A Simple Method for Calibrating GPS-Disciplined Clocks via Direct Comparison to a UTC(k) Time Scale

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Abstract

Global-Positioning-System-(GPS) disciplined clocks are referenced to the Coordinated Universal Time (UTC) scale at the United States Naval Observatory (USNO), known as UTC(USNO). The UTC(USNO) time scale is a very close approximation of UTC, the official world time scale. GPS-disciplined clocks therefore provide a very close approximation of UTC when they are properly calibrated to compensate for equipment and cable delays. This paper provides a simple method for calibrating GPS-disciplined clocks with respect to UTC so they can replicate UTC with uncertainties of < 10 ns. The method involves directly comparing the device under test to any UTC(k) time scale listed on the weekly Rapid UTC (UTCr) reports published by the Bureau International des Poids et Mesures (BIPM).

1. Introduction

GPS-disciplined clocks (GPSDCs) produce a one-pulse-per-second (pps) output that is referenced to UTC(USNO), the Coordinated Universal Time (UTC) scale operated by the United States Naval Observatory (USNO). In nearly all cases – unless the antenna cable is unusually long or if the antenna coordinates have a very large error – an uncalibrated GPS-disciplined clock should agree with UTC(USNO) to within less than 1 μs, with ~100 ns accuracy being a typical specification. However, even when short antenna cables are used and even when the antenna coordinates have been determined to within less than 1 m, getting the best available accuracy from a GPS-disciplined clock still requires measuring and calibrating all hardware delays. The antenna cable typically introduces the largest delay, but the GPS receiver and its associated electronics and firmware, as well as the antenna itself, will also introduce delays. Once these delays are determined, a delay constant can be keyed into the GPS-disciplined clock to correct the 1 pps output. Within the semantic framework of the *International Vocabulary of Metrology* [1], the determination of the delays and their respective uncertainties is accomplished by calibration, and the correction of the delay in the GPS-disciplined clock is accomplished by adjustment. After a GPS-disciplined clock has been both calibrated and adjusted, it can be utilized as a true UTC synchronization source.

Several methods of GPS delay calibrations are routinely practiced. Some methods involve measuring the cable, receiver, and antenna delays separately, and then appropriately accounting for all delays, but some parts of this process can be difficult. For example, methods for measuring cable delays are well established [2, 3], but measuring receiver delays might require the use of a GPS simulator [4, 5]. Antenna-delay measurements might be even more involved, requiring the use of a network analyzer and an anechoic chamber. For these reasons, it is common and usually preferable to calibrate a GPS-disciplined clock as a system that includes the receiver, antenna, and antenna cable, and obtaining a single delay value that accounts for the entire system. This is usually done by simultaneously comparing both the GPS-disciplined clock under test and a previously calibrated reference GPS-disciplined-clock system, each connected to antennas separated by a short distance, to the same clock [6]. The comparisons usually last for about three to 10 days, with one week being typical. The reference GPS-disciplined-clock system is assumed to be correct, and thus the average time difference between the two units is attributed to the GPS-disciplined clock under test.

This paper introduces a simple method that retains the advantage of calibrating GPS-disciplined clocks as a system, but that eliminates the need for a reference GPS-disciplined clock. This is advantageous because the reference GPS-disciplined clock contributes uncertainty to the measurement, and because in many cases the reference GPS-disciplined clock is a unit maintained elsewhere that must travel to the site of the calibration. The method described here consists of directly comparing the GPS-disciplined clock under test to any UTC(*k*) time scale listed on the Rapid UTC (UTCr) reports that are published weekly by the Bureau International des Poids et Mesures (BIPM), and then applying a UTCr correction to the measurement. The new method works because of the very close agreement between UTC, UTCr, UTC(USNO), and GPS time. It is viable to implement because the UTC(USNO) time scale provides the time reference for GPS, and because daily UTCr – UTC(USNO) results are freely available via the BIPM. For the purposes of this paper, the method is called the GUC method, an acronym for GPS/UTC calibration.

Section 2 provides a short discussion of the close relationship between UTC, UTCr, UTC(USNO), and the time broadcast by the GPS satellites. Section 3 explains the GUC method, including the application of UTCr data. Section 4 provides and discusses measurement results from several GPS-disciplined clocks calibrated with this method at the National Institute of Standards and Technology (NIST). Section 5 discusses the measurement uncertainty of the delay calibrations, and Section 6 provides a summary.

2. The Relationship Between UTC, UTCr, UTC(USNO), and the Time Broadcast by GPS

Time calibrations, as is the case with nearly all metrological calibrations, should be referenced and traceable to the International System of Units (SI). The base unit for time is the second (s), one of the seven base units of the SI. The ultimate reference for establishing traceability for time calibrations is Coordinated Universal Time (UTC), an atomic time scale that provides the world's best approximation of the SI second [7]. UTC is computed by the BIPM from a weighted average of data collected from local time scales, known as UTC(*k*), that are located at national metrology institutes or other facilities that have legal or technical timekeeping responsibilities.

The UTC(*k*) time scales produce signals that can serve as a reference for physical measurements, whereas UTC does not. Instead, UTC is defined by regularly publishing the time difference, UTC – UTC(*k*), for each institution that contributes to UTC. The official time differences are published monthly in the BIPM *Circular T*,

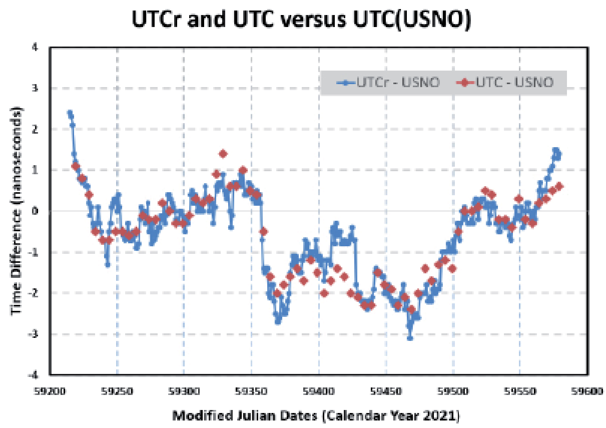

Figure 1. Time differences for UTC – UTC(USNO) and UTCr – UTC(USNO) for calendar year 2021.

which provides UTC – UTC(*k*) values at five-day intervals [8]. Since July 2013, the BIPM has also published weekly UTCr – UTC(*k*) time differences, with values provided at one-day intervals [9].

The UTC(USNO) time scale is the largest contributor to UTC, and as previously noted, provides a very close approximation of UTC. This is illustrated in Figure 1, which shows both the UTC – UTC(USNO) and UTCr – UTC(USNO) time differences for the calendar year 2021. The graph indicates UTC(USNO) did not depart from UTC by more than ±4 ns at any point during 2021, with the average UTC – UTC(USNO) and UTCr – UTC(USNO) time differences being -0.58 ns and -0.62 ns, respectively. The largest difference between the UTC and UTCr comparisons, for the days spaced at five-day intervals when both values were available, was 1.4 ns, with the average difference being -0.03 ns. This suggests that the results of UTCr, while non-official, agree closely with the results published later in the *Circular T* and are suitable for use for GPS-disciplined clock-delay calibrations, as described in Section 3.

The close approximation of UTC(USNO) to UTC is advantageous to GPS, because GPS broadcasts a prediction of UTC(USNO). Even though GPS has its own time scale, known as GPS time, the satellites broadcast UTC correction parameters in subframe 4, page 18, of the navigation message. Nearly all GPS-disciplined clocks apply these parameters and convert GPS time to a prediction of UTC(USNO). The UTC offset correction, Δt_{UTC}, is computed as [10]

$$\Delta t_{UTC} = \Delta t_{LS} + A_0 + A_1\left[t_E - t_{0t} + 604800\left(WN - WN_t\right)\right] \quad (1)$$

where

Δt_{LS} is the number of leap seconds introduced into UTC since the beginning of the GPS epoch (January 6, 1980),

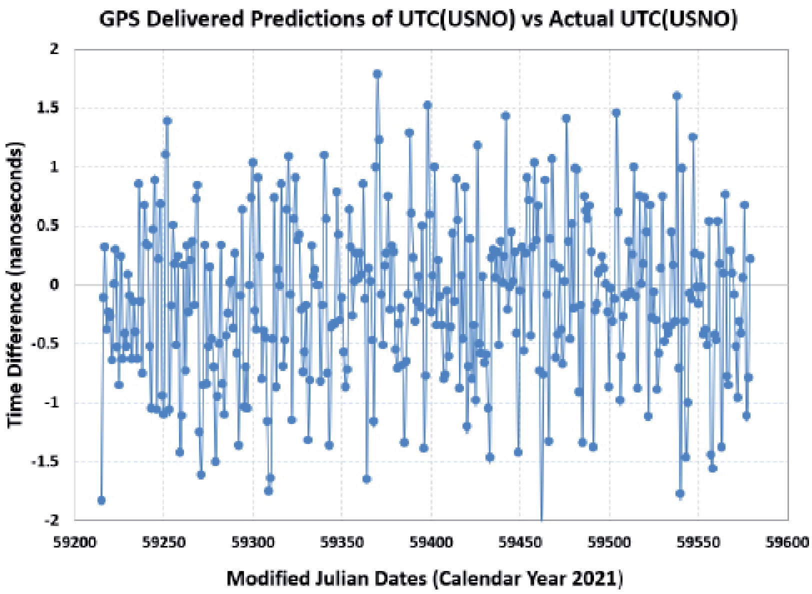

Figure 2. GPS delivered predictions of UTC(USNO) compared to actual UTC(USNO) for 2021.

A_0 is the constant UTC offset parameter expressed in seconds,

A_1 is a dimensionless frequency-offset value that allows the correction of the time error accumulated since the UTC reference time, t_{0t}, which is when A_0 was last determined,

t_E is GPS system time (the time to be converted to UTC(USNO)),

604800 is a constant that equals the number of seconds in one week,

t_{0t} is the reference time for UTC data,

WN is the GPS week number, and

WN_t is the UTC reference week number.

The Δt_{LS} term is the large, integer-second part of the correction, equal to the number of leap seconds that have occurred since the beginning of the GPS epoch (January 5, 1980). The A_0 term is the small, nanosecond part of the correction, equal to the difference between the GPS and UTC(USNO) second markers. It is broadcast in units of seconds, but is typically $< 1 \times 10^{-8}$ s, or < 10 ns in magnitude. The fine tuning of the UTC output of a GPS-disciplined clock is accomplished by applying a dimensionless frequency offset, provided by A_1, as a drift correction for the interval between the time specified by t_{0t} and WN_t and the current time. This is normally a sub-nanosecond correction, because A_0 is normally updated in the GPS broadcast more than once per day and the drift correction supplied by A_1 is typically near 1 ns per day.

The UTC(USNO) prediction broadcast by GPS is based upon an extrapolation of the observed difference

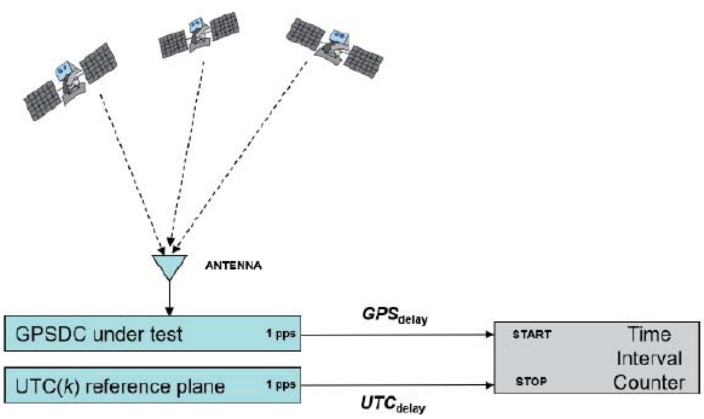

Figure 3. The GPSDC – UTC(k) time-difference measurement.

Table 1. The sample calibration of a GPS-disciplined clock at NIST using UTCr report 2152 (values are in nanoseconds).

MJD	UTCr – UTC(USNO)	UTCr – UTC(NIST)	GPSDC – UTC(k)	GPSDC delay bias, GPS_{db}
59575	1.3	1.3	85.5	85.5
59576	1.5	1.4	86.8	86.9
59577	1.5	1.4	87.1	87.2
59578	1.3	1.4	84.1	84.0
59579	1.4	1.6	84.5	84.3
59580	1.4	1.4	86.8	86.8
59581	1.5	1.6	85.3	85.2
Average delay (applied to GPSDC after calibration)				85.7

between GPS and UTC(USNO) from the start to the end of the previous day. Figure 2 shows the differences between the GPS-delivered predictions of UTC(USNO) and actual UTC(USNO) daily averages for calendar year 2021. The GPS – UTC(USNO) time differences fall within a ±2 ns range and have a mean value of -0.1 ns . The very close agreement between UTC(USNO) and the time broadcast by the GPS satellites provides the foundation for the simple GUC method described in Section 3.

3. The GUC Calibration Method

The GUC delay calibration method consists of comparing the 1 pps output of the GPS-disciplined clock under test to a 1 pps output of any UTC(k) time scale included on the weekly UTCr reports. Because the GPS-disciplined clock is being calibrated as a system (receiver, antenna, antenna cable), it should be measured with its delay compensation equal to 0, in other words, no previously measured or estimated delay values should be keyed into the unit. The comparisons are typically done with a time-interval counter (TIC), as shown in Figure 3.

If the two cables that connect the GPS-disciplined clock to the time-interval counter and the UTC(k) reference plane to the time-interval counter do not have equivalent delays, then the raw time-interval counter readings must be corrected. Assuming that the GPS-disciplined clock is connected to the start channel on the time-interval counter and UTC(k) is connected to the stop channel, as shown in Figure 3, each time-interval counter reading is corrected as

$$TIC_{corr} = TIC_{raw} - UTC_{delay} + GPS_{delay}, \tag{2}$$

where TIC_{raw} is the uncorrected reading from the time-interval counter, UTC_{delay} is the cable delay between the UTC(k) reference plane and the time-interval counter, and GPS_{delay} is the cable delay between the 1 pps output of the GPS-disciplined clock and the time-interval counter. The corrected time-interval counter readings (TIC_{corr}) are a measurement of GPS-disciplined clock – UTC(k). These readings can be recorded every second, but the data points used for the calibration should be one-day averages. Daily averages are required because UTCr only provides one value per day, and also have the benefit of attenuating diurnal variations caused by ionospheric or environmental factors. The daily averages should ideally be recorded for a period of at least seven days that corresponds to the period of a UTCr report. The last day of a UTCr report is always a Sunday and the reports are published on Wednesdays. This means that the GPS-disciplined clock delay calibration cannot be completed until a few days after the most recent measurement when the UTCr report becomes available.

The equation to compute the delay bias in a GPS-disciplined clock, GPS_{db}, for a given day is

$$GPS_{db} = \left[\left(UTCr - UTC_{USNO} \right) - \left(UTCr - UTC_k \right) \right] + \left(GPSDC - UTC_k \right), \tag{3}$$

where $\left(UTCr - UTC_{USNO} \right)$ and $\left(UTCr - UTC_k \right)$ are obtained from the UTCr report, and $\left(GPSDC - UTC_k \right)$ is obtained from the time-interval counter measurements.

Table 1 provides an example where a GPS-disciplined clock was measured at NIST in Boulder, Colorado. In this example, the UTC(NIST) time scale served as UTC(k), and correction data were obtained from UTCr report 2152 (published on January 5, 2022). A delay constant of 0 was used in the GPS-disciplined clock under test, meaning that all hardware delays, including antenna cable delays, were unaccounted for and ignored. The purpose of the calibration was to obtain a single delay value that could be keyed into the GPS-disciplined clock to compensate for its hardware delays, and therefore make the GPS-disciplined clock produce signals that agreed with UTC as closely as possible.

Note that in this example, the close agreement between the UTC(USNO) and UTC(NIST) time scales meant the corrections obtained from UTCr and applied to the

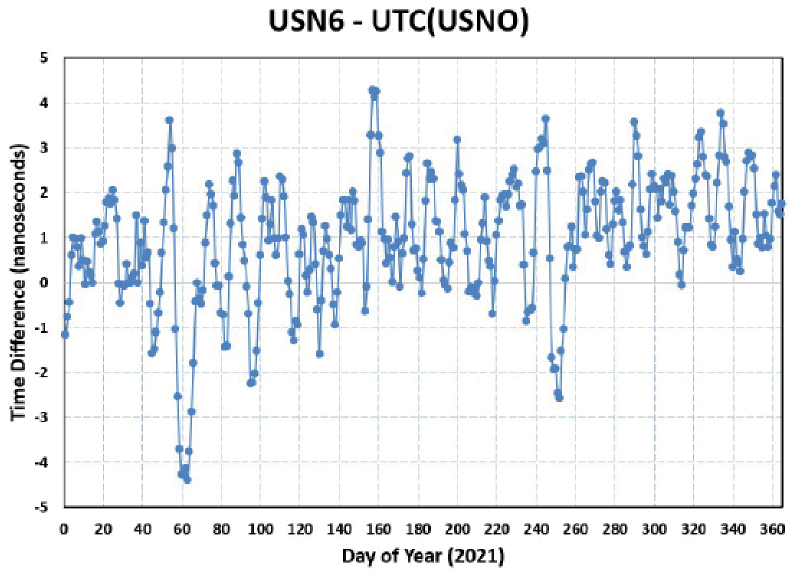

Figure 4. Time differences for the reference USN6 GPS receiver – UTC(USNO) for calendar year 2021.

calibration were sub-nanosecond. A direct comparison of the GPS-disciplined clock to UTC(NIST) would therefore have produced nearly the same results. For GUCs performed at other UTC(*k*) laboratories, the UTCr corrections would likely be larger. For example, if UTC(USNO) – UTC(*k*) as obtained from the UTCr report had an average time difference of 20 ns, then a GPS-disciplined clock compared to UTC(*k*) should also have an average time difference of 20 ns after it had been calibrated. The GUC method calibrates devices to agree with UTC and not with UTC(*k*), and the method is viable because the differences between UTC and UTC(*k*) are published.

4. Calibration Results

One way to test the validity of the GUC method is to use it to check for biases in reference GPS time-transfer receivers that have been previously calibrated with other more-established methods. The first, most-basic check, was done by comparing USN6, a reference GPS receiver at the USNO, to the UTC(USNO) time scale for the entire year of 2021 (Figure 4). The average time difference of this comparison should be near 0 (as previously indicated by the data shown in Figure 2) if the GPS device has been properly designed and calibrated. As expected, despite daily peak-to-peak variations that sometimes reached ±4 ns, the

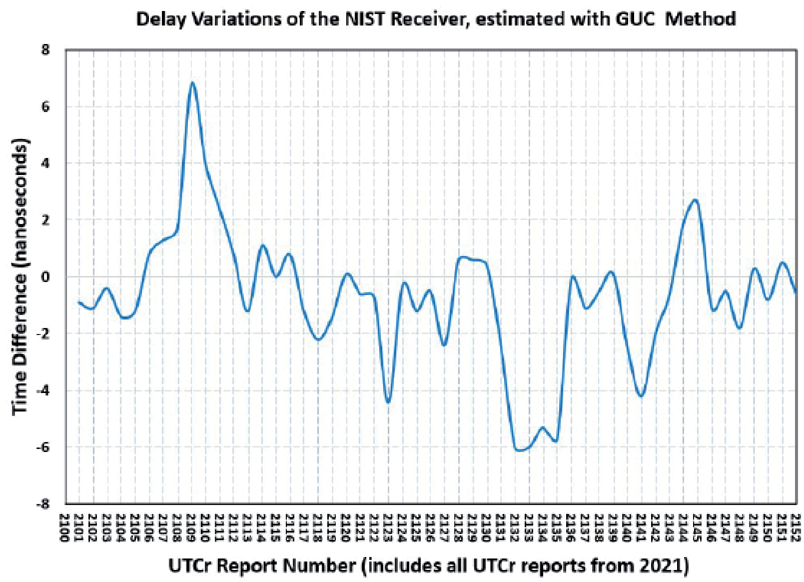

Figure 5. Post-calibration delay variations of the NIST receiver, estimated with the GUC method, for calendar year 2021.

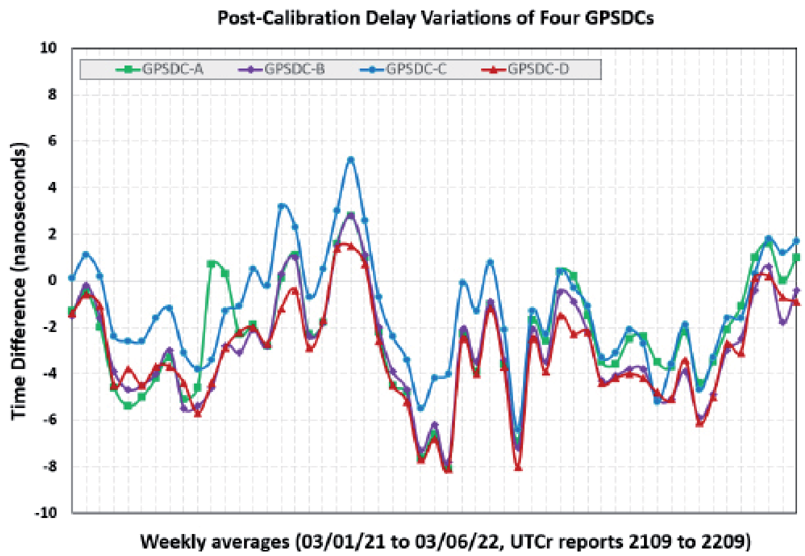

Figure 6. Post-calibration delay variations of four single-frequency (L1 band) GPS-disciplined clocks during a one-year interval.

average time difference for 2021 was just 1.0 ns, indicating only a small calibration bias.

A second test of the GUC method involved a comparison for the year 2021 involving a reference receiver at NIST, which was previously (and periodically) calibrated via the BIPM calibration program. Here, the GUC method was applied to estimate post-calibration delay biases as shown in Figure 5. Daily variations occasionally exceeded ±5 ns, but the average time difference for 2021 was 0.7 ns.

During the first two months of 2021, the GUC method was used to calibrate four low-cost single-frequency (L1 band) GPS-disciplined clocks at NIST. These calibrations were not all performed simultaneously – no more than two were in progress at one time – and each lasted for about two weeks (at least two full UTCr reports were used for each calibration). After these calibrations were completed, the four GPS-disciplined clocks were allowed to run undisturbed for an entire year, and the GUC method was then again applied to the collected data. Figure 6 shows the post-calibration delay variations of the four GPS-disciplined clocks for the approximate one-year period from March 1, 2021,through March 6, 2022 (via UTCr reports 2109 through 2209; 53 full one-week reports were used). Although the daily delay variations were different, the average delay bias for GPS-disciplined clock-C (which utilized different receiver hardware than the other three units), was −0.8 ns for the approximate one-year interval, differing by just 0.3 ns from the NIST reference receiver over the same interval. The other three units (A, B, and D) used identical hardware. As such, their daily values closely tracked each other, and their average values for the one-year interval were nearly the same: −3.1 ns, −3.3 ns, and −3.4 ns, respectively. Units A, B, and D each differed by nearly 3 ns from the NIST reference receiver.

Figure 6 indicates that after the original approximate two-week GUC calibration, all four of the GPS-disciplined clocks under test were still "biased low" with respect to UTC, with GPSDC-C having the smallest bias and the other three units grouped closely together. No calibration can eliminate the inevitable daily delay fluctuations, but long-term calibrations can do a better job of centering the fluctuations around zero by removing the bias of seasonal effects. The delay values of each of the four GPS-disciplined clocks in Figure 6 were thus subsequently adjusted to remove the revealed calibration biases, allowing the one-year observation period to serve as a lengthy recalibration with a smaller uncertainty. This is shown in Figure 7, which includes data for the next six UTCr reports (2210 to 2215) after the calibration biases were removed. The average values for this six-week period then all showed sub-nanosecond agreement with UTC, with values of 0.7 ns, −0.5 ns, 0.1 ns, and −0.5 ns for units A, B, C, and D, respectively.

5. Factors that Limit the Measurement Uncertainty of GUC Calibrations

The time differences of the GPS time signal with respect to UTC(USNO) are tightly controlled, as shown in Figure 2, but GPS-disciplined clocks will produce larger time deviations, as indicated by the data shown in Figures 4, 5, and 6. These deviations can be attributed either to factors that affect the GPS signal as it propagates from the satellite to the receiving system's antenna, or to delay changes within the receiving system itself. For the purposes of this brief discussion, uncertainties related to GPS signal propagation can be labeled as U_P. These include uncertainties in the broadcast ionospheric delay corrections

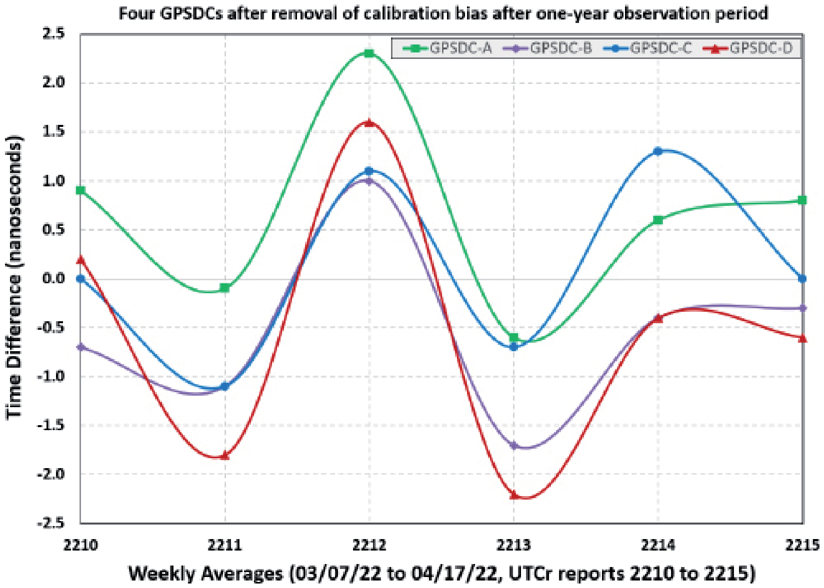

Figure 7. Delay variations of four single-frequency (L1 band) GPS-disciplined clocks during a six-week interval after calibration bias was removed following a one-year observation period.

that are applied by GPS-disciplined clocks, as well as multipath signal reflections, solar activity, tropospheric activity (weather), and other factors that can cause propagation delays to change. Environmental conditions, such as changes in both indoor and outdoor temperature that can cause hardware delay changes, are labeled as U_E. For example, outdoor temperature and conditions such as snow on the antenna can change the delay of receiving antennas or cables. Indoor temperature often has a larger effect, especially in the case of GPS-disciplined clocks with low-cost local oscillators, such as those graphed in Figure 6. In fact, both indoor and outdoor temperatures likely contributed to the approximate 3 ns bias in units A, B, and D, as they were calibrated during the months of January and February, winter months in Boulder, Colorado. Some amount of delay variation both during and after the calibration is inevitable in all GPS-disciplined clocks, regardless of whether the GUC method or another calibration method is utilized, and it seems reasonable and pragmatic to evaluate both U_P and U_E as Type B uncertainties, and to conservatively assign a value of 3 ns to each

The other factors that must be considered in the uncertainty analysis of the GUC method are the uncertainties of the links that UTC(USNO) and UTC(*k*) utilize to send their data to the BIPM, which can be labeled as U_{USN} and U_{UTK}, respectively. These uncertainties are provided by the BIPM in Section 1 of the *Circular T*. For purposes of example, we used the January 2022 *Circular T* where U_{USN} was reported as 1.6 ns. If we selected UTC(NIST) as UTC(*k*), then U_{UTK} was reported as 2.3 ns. We could also assign an uncertainty of 1 ns to U_{UG}, or the difference between UTC(USNO) and the prediction of UTC(USNO) broadcast by GPS (obtained by rounding up from the average time difference shown in Figure 2). Using standard methods for uncertainty analysis [11] and the aforementioned values, we estimated the combined uncertainty, U_c, of the calibration method via UTC(NIST) as

$$U_c = k\sqrt{U_P^2 + U_E^2 + U_{USN}^2 + U_{UTK}^2 + U_{UG}^2}$$

$$= 2\sqrt{3^2 + 3^2 + 1.6^2 + 2.3^2 + 1^2} = 10.4 \text{ ns.} \tag{4}$$

The coverage factor of $k = 2$ indicates that the coverage probability was 95.45%. This represents the probability that a daily measurement value was within the coverage interval. All of the post-calibration delay variations shown in Figures 4, 5, and 6 were well within a ±10 ns interval, suggesting that an approximate 10 ns uncertainty for a one-day average value was probably overestimated. The statistical (Type A) measurement uncertainty from a GUC calibration averaged across multiple days should improve by a factor of $\sqrt{N}$, where *N* is the number of days of the calibration. However, the systematic (Type B) uncertainties, such as the uncertainties in the BIPM links or GPS-disciplined clock antenna coordinate errors, will not be reduced by a longer calibration. Even so, for a seven-day calibration based on a single UTCr report, U_c should be ~5 ns ($k = 2$) in this example, and if three UTCr reports are used for a 21-day calibration, U_c should be reduced to < 4 ns ($k = 2$).The combined uncertainty, U_c, is with respect to UTC. As previously noted, the actual number of days required to complete the calibration will be *N* plus the days elapsed before UTCr is published (the last value shown on a UTCr report is for Sunday and publication is on Wednesday).

6. Summary

This paper has presented a simple method for the delay calibration of GPS-disciplined clocks, introduced here as the GUC method (GPS/UTC calibration). The GUC method can be utilized by any laboratory with a UTC(*k*) time scale. It has the advantage of not involving a reference GPS-disciplined clock, which potentially reduces the uncertainty of the measurements. The foundation for the method is the very close agreement between UTC, UTCr, UTC(USNO), and the time broadcast by the GPS satellites. The GUC method allows UTC(*k*) providers to calibrate GPS-disciplined clocks with respect to UTC, and to establish calibration services for industrial partners. It may also be useful for periodic recalibration of any secondary time-transfer receivers located at UTC(*k*) laboratories that do not serve as their primary link to UTC, and for continuous post-calibration verification of primary links.

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