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Research Notes

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Nonlinear Diffusion with Recombination in an Electron Beam Excited Plasma

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The radial distributions of electron density and light intensity are calculated from the nonlinear diffusion equations with a recombination term added. Two-body and three-body recombination mechanisms are considered in addition to the intermediate case of collisional-radiative recombination.

THE complete nonlinear equations describing diffusion in an isothermal plasma[1,2] have been modified to describe the conditions of an electron beam excited (brush-cathode) plasma[3] by substituting a source term which is independent of both electron density and position within the tube. A recombination term has been added and the equations have been solved in cylindrical geometry for zero magnetic field. The recombination coefficient $\alpha\ (n)$ is introduced into the equations in two-body form but without any basic commitment to a two-body recombination mechanism since arbitrary dependence on the electron density n is allowed. The modified equations are

$$\frac{d\kappa}{dy} = \left\{A\kappa^2 + \frac{1+\kappa^2}{\eta_0^2}\frac{\kappa}{W} \cdot \left[1 - \left(\frac{W}{\kappa}\right)^2 d(n)\right] - \beta\frac{\kappa}{y}\right\}(1-\kappa^2)^{-1}, \quad (1)$$

$$\frac{dW}{dy} = -\beta\left(\frac{W}{y}\right) + \eta_0^{-2}\left[1 - \left(\frac{W}{\kappa}\right)^2 d(n)\right], \quad (2)$$

$$d(n) = d(n_0)\left[\frac{\alpha(n_0\Phi)}{n_0\Phi}\right]\left[\frac{n_0}{\alpha(n_0)}\right]\Phi, \quad (3)$$

$$d(n_0) = \frac{R}{1+R}, \quad (4)$$

where

$$A = \left[\frac{M}{M+m}\right]\frac{\nu_M}{\nu_m} + \left[\frac{m}{M+m}\right]\frac{\nu_m}{\nu_M + \nu_m}, \quad (5)$$

$$R \equiv \frac{\alpha(n_0)n_0\Lambda^2}{D}, \quad (6)$$

and

$$\eta_0^2 = \frac{n_0\nu_m}{s}. \quad (7)$$

The definitions of the symbols are: $\kappa = u/v$ is the ambipolar drift velocity normalized to the ambipolar thermal velocity; $y = \nu_m r/v$, a dimensionless variable proportional to position within the plasma; $\Phi = \rho/\rho_0$, the mass density of the ionized component of the plasma normalized to its value at the center of the plasma; $W = \Phi\,\kappa$, a dimensionless variable proportional to the mass flow of the ionized component; n_0, the electron density at the center of the plasma; ν_m, the electron–neutral collision frequency; ν_M, the ion–neutral collision frequency; m and M, the masses of electron and ion, respectively; s, the source frequency, a constant over the volume; and β, a configuration parameter equal to 0 or 1 for plane parallel and cylindrical geometry, respectively.

The equations were solved, by a Runge–Kutta method, for the variables κ and $\Phi = W/\kappa$, which were then plotted as functions of y/y_{max}, where $y/y_{max} = 1$ at the boundary of the plasma (cf. Refs. 1 and 2). The function Φ is the relative radial electron density. The relative light intensity I is calculated from the equation

$$I \propto \alpha(n)n^2 \propto \left[\frac{d(n)}{d(n_0)}\right]\Phi^2. \quad (8)$$

Solutions were obtained for the cases of two-body and three-body recombination mechanisms as well as the intermediate case of collisional–radiative recombination. Values for the collisional–radiative recombination coefficient were taken from Table 2A of Bates, Kingston, and McWhirter[4] and were

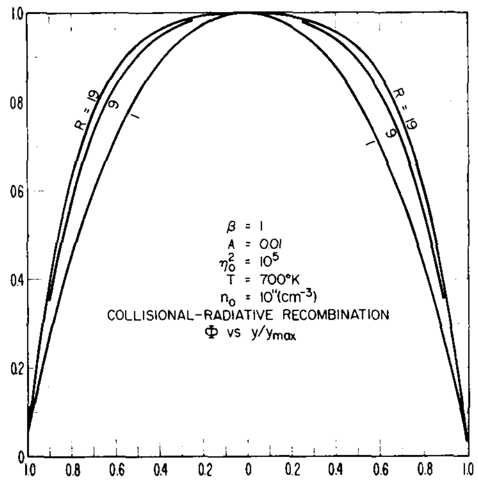

FIG. 1. Curves of relative electron density as a function of radius within the tube for collisional–radiative recombination at 700°K.

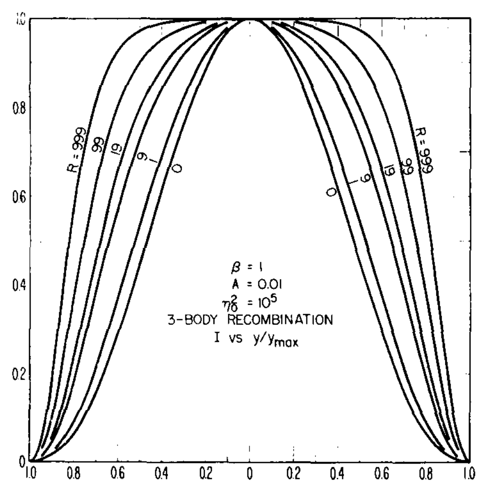

FIG. 3. Curves of relative luminous intensity as a function of radius within the tube for three-body recombination.

interpolated by a four point Lagrangian method applied to the logarithms.

Solutions of these equations for cylindrical geometry with axial symmetry are given in Figs. 1–3. There was a noticeable difference between the light intensity distributions for two- and three-body recombination but these differences were much smaller than those produced by changes in the ratio R of the recombination loss rate to the diffusion loss rate at the center of the plasma [Eq. (6)]. The light distributions for collisional–radiative recombination at 700°K were indistinguishable from those for three-body recombination. Changing the parameter η_0^2 from 10^5 to 10^6 produced only a small change in the light distributions, shifting the $I = 0.5$ points of the curves slightly toward the center. Note that the electron density at the boundary does not vanish in Fig. 1. The departure from the linear diffusion theory near the wall is greater for the brush-cathode plasma than for the normal positive column due to the fact that the source function remains high near the wall.

The values of the average relative electron density and light intensity were also calculated as given by the integrals

$$\int_0^1 \Phi \, d(y/y_{\max}) \quad \text{and} \quad \int_0^1 I \, d(y/y_{\max}).$$

In Fig. 4 the reciprocal of the Φ-integral is plotted as a function of the ratio R. Curves of this reciprocal versus R for two-body and three-body recombination are totally indistinguishable from each other. Corresponding curves for the I-integral show a noticeable but small ($\lesssim 15\%$) dependence on the particular recombination mechanism used.

It is expected that comparison of these theoretical radial distributions with experimentally determined

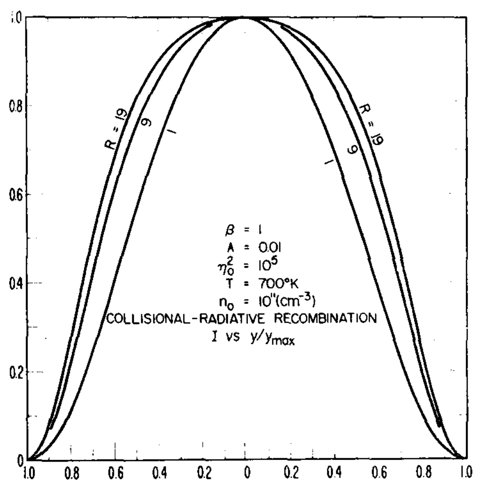

FIG. 2. Curves of relative luminous intensity as a function of radius within the tube for collisional–radiative recombination at 700°K.

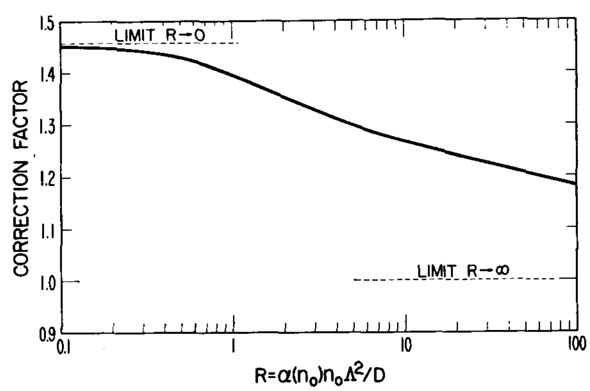

FIG. 4. Value of the factor, $1/\int \Phi \, d(y/y_{\max})$, as a function of the ratio R, of recombination loss rate to diffusion loss rate at the center of the plasma.

ones will prove useful in estimating the ratio R for laboratory plasmas of this type.

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