Motional Squeezing for Trapped Ion Transport and Separation

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Transport, separation, and merging of trapped ion crystals are essential operations for most large-scale quantum computing architectures. In this Letter, we develop a theoretical framework that describes the dynamics of ions in time-varying potentials with a motional squeeze operator, followed by a motional displacement operator. Using this framework, we develop a new, general protocol for trapped ion transport, separation, and merging. We show that motional squeezing can prepare an ion wave packet to enable transfer from the ground state of one trapping potential to another. The framework and protocol are applicable if the potential is harmonic over the extent of the ion wave packets at all times. As illustrations, we discuss two specific operations: changing the strength of the confining potential for a single ion and separating same-species ions with their mutual Coulomb force. Both of these operations are, ideally, free of residual motional excitation.

A suitable platform for quantum information processing must enable the precise control of many-body quantum states [1,2]. Trapped ions are promising in this regard due to their long coherence times, high-fidelity gate operations, and potential for all-to-all connectivity between qubits [3–11]. One way to address the challenge of scaling to larger trapped-ion systems is the so-called quantum charge-coupled device (QCCD). In the QCCD architecture, ions are shuttled between different trap "zones" that can have designated functions, such as gate operations, memory, or readout [5,12,13]. To be as efficient as possible, separation and transport of same- and mixed-ion species should be fast and minimize residual motional excitation. While theoretical work has explored various shuttling protocols [14–20], only single-ion and same-species ion transport have been demonstrated on timescales comparable to the ion's motional period; experimentalists have performed other operations, but only adiabatically with respect to the motional period [19,21,22].

In this Letter, we develop a new theoretical framework to analyze the motional states of ions in a linear trap with time-varying potentials. Specifically, we consider the case of ions starting and ending in the ground states of a set of harmonic wells with frequencies and equilibrium positions $\{\omega_j(0), c_j(0)\}$ to $\{\omega_j(t_f), c_j(t_f)\}$ over duration t_f, where j indicates the motional mode (see Fig. 1). This framework can be applied if, at all times, the effective potential can be approximated as quadratic over the spatial extent of the ion motional wave packets. Under this condition, the wave packets remain Gaussian and follow classical trajectories [23–26]. This fact allows us to define a transformation into

a frame of reference that follows the "classical" position and momentum of the ions. In this "classical frame of reference," the Hamiltonian can be cast in a basis that is represented by generators of the SU(1,1) Lie algebra [27–31], allowing us to reduce the remaining dynamics to Euler rotations [28,32,33]. Once the angles of these rotations are determined, the effect of the entire operation is equivalent to a squeezing operation, followed by a coherent displacement; this operator parametrization of Gaussian trajectories has been used in other contexts [26,34], but its use in QCCD operations has not been explored. Further, we can add an additional squeezing operation per mode, before or after the main dynamical
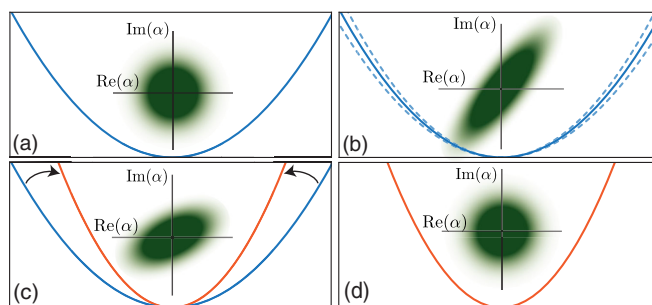

FIG. 1. We illustrate a change of the potential of a harmonic trap, showing the motion's Wigner distribution $W(\alpha)$ [35] for each step (not to scale). We (a) initialize the motion to the ground state of a potential well, (b) squeeze the motion (shown as parametric modulation of the potential), and (c) increase the confinement strength. Finally, (d) the motion finishes in the ground state of the final potential.

operation, so that the system finishes in the ground states of a set of final trapping potentials with arbitrary frequencies $\omega_j(t_f)$. Taken together, one can transport ions from the ground states of one set of wells to the ground states of another, with no motional excitation. We present two applications of this technique: changing the frequency of an ion's motion and separating two same-species ions. Using experimentally realistic parameters, we show that the latter could potentially be conducted more than an order of magnitude faster than in previous demonstrations.

We consider N ions in a linear trap, described by "lab-frame" motional wave function $|\psi(t)\rangle$ and Hamiltonian

$$\hat{H}_l(t) = \sum_j \frac{\hat{p}_j^2}{2m_j} + V_j(\hat{x}_j, t), \tag{1}$$

where $\hat{p}_j$ is momentum, $\hat{x}_j$ is position, m_j is mass, and t is time. The index j can here indicate the coordinates of an individual ion or a collective mode, depending on the problem. We assume negligible coupling between different degrees of freedom; this approximation means that for systems with more than one ion we can factor $|\psi(t)\rangle$. Therefore, each degree of freedom j can be considered separately, and we drop this subscript unless stated otherwise. We can now transform $\hat{H}_l(t)$ into a frame of reference that is centered around the expectation values of the position $c(t)$ and momentum $m\dot{c}(t)$. This unitary transformation is represented by the displacement operator $\hat{D}(t) \equiv \exp\{[i/\hbar][m\dot{c}(t)\hat{x} - c(t)\hat{p}]\}$. This gives [36] $\hat{D}^\dagger(t)\hat{x}\hat{D}(t) = \hat{x} + c(t)$ and $\hat{D}^\dagger(t)\hat{p}\hat{D}(t) = \hat{p} + m\dot{c}(t)$. We consider the transformed wave function $|\phi(t)\rangle = \hat{D}^\dagger(t)|\psi(t)\rangle$ and Hamiltonian $\hat{H}_t(t) = \hat{D}^\dagger(t)\hat{H}_l(t)\hat{D}(t) + i\hbar\dot{\hat{D}}^\dagger(t)\hat{D}(t)$, which gives [36–38]

$$\hat{H}_t(t) = \frac{\hat{p}^2}{2m} + V(\hat{x}, t) - \hat{x}\frac{\partial V}{\partial \hat{x}}\bigg|_{\hat{x}=0}. \tag{2}$$

If we assume the potential $V(\hat{x}, t)$ is quadratic around $c(t)$, we can write the above equation as

$$\hat{H}_t(t) = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2(t)\hat{x}^2. \tag{3}$$

Here, we have set $m\omega^2(t) \equiv \partial^2 V/\partial\hat{x}^2|_{\hat{x}=0}$. Notice that Eq. (3) does not have a $\propto \hat{x}$ force term, as its effect is now encompassed by $\hat{D}(t)$. This gives a free harmonic oscillator with a time-dependent potential centered at $\langle\hat{x}\rangle = 0$.

We are interested in modes that begin in a $\omega(0) \equiv \omega_0$ potential well. We set $\omega(t) \equiv [1 + \gamma(t)]\omega_0$, where $\gamma(t)$ is a dimensionless time-dependent function. We rewrite Eq. (3) in terms of ladder operators $\hat{a}(\hat{a}^\dagger)$ acting on the mode defined by $\omega(0)$. Doing this, and ignoring global phases, gives [37]

$$\hat{H}_t(t) = \hbar\omega_0\left\{\hat{a}^\dagger\hat{a} + \frac{\gamma(t)}{2}\left[1 + \frac{\gamma(t)}{2}\right][\hat{a}^\dagger + \hat{a}]^2\right\}$$
$$= 2\hbar\omega_0\{[1 + \alpha(t)]\hat{K}_3 + \alpha(t)\hat{K}_1\}, \tag{4}$$

where we have substituted $\hat{x} \equiv (\hbar/2m\omega_0)^{1/2}(\hat{a}^\dagger + \hat{a})$, $\hat{p} \equiv i(\hbar m\omega_0/2)^{1/2}(\hat{a}^\dagger - \hat{a})$, and $\alpha(t) \equiv \gamma(t)[1 + \frac{1}{2}\gamma(t)]$. Importantly, Eq. (4) introduces the generators of the SU (1,1) Lie algebra [27,28,39]

$$\hat{K}_1 \equiv \frac{1}{4}(\hat{a}^{\dagger 2} + \hat{a}^2), \qquad \hat{K}_2 \equiv \frac{1}{4i}(\hat{a}^{\dagger 2} - \hat{a}^2),$$
$$\hat{K}_3 \equiv \frac{1}{2}\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right). \tag{5}$$

Because Eq. (4) depends only on these generators, we may represent the propagator associated with $\hat{H}_t(t)$ as $\hat{U}_s$ with three Euler rotations in SU(1,1) space [28,32],

$$\hat{U}_s = e^{i\theta_a\hat{K}_3}e^{2ir_s\hat{K}_2}e^{i\theta_b\hat{K}_3}, \tag{6}$$

where we have defined the angle r_s such that its value corresponds to the squeeze parameter [35]. We designate the position and velocity coordinates of the final potential minimum (in the lab frame) as c_f and $\dot{c}_f$, respectively, distinct from the ion coordinates $c(t_f)$ and $\dot{c}(t_f)$, to encompass residual displacements in our framework; when $c(t_f) = c_f$, the packet is centered at the final potential minimum, and, when $\dot{c}(t_f) = \dot{c}_f$, the ion is stationary with respect to it. Transforming into the reference frame centered at, and stationary with respect to, the minimum of the final potential after duration t_f gives $|\phi_f(t_f)\rangle = \hat{D}'_f\hat{U}_s|\phi(0)\rangle$, where the final frame change is represented by the displacement operator $\hat{D}'_f \equiv \exp(\{i/\hbar\}\{m[\dot{c}(t_f) - \dot{c}_f]\hat{x} - [c(t_f) - c_f]\hat{p}\})$ (see Supplemental Material [37]). Under the approximation that $V(\hat{x}, t)$ is quadratic over the spatial the extent of $|\phi(t)\rangle$ $\forall$ t, we can thus represent ion motional dynamics with a squeeze, followed by a displacement, operator. While there is a broad set of transport, separation, and mode frequency change operations this framework could analyze, for this Letter we consider only the subset of operations where $c(t_f) = c_f$ and $\dot{c}(t_f) = \dot{c}_f = 0$, from which it follows that $\hat{D}'_f = \hat{I}$; this framework could, however, straightforwardly describe protocols where ions are caught by moving potentials ($\dot{c}_f \neq 0$) or are displaced at t_f ($\hat{D}'_f \neq \hat{I}$).

As an example, we can study the case of taking the system from an initial Fock state of a well with frequency and coordinate $\{\omega(0), c(0)\}$ to the same Fock state of $\{\omega(t_f), c_f\}$. Finding experimentally realistic functions that efficiently take the system from $c(0)$ to c_f while *simultaneously* taking the system from the ground state of $\omega(0)$ to the ground state of $\omega(t_f)$ is a difficult task in general; we

can, however, guarantee the latter requirement by introducing an additional step, either before or after the transport, separation, or frequency change operation, that squeezes the motion so it ends in the ground state of the $\omega(t_f)$ potential well. This works so long as $c(t_f) = c_f$ and $\dot{c}(t_f) = \dot{c}_f$. The operator that describes changing the mode from the ground state of $\omega(0) = \omega_0$ to that of $\omega(t_f) = [1 + \gamma(t_f)]\omega_0$ is [37]

$$\hat{U}_c = e^{2ir_c\hat{K}_2}, \qquad r_c \equiv -\frac{1}{2}\ln[1 + \gamma(t_f)]. \qquad (7)$$

We want to find a squeezing operation $\hat{U}_p$ that, when applied before or after the main dynamical operation, gives the desired mode frequency change $\hat{U}_c$. We can express this as $\hat{U}_s\hat{U}_p = \hat{U}_c$ or $\hat{U}_p\hat{U}_s = \hat{U}_c$, depending on whether squeezing is applied before or after the main dynamical operation, respectively. We can decompose $\hat{U}_p$ into Euler rotations as

$$\hat{U}_p = e^{i\theta_k\hat{K}_3}e^{2ir_p\hat{K}_2}e^{i\theta_{k'}\hat{K}_3}. \qquad (8)$$

Assuming we can generate a squeezing operator of the form $\hat{S}_{k,k'} = \exp[(r_p/2)(\hat{a}^{\dagger 2}e^{i\theta_{k,k'}} - \hat{a}^2 e^{-i\theta_{k,k'}})]$, where the index $k(k')$ indicates whether the squeeze operation is rendered before (after) $\hat{U}_s$, we find that

$$\hat{U}_s\hat{S}_k = \hat{U}_c e^{-i(\theta_k + \theta_{k'})\hat{K}_3},$$
$$\hat{S}_{k'}\hat{U}_s = e^{-i(\theta_k + \theta_{k'})\hat{K}_3}\hat{U}_c. \qquad (9)$$

In the Fock basis $\{|n\rangle\}$, the operator $e^{-i(\theta_k + \theta_{k'})\hat{K}_3}$ just gives an n-dependent global phase, as $\hat{K}_3|n\rangle = \frac{1}{2}(n + \frac{1}{2})|n\rangle$. Thus $\hat{S}_{k,k'}$ effectively realizes $\hat{U}_p$, provided that the initial motional state can be described by a diagonal density matrix in the Fock basis, a requirement that is fulfilled by both pure Fock states and thermal states.

Squeezing can be induced with parametric modulation or a diabatic change of the trapping potential using applied voltages [40–42] or with lasers [43–46]. The focus of this Letter is not how squeezing is generated, and, importantly, the validity of the above scheme does not depend on the timescales of the squeezing generation. We, therefore, assume a general squeezing Hamiltonian $\hat{H}_I$ considered in the interaction picture of $\hbar\omega_0\hat{a}^\dagger\hat{a}$,

$$\hat{H}_I = i\hbar\frac{g}{2}(\hat{a}^{\dagger 2}e^{i\theta_I} - \hat{a}^2 e^{-i\theta_I}), \qquad (10)$$

which describes the squeezing induced by frequency modulation of the trap frequency at $2\omega_0$ (see Fig. 1). After a duration t_p, the propagator representing the lab-frame action of $\hat{H}_I$ is, up to a phase,

$$\hat{U}_I|n\rangle = \exp\left[\frac{gt_p}{2}(\hat{a}^{\dagger 2}e^{i(\theta_I - 2\omega t_p)} - \hat{a}^2 e^{-i(\theta_I - 2\omega t_p)})\right]|n\rangle, \qquad (11)$$

which is equivalent to $\hat{U}_p$ when $gt_p = r_p$ and $\theta_I = 2\omega_0 t_p + \theta_k$; this means that squeezing the ground state of $\{\omega(0), c(0)\}$ can prepare it to end in that of $\{\omega(t_f), c_f\}$ after a change of the external potential. We provide two examples of how this technique may be used below.

We first discuss how to use squeezing to change the motional frequency of an ion in a harmonic potential without residual motional excitation. We choose this as an initial example because it is a simple illustration of how squeezing can not only transform a wave packet from the ground state of one well to another, but also account for the change of the external potential, where $\hat{H}_l(0) \to \hat{H}_l(t_f)$ over a finite duration. Figure 1 pictorially illustrates this scheme. In this example, we increase the frequency of a mode, squeezed beforehand according to $\hat{U}_p$, following the function $\gamma(t) = \sin^2(\pi t/2t_f)$, in which the trap frequency is doubled from ω_0 to $2\omega_0$ over a time period t_f. The functional form of $\gamma(t)$ can be chosen arbitrarily; different functional choices would not qualitatively affect the results, so long as boundary conditions remain satisfied. Here, the classical position of the particle remains at rest, trivially meeting the requirement that $c(t_f) = c_f$ and $\dot{c}(t_f) = \dot{c}_f$. Figure 2 shows the phonons $\langle\hat{n}_\omega\rangle$ (as defined by the ladder operators of the initial $\omega_0/2\pi = 1$ MHz and final $\omega(t_f)/2\pi = 2$ MHz trapping frequencies [37]) versus time for a ramp time of $t_f = 0.5$ μs. Here we see that the motion is not in the ground state of either basis after the initial squeezing operation; after doubling the frequency, however, the final well is.

The inset of Fig. 2 shows the residual phonons in the $2\omega_0$ mode versus t_f *without* a squeezing step $\hat{U}_p$. It is interesting
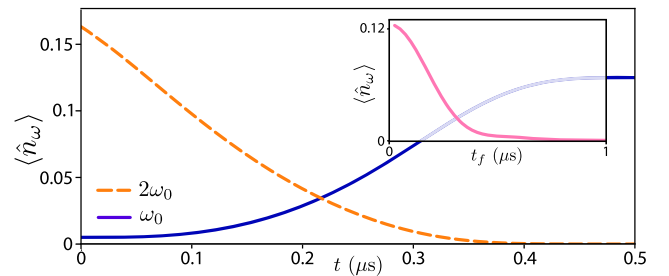

FIG. 2. Changing the frequency from ω_0 to $2\omega_0$ ($\omega_0/2\pi = 1$ MHz) of a harmonic trap after a squeezing operation. We show the phonon number $\langle\hat{n}_\omega\rangle$ in the eigenbases of the original mode at ω_0 (solid blue) and the final mode at $2\omega_0$ (dashed orange). The inset shows $\langle\hat{n}_\omega\rangle$ versus t_f in the eigenbasis of the final well when no squeezing is performed; if squeezing were performed, $\langle\hat{n}_\omega\rangle$ would remain zero for all t_f.

to note that, when $t_f \to 0$, the number of residual phonons approaches that of the ω_0 mode in the main figure. This is because, in this regime, $\hat{U}_s \to \hat{I}$; this means the $\hat{U}_p$ operation, that *would* transform the wave function from the ground state of ω_0 to that of $2\omega_0$, converges to $\hat{U}_c$. Since experimental techniques for squeezing typically do not operate in time frames shorter than $2\pi/\omega_0$, the inset indicates that our squeezing scheme is unlikely to be more time efficient than just adiabatically changing the potential, so this offers only a simple example of the protocol. Ion separation, on the other hand, could potentially be expedited with squeezing.

We now discuss a protocol that uses motional squeezing to diabatically separate two same-species ions. We begin with two ions, here taken to be $^9\mathrm{Be}^+$, in an initial potential well with frequency ω_0 at the equilibrium positions of ions 1 and 2 at $\sqrt[3]{ke^2/(4m\omega_0^2)}$ and $-\sqrt[3]{ke^2/(4m\omega_0^2)}$, respectively, where k is the electrostatic constant and e is charge. We can then take the usual coordinate system $\hat{x}_c = \frac{1}{2}(\hat{x}_1 + \hat{x}_2)$ and $\hat{p}_c = \hat{p}_1 + \hat{p}_2$ for the in-phase mode, labeled "c.m.," mode, $\hat{x}_s = \frac{1}{2}(\hat{x}_1 - \hat{x}_2)$ and $\hat{p}_s = \hat{p}_1 - \hat{p}_2$ for the out-of-phase mode, labeled "STR," mode, each with effective mass $M = 2m$. This allows us to analyze the system in an uncoupled basis, $\hat{H}_t(t) = \hat{H}_{t,c}(t) + \hat{H}_{t,s}(t)$. Dropping terms $\propto \hat{I}$, we get

$$\hat{H}_{t,c}(t) = \frac{\hat{p}_c^2}{2M} + \frac{1}{2}M\omega_0^2[1 + \gamma(t)]^2\hat{x}_c^2, \qquad (12)$$

$$\hat{H}_{t,s}(t) = \frac{\hat{p}_s^2}{2M} + \frac{1}{2}M\omega_0^2[1 + \gamma_s(t)]^2\hat{x}_s^2. \qquad (13)$$

We have Taylor expanded the Coulomb potential up to $\hat{x}_s^2$, encompassing this term's dynamics in [37],

$$\gamma_s(t) \equiv \left([1 + \gamma(t)]^2 + \frac{2c_s^3(0)}{c_s^3(t)}\right)^{1/2} - 1, \qquad (14)$$

in order to cast $\hat{H}_{t,c}(t)$ and $\hat{H}_{t,s}(t)$ in the same form. Initially, the STR mode frequency is $\omega_s(0) = \sqrt{3}\omega_0$, so we measure the phonon number $\langle \hat{n}_\omega \rangle$ as defined by these ladder operators, while measuring the c.m. modes in terms of $\omega_c(0) = \omega_0$ operators. When separating into different trap zones, however, the $\propto c_s^{-3}(t)$ Coulomb term in $\gamma_s(t)$ becomes negligibly small, giving $\omega_s(t_f) \simeq \omega_0$; this makes $\langle \hat{n}_\omega \rangle = 0$, defined by $\omega_s(t_f) \simeq \omega_c(t_f) = \omega_0$ ladder operators, the target for both modes.

The separation of the classical trajectory from SU(1,1) dynamics allows us to isolate each when designing a protocol. Therefore, we discuss individual positions for the former and modes for the latter. We first choose a protocol that separates the particles from $\{c_1(0), c_2(0)\}$ to a desired $\{c_{f,1}(t_f), c_{f,2}(t_f)\}$, such that the particles finish in equilibrium. After the parametric modulation

sequence lasting t_p, we release the ions from confinement, ramping the potential to zero over a duration $t_{s,1}$ according to $\gamma(t) = -\sin^2[\pi(t - t_p)/2t_{s,1}]$. Subsequently, for a duration $t_{s,2}$ we leave the particles unconfined [$\gamma(t) = -1$], after which we apply separate catching potentials over duration $t_{s,3}$ according to $\gamma(t) = -\cos^2\{\pi[t - (t_p + t_{s,1} + t_{s,2})]/2t_{s,3}\}$; $\gamma(t) = 0$ everywhere else. We set the centers of the catching potentials to be $c_{f,j}(t) = c_j(t) - \eta\dot{c}_j(t)$, where η is a constant with dimensions of time. This ensures that $c_j(t_f) \simeq c_{f,j}(t_f)$ and $\dot{c}_j(t_f) \simeq \dot{c}_{f,j}(t_f) \simeq \ddot{c}_j(t_f) \simeq \ddot{c}_{f,j}(t_f) \simeq 0$, whereby $\hat{D}'_{f,j} \to \hat{I}$. Figure 3(a) shows the classical trajectories of two ions being separated by $c_1(t_f) - c_2(t_f) \simeq 100~\mu\mathrm{m}$ in $t_{s,1} + t_{s,2} + t_{s,3} \simeq 2.17~\mu\mathrm{s}$, not including the squeezing period. Here we have set $\omega_0/2\pi = 1$ MHz, $t_p = 3~\mu\mathrm{s}$, $t_{s,1} = 0.5~\mu\mathrm{s}$, $t_{s,2} \simeq 0.67~\mu\mathrm{s}$, $t_{s,3} = 1~\mu\mathrm{s}$, and $\eta = 0.5~\mu\mathrm{s}$—the values of g for both modes are determined after the values of r_p are calculated (discussed below). The ions remain at $c_j(0)$ during the squeezing stage, then quickly separate when their initial confinement is dropped,
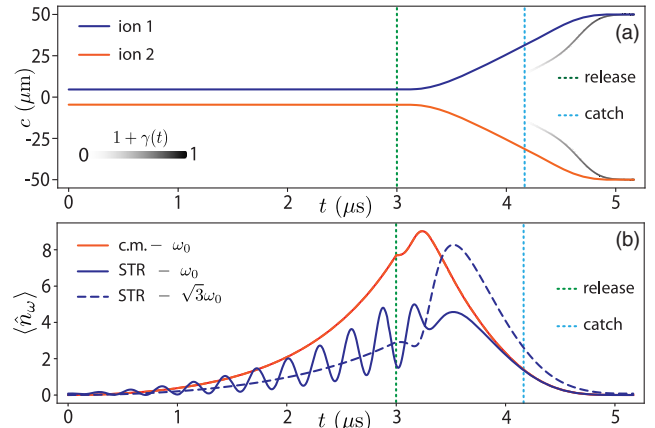

FIG. 3. Illustration of same-species separation using squeezing. Both figures are for the same run, using parametric modulation with amplitudes of $g_c/2\pi \simeq 92.6$ and $g_s/2\pi \simeq 69.2$ kHz acting simultaneously on the c.m. and STR modes, respectively. Here $\omega_0/2\pi = 1$ MHz. This example protocol is composed of a $t_p = 3~\mu\mathrm{s}$ parametric modulation step, followed by $t_{s,1} = 0.5~\mu\mathrm{s}$ of ramping down the original confining potential to $\omega(t_p + t_{s,1}) = 0$, $t_{s,2} \simeq 0.67~\mu\mathrm{s}$ of only Coulomb repulsion, followed by $t_{s,3} = 1~\mu\mathrm{s}$ of ramping up the "catching" potential. This gives a total separation of $c_1(t_f) - c_2(t_f) \simeq 100~\mu\mathrm{m}$ over $t_f \simeq 5.17~\mu\mathrm{s}$. (a) Shows the respective position of ions 1 and 2 versus time t in microseconds. The green and blue dotted lines mark the beginnings of the release and catch steps, respectively. The black lines show the position of the catching potential well versus t and the lines' shade is proportional to the fraction of its final strength, $1 + \gamma(t)$. (b) Shows the phonons in the c.m. mode and both of the relevant STR modes versus time. At the end of the trajectory, both ions are in the ground state of their spatial separated $\omega(t_f) = \omega_0$ potential wells, while the $\sqrt{3}\omega_0$ mode is not.

coming to rest after their catching potentials reach their full value at t_f.

The design of a scheme where both ions come to rest at their respective potential minimums requires many adjustable parameters. The squeezing needed to prepare each mode for separation, however, is virtually identical to that in our discussion of changing the mode frequency, only with different $\gamma(t)$. Here, we squeeze both modes simultaneously for a fixed t_p, making the required values of g_c and g_s different. For the values of $r_{p,c}$ and $r_{p,s}$ in the example shown in Fig. 3, we find that $g_c/2\pi \simeq 93$ and $g_s/2\pi \simeq 69$ kHz. These values of $g_{s,c}$ were chosen to correspond to current state-of-the-art experiments [41,47], but are not necessary for this scheme to work; the use of stronger or weaker squeezing $g_{s,c}$ would simply cause t_p to scale as $1/g_{s,c}$. For this calculation, $\omega_c(0) = \omega_c(t_f)$, but $\omega_s(0) \neq \omega_s(t_f)$. To finish in the ground state of $\omega_s(t_f) = \omega_0$, we incorporate this change of frequency into $\hat{U}_{p,s}$, such that the wave packet changes from the ground state of $\sqrt{3}\omega_0$ to that of ω_0. In Fig. 3(b), we show $\langle \hat{n}_\omega \rangle$ for the modes defined by $\omega_c(0) = \omega_c(t_f)$, $\omega_s(0)$, and $\omega_s(t_f)$ versus time for the same separation shown in Fig. 3(a). This shows that both modes end in the ground state of their final potentials. For this protocol, we see the largest squeezing parameter is $r_{p,c} \simeq 1.8$, which is experimentally feasible [41].

In conclusion, this Letter presents a new, general method for analyzing the behavior of ions in time-varying potentials and for designing improved ion transport, separation, and merging protocols by using motional squeezing. First, we show that, when the Hamiltonian of an ion or ions in a time-varying potential takes the form of Eq. (4), after a frame transformation that accounts for the classical trajectory of each ion, the remaining dynamics of the system can be described by three Euler rotations in SU(1,1) space. When acting on the ground state of a motional mode, we show that one can use a single squeezing operation per mode such that the wave packet finishes its trajectory in the ground state of the final potential. It is important to note that the frequency change and separation protocols shown above represent specific examples of a wide range of feasible transport, separation, or merging schemes described by Eq. (4). It is reasonable to expect that variants of these two examples would perhaps better suit a particular experimental setup, or that other types of transport, separation, or mode frequency change operations may be catalyzed by this concept. This work could, therefore, open many new options for designing schemes that use motional squeezing in QCCD operations.

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