LETTER TO THE EDITOR

Influence of ion dynamics on Hα and Hβ at low densities

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Abstract. Calculations for the Stark broadening of Hα and Hβ using a unified theory
for both ions and electrons are presented in the density range 10^{14} to 10^{18} \text{cm}^{-3} at a
temperature of 10^4 K. Although the theory is of marginal validity at these densities,
it shows quite conclusively the influence of ion dynamics in reducing the dip at the
centre of the Hβ profile and the peak in the centre of the Hα profile.

Recent experiments (Wiese et al 1972, Burgess and Mahon 1972) on the broadening of
hydrogen Balmer lines have shown discrepancies with theoretical calculations (Kepple
and Griem 1968, Vidal et al 1973) close to line centre, although good agreement is
obtained with Vidal et al (1973) in the line wings. In particular, for Hβ, at electron
densities n_e \sim 8 \times 10^{16} \text{cm}^{-3}, the calculations yield central minima about 35% down from
the peak intensities, in contrast to a measured dip of only 15% (Wiese et al 1972) and
at densities around 10^{18} \text{cm}^{-3} the central dip appears to be absent (Burgess and Mahon
1972). Similarly, for Hα, the unshifted component in the line centre shows up more
strongly in the calculation than it does in the experiments (Wiese et al 1972). By
approximate inclusion of effects due to inelastic collisions by electrons, Hill et al (1971)
have found a relatively small reduction in structure at line centre. The calculations of
Kepple and Griem (1968 and Vidal et al 1973) both treat the ions as stationary in the
quasi-static approximation. However, it is known that the quasi-static approximation
breaks down in the line centre and the dynamic properties of the ions could be respon-
sible for some of the discrepancy. Ion–ion correlations have been shown to be import-
ant in microfield calculations (Mozer and Baranger 1960) and therefore should be
included in any theory for ion dynamics. In fact, the theory of Dufty (1969) includes
many-body effects but calculations (Lee 1973) thus far have been restricted to a second
order evaluation of the desired resolvent operator. While the validity of these second
order calculations is open to question, they nonetheless produce an observable effect
on Hβ reducing (at 10^{16} \text{cm}^{-3}) the dip at line centre from about 64% to about 43% (before Doppler folding).

The purpose of this note is to present calculations of dynamic ion broadening
effects using a unified theory for both ions and electrons. The ion–ion correlations are
approximated by considering the ions and electrons as independent shielded quasi-
particles, with a shielding length of order of the Debye length \rho_D. This procedure is
certainly consistent to second order in the electric field (see Chappell et al 1970). In addition, it is found by Mozer and Baranger (1960) that the low frequency microfield, as required by a static theory, can also be closely approximated by considering the ions interacting only via Debye-shielded fields (with shielding length in vicinity of $\rho_D/\sqrt{2}$ or $\rho_D/\sqrt{1.5}$) and neglecting other ion-ion correlations.

Since the unified theory (Smith et al 1969, Voslamber 1969, Vidal et al 1970, 1971) is essentially a binary collision theory, a necessary condition for its validity is that strong collisions be separated in time. Using the following formula for the strong collision radius $\rho_s \approx (n^2 - n'^2) h/m_e v_i$ (Kepple and Griem 1968) (where $v_i$ is the relative ion velocity, $m_e$ is the electron mass, and $n$ and $n'$ are the principal quantum numbers for upper and lower states respectively), this condition is

$$n_e v_i \rho_s^2 < v_i \rho_s^{-1}$$

which gives $n_e \leq 7 \times 10^{14}$ cm$^{-3}$ for H$\beta$ for hydrogen ion perturbers at $10^4$K (and a factor of about two less for helium perturbers). Since the time between strong electron collisions is greater than $n_e v_i \rho_s^2$ by a factor of $\mu'(=(m_e/m_i)^{1/2}$ where $m_i$ is the reduced ion mass and $m_e$ the electron mass) and the duration of a strong electron collision is less than $t_s = v_i / \rho_s$ by the same factor $\mu'$, all strong collisions are separated in time when inequality (1) is satisfied. Therefore, the unified theory is used for both ions and electrons. It has been argued, however, by Capes and Voslamber (1972) that the condition (1) is not sufficient and that the cumulative effects of weak collisions also have to be taken into account. The unified theory neglects overlapping (in time) weak collisions beyond second order in the electric field (Smith et al 1973). The condition obtained by Capes and Voslamber (1972) (their condition (c)) may be rewritten as

$$\frac{\rho_{\text{max}}}{v_i} w_W < 1$$

with $w_W = n_e v_i \rho_s^2 \ln (\rho_{\text{max}}/\rho_s)$ and $\rho_{\text{max}}$ is the maximum impact parameter of order of the Debye length $\rho_D$ (Chappell et al 1970 show $\rho_{\text{max}} = 0.68 \rho_D$). Since $w_W$ is essentially the effective frequency of weak collisions and $\rho_{\text{max}}/v_i$ is the maximum duration of a collision, this condition (2) is obviously sufficient, since it implies all collisions are separated in time. For electron perturbers it is almost always satisfied. At low frequencies (less than the ion plasma frequency $\omega_{\text{pi}} = v_i / \rho_D$) shielding by the ions becomes important as well as by the electrons and a shielding length of about $\rho_D/\sqrt{2}$ is appropriate (Mozer and Baranger 1960). In this case condition (2) requires for H$\beta$ for hydrogen ion perturbers $n_e \leq 10^{14}$ cm$^{-3}$ at $10^4$K. In the density range of interest for H$\beta$ from $10^{14}$ cm$^{-3}$ to $10^{15}$ cm$^{-3}$ at $10^4$K this condition is obviously too restrictive. At $10^{14}$ cm$^{-3}$ the Doppler width, $\Delta \omega_{\text{D}}$, is over ten times $\omega_{\text{pi}}$. Therefore, if we take $1/\Delta \omega_{\text{D}}$ as the time of interest (since convolution with the Doppler profile smears out all structure within $\Delta \omega_{\text{D}}$), then the distant collisions at about $\rho_D$ are no longer effective and the maximum effective impact parameter is $\rho_{\text{max}} \sim v_i / \Delta \omega_{\text{D}}$ (see Lewis 1961). Since now, $\rho_{\text{max}}$ and $\rho_s$ are comparable, condition (2) is no longer more restrictive than condition (1).

It is also of interest to note that numerical calculations for helium lines (Barnard et al 1974) (where, however, distant collisions are less important) indicate that condition (1) is, within a factor of two or so, a reasonable criterion.

In spite of the uncertain (but admittedly marginal) validity it was decided that the unified results were worth presenting for comparison with experiment. To modify the
calculations of Vidal et al (1970, 1971, 1973), it is only necessary to add a quantity $i(\Delta \omega_{\text{ions}})$ for the ions to the similar quantity for the electrons and omit the ion field integration. In $i(\Delta \omega_{\text{ions}})$ the quantity $p_2$ is unchanged, $p_2$ is changed to

$$p_2 = -(4\mu^2)n_dD^2C^2[B - \ln(4C^2\mu^2)]$$  \hspace{1cm} (3)

where $\mu' = (m_e/m_i)^{1/2}$ (with $m_i$ the reduced mass for interacting ion–hydrogen atom pair) and

$$z = 3(nq-n'q)(\lambda/\mu + \frac{3}{5}(nq-n'q)a_0)^{-1}$$  \hspace{1cm} (4)

with $\mu = (m_i/m_e)^{1/2}$; all other quantities refer to electrons as in Vidal et al (1970, 1971, 1973). It should be noted that this relation (equation (3)) for $p_2$ is only correct as long as $C\mu' \ll 1$. This is generally well fulfilled for the electrons but not necessarily for the ions, in particular for the outermost Stark components and with increasing electron density. When this inequality is not completely fulfilled the constant $p_2$ has been calculated according to (Vidal et al 1970)

$$p_2 = -8\sqrt{\pi}n_dD^2C \int_0^\infty du u^2 e^{-u^2} \int_0^\infty \sin \left(\frac{2C\mu'}{\mu}z\right)(1+z^2)^{-1} dz.$$  \hspace{1cm} (5)

It should be stressed however when $C\mu' \sim 1$, the condition of equation (1) can be violated since $p_2 \approx C\mu'D_0$ and $p_2/p_0 \geq 1$ (with $4\pi/3p_0^3n_e = 1$). In addition, in the calculations the electron Debye length $\rho_D$ was used, rather than $\rho_D/\sqrt{2}$ which is appropriate for shielding by both ions and electrons, since at frequencies of interest (greater than $\omega_\text{pp}$) shielding by ions is small (i.e. the Lewis cutoff is appropriate).

Figure 1 compares the H$\beta$ results for hydrogen ion perturbers and for helium ion perturbers with the static results at $n_e = 10^{14}$ cm$^{-3}$ and $T = 10^{4}$K. $\Delta \alpha$ is the usual reduced wavelength (Kepple and Griem 1968, Vidal et al 1973). Notice the dramatic

**Figure 1.** Comparison of static and unified theories for ion broadening of H$\beta$ at $10^{14}$ cm$^{-3}$. Profile not convolved with Doppler profile.
Figure 2. Comparison of static and unified theories for ion broadening of Hβ at 10¹⁴ cm⁻³, with profile convolved with Doppler profile corresponding to 10⁴ K. $\Delta \alpha_s$ and $\Delta \alpha_c$ represent Doppler width and Weisskopf frequencies.

Figure 3. As in figure 2, but at 3.2×10¹⁴ cm⁻³.

Figure 4. As in figure 2, but at 10¹⁵ cm⁻³.

Figure 5. Comparison of static and unified theories for ion broadening of Hα at 3.2×10¹⁴ cm⁻³ with profile convolved with Doppler profile corresponding to 10⁴ K.
filling in of the dip at line centre. This filling in of the dip is also apparent at lower densities where the condition (2) (corresponding to \( n_e \leq 10^{14} \text{ cm}^{-3} \)) is better satisfied. The differences are much reduced after a convolution with the Doppler profile is performed, and results (after convolution with a profile corresponding to \( 10^4 \text{K} \)) at densities of \( 10^{14} \text{ cm}^{-3}, 3.2 \times 10^{14} \text{ cm}^{-3}, \) and \( 10^{15} \text{ cm}^{-3} \) are shown in figures 2, 3 and 4 respectively. In addition, profiles with helium ion perturbers were calculated; they show a slightly more pronounced dip at line centre (roughly \( 10 \% \) more at \( 10^{14} \text{ cm}^{-3} \)), but after convolution the profiles are essentially indistinguishable. Figure 5 shows the corresponding line profiles for \( \text{H} \alpha \) at an electron density of \( 3.2 \times 10^{14} \text{ cm}^{-3} \) where the peak in the line centre due to the unshifted Stark component is noticeably reduced.

The quantities \( \Delta x_D \) and \( \Delta x_0 \) in figures 2 to 5 correspond to the Doppler width \( \Delta x_D \) and Weisskopf frequency \( (= v/\mu) \) respectively. We do not expect a large variation with perturber mass since \( \mu' \) only changes by a factor of \( \sqrt{2} \) in going from hydrogen perturbers to very massive perturbers. Figures 2 to 5 are for equal ion and electron temperatures (10 000 K); calculations with electron temperature of 10 000 K and an ion temperature of 5 000 K yielded only minor differences. Furthermore, collisional effects on the Doppler profile (ie Doppler narrowing) will be unimportant at the densities considered because the mean free path for 90° deflection is much greater than the wavelength of the radiation (Rautian and Sobel'man 1967).

We conclude from these results that ion dynamics will considerably reduce the dip at the centre of \( \text{H} \beta \) and that these dynamic effects appear to explain part of the discrepancies between the theory and the results of Burgess and Mahon (1972) (although at about \( 10^{18} \text{ cm}^{-3} \) they are in a region where this theory is particularly of marginal validity). Although Lee (1973) also takes into account ion dynamics, his second order (Dufty 1969) operator \( H^{(2)}(\Delta \omega) \) actually gets quite large as \( \Delta \omega \rightarrow 0 \) (and diverges if natural damping is not included). This divergence is a manifestation of the breakdown of the second-order approximation in the Lee-Dufty theory (Barnard et al 1974) and could account for too large a dip at line centre in his computations (since \( H^{(2)}(\Delta \omega) \) occurs in the denominator of the expression determining the intensity).

Further work on ion dynamics will of course be necessary before our tentative conclusions relating problems with hydrogen profiles to ion dynamics are confirmed.

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