder, Colorado, are represented by the squares marked CBS in Fig. 11. Typically the stability was a factor of two or three times worse than this. The stability was well modeled by a $\tau^{-2/3}$ power law and persisted regardless of which of the three networks was used. Calculating $\sigma_x(\tau) \equiv \tau \cdot \sigma_y(\tau)$ for an estimate of the time dispersion of the color subcarrier for the data plotted gives the very impressive result that:

$$\sigma_x(\tau) = 0.3\ \mathrm{ns} \cdot \tau^{1/3}\,\mathrm{s}^{-1/3} \tag{17}$$

with τ in the range $12\ \mathrm{s} \leq \tau \leq 384\ \mathrm{s}$. The frequency stability of the color subcarrier was also measured in the range $1\ \mu\mathrm{s} \leq \tau \leq 1\ \mathrm{s}$ with the resultant value for $\sigma_x(\tau)$ of 1 nanosecond.

Some measurements of the stability of the horizontal sync pulse for short sample times, τ, in the range from 1 s to 64 s were made. It was determined that a reasonable model ($\sim$30% confidence) of the time dispersion for both this short term measurement and the combined three network stability plotted in Fig. 4 was as follows:

$$\sigma_x(\tau) = 5\ \mathrm{ns} \cdot \tau^{1/3}\,\mathrm{s}^{-1/3}\,. \tag{18}$$

We have no explanation for the unusual apparent power law of $\tau^{1/3}$.

It should be noted that for sample times longer than a few minutes one almost always observes step changes of at least several nanoseconds in a particular network's delay time. These changes apparently are caused as a network changes TV cameras for commercials, new programs, etc. Since these steps are discrete and non-random, they can usually be identified, and the data could be corrected accordingly.

The fractional frequency stability of the Loran-C signal received in Boulder, Colorado, from Dana, Indiana, was also analyzed for sample times, τ, of several seconds and is plotted as the circles labeled Dana to Boulder in Fig. 11. This is a very long ground-wave path for Loran-C, and the stability is probably much better than this in stronger signal areas. Also plotted in Fig. 11 as the triangles are fractional frequency stability data for a typical cesium beam portable clock.

It is interesting to compare the relative stabilities (precision) of the three methods for a sample time, τ, of about 200 s. The values of $\sigma_y(\tau)$ are about 10^{-10}, 10^{-11}, 4×10^{-12} for Loran-C, TV color subcarrier, and cesium portable frequency standard, respectively. The TV color subcarrier provides a very inexpensive and precise method for frequency calibration. The accuracy of a measurement in all three cases is limited by the accuracy of the reference standard employed as well as by the precision of the measurement. To improve the usefulness of the TV color subcarrier method NBS now publishes weekly measurements of the absolute frequencies of the above three TV networks' rubidium gas cell frequency standards [18].

Fig. 11 also shows for comparison purposes the previous data discussed (see Figs. 4 and 9) comparing AT (USNO) and AT (NBS) via Loran-C and via TV line-10 time transfer system. Two additional stability points are plotted for the line-10 TV time transfer system for the Washington, Boulder path at τ equal 1 and 2 days. The circles at the right are the stabilities via Loran-C, and the squares at the right are the stabilities via line-10 TV.

Conclusions

It may be inferred from the results of this paper that the three network TV line-10 timing system properly filtered may be used in a large majority of the United States to keep clocks synchronized to within an rms precision of about:

$$\sigma_x(\tau) = 5\ \mathrm{ns} \cdot \tau^{1/3}\,\mathrm{s}^{-1/3} \tag{19}$$

where τ has at least the range of 86400 s to about 2×10^7 s (1 day to 224 days). The clocks are assumed to have been synchronized previously. The TV color subcarrier may be used as a frequency reference with a precision capability of about:

$$\sigma_y(\tau) = 3.5 \times 10^{-10}\, \tau^{-2/3}\, s^{2/3} \tag{20}$$

where τ has at least the range of values from $12\,s \leq \tau \leq 384$ s.

The long-term fractional frequency stabilities of Loran-C and of the three network TV line-10 system as received in Boulder, Colorado, are comparable at a level of about:

$$\sigma_y(\tau) = 2 \times 10^{-12} \cdot \tau^{-2/3}\, \mathrm{day}^{2/3} \tag{21}$$

where τ has at least the range from $1\ \mathrm{day} \leq \tau \leq 224$ days. Both systems provided precision capabilities of a few parts in 10^{14} for sample times of one-half year and longer and with rms time dispersions of about 1 μs for $\tau = 1$ year.

Acknowledgements. We wish to thank Dr. James A. Barnes, Dr. Donald Halford, Mr. James E. Gray, and Mr. Gunther Kramer for the helpful criticisms and comments of this paper. The assistance of Mrs. Glynetta Perrymore, Dr. Bernard Wieder, and Mr. James Barnaba in the data-taking and processing and of Mrs. Eddyce Helfrich in manuscript preparation has been invaluable.

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