A Digital Time Scale at the National Institute for Standards and Technology

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Abstract

A novel, all-digital measurement system has been developed at the National Institute of Standards and Technology (NIST) as an alternative to the traditional analog approach to measuring time differences between the outputs of atomic standards used in the implementation of an ensemble time scale. The novel approach has been demonstrated viable for time scale applications and we are now working toward a full implementation of the system in an all-digital time scale.

1. Introduction

If the simplest time scale is one’s wristwatch, the availability of several clocks allows for averaging of their outputs, improving the overall performance of the resulting time scale, which is also called “ensemble clock”, being the weighted average of all the available clocks.

The implementation of an ensemble time scale, therefore, requires the ability to compare the time evolution of several clock signals: this information is then fed to a weighting algorithm that computes the output of the time scale. The comparison is generally realized by inferring the time differences between all clocks from measurements of their phase differences, starting at the conventional origin of the scale in question \(t_0\) as shown in Figure 1 below, where the signals from the i-th clock (clock\(_i\)) and from the clock elected to be the reference for the ensemble (clock\(_r\)) are not drawn to scale. The basic idea for comparing the phase of two signals is illustrated in Figure 1 and uses a time interval counter to measure the time between zero-crossings of the two waveforms under consideration. The phase difference \(\Phi_i - \Phi_r\) is then calculated using \(\bar{\nu}_i\), the average frequency of clock\(_i\).

![Figure 1. Illustration of the basic idea for measuring the phase difference between the signals of the i-th clock and the reference clock in a time scale ensemble. A time interval counter measures the time between the START and STOP zero-crossings, and the phase difference is then inferred according to the relation written above.](image)

The set of measured phase differences between each clock and the reference clock is then used to compute the evolving phase of the “ensemble clock” which represents the time scale’s output [1,2].

The high stability of the atomic standards used as clocks in timekeeping operations requires high-resolution measurements that cannot be performed directly by presently available time interval counters. The best clocks used to contribute to a time scale are generally hydrogen masers, whose relative frequency stability is approximately...
\(10^{13} \tau^{-1/2}\) for measurement times of \(\tau\) second (in terms of Allan deviation, up to several thousands of seconds), that is translated to a time stability of \(6 \cdot 10^{-14} \tau^{-1/2}\). Even for measurement times of 500 seconds or more (typically used for time scale computation) the time resolution required to measure time differences between hydrogen masers is about 1 picosecond (ps), approximately 10 times better than state-of-the-art time interval counters.

The most widely used sensitivity enhancement technique heterodynes the signals from two clocks in a scheme that is called Dual-Mixer Time–Difference (DMTD) measurement system, illustrated in Figure 2 (for the case of 5 MHz clock signals) [3,4]. The intermediate frequency is synthesized from one of the two signals under measurement and its spectral purity characteristics do not impact the measurement, because this synthesized signal is common to the downconversion process for both clock signals. It is also important to remember that the phase information of a signal is preserved through the downconversion process. The measured phase differences between all the downconverted signals can therefore be used for the computation of the timescale as a valid representation of the phase differences between the original 5 MHz clock signals.

The same basic technique described in Figure 1 is now applied to the downconverted signals, resulting in a sensitivity enhancement factor equal to the frequency downconversion ratio \(\frac{v_{\text{clock}}}{v_{\text{beat}}}\) (for \(v_{\text{clock}} >> v_{\text{beat}}\)). The typical downconversion factor used in time scale systems produces a beat-note of approximately 10 Hz, for which even a time-interval counter with a 100-picosecond time resolution can provide a phase resolution of 0.6 nanoradians on a 10-Hz signal, corresponding to a time resolution of 0.2 femtoseconds. In practice, the sensitivity limitations of a DMTD measurement system come from the electronic noise of the system’s components, setting it around several tenths of a picosecond: well within the requirements for the characterization of hydrogen masers.

**Figure 2.** Illustration of the principles of time-difference measurement using the traditional approach, also known as Dual Mixer Time Difference Measurement System.

### 2. The new measurement system

At NIST a novel, all-digital measurement system has been developed as an alternative to the traditional DMTD approach [5], and we are working towards a full implementation of an all-digital time scale. This all-digital system measures the time difference between clocks without the use of mixers, comparing the waveforms obtained by sub-sampling the clocks output signals. A graphical representation (not to scale) of the sub-sampling process is shown in Figure 3, where the dashed sinusoid represents the signal generated by one of the atomic standards and the small squares indicate the values sampled from it at a rate that is significantly lower than its frequency. The solid-line waveform that fits the samples is a sinusoid that has the same initial phase of the original clock’s signal, but has a much lower frequency. This waveform is equivalent to the beat-note resulting from the downconversion process in DMTD systems. In the implementation at NIST, The clocks’ frequency is 5 MHz, the rate of the sampling timebase (indicated by the square-wave in Figure 3) is approximately 78 kHz and the resulting “beat-note” has a frequency of about 4 Hz. The sampling timebase is synthesized from one of the clocks of the ensemble, providing a reference for all the measurements in the same way it is provided by the designated reference clock, in the DMTD measurement systems.

The solid-line waveform of Figure 3 is the result of numerical fitting of the batch of samples acquired during each measurement instance starting at time \(t_i\) as illustrated in the drawing (not to scale) in Figure 4. The fitting routine is a standard nonlinear least-square routine, implemented using the Levenberg-Marquardt method, and the model used for the fitting includes frequencies up to the 3rd harmonic of the fundamental frequency. It appears that
the inclusion of higher order harmonics doesn’t significantly improve the quality of the fitting (at least in our system), in terms of the initial phase at time \( t_i \), which is the quantity of interest.

![Figure 3](image)

**Figure 3.** A graphical representation of the waveform (here is at 4 Hz) fitting the samples generated by the sub-sampling process for each clock signal (here 5 MHz).

In order to quantify and compare the performance of this class of measurement systems, either in the traditional implementation or as the all-digital equivalent, the same 5 MHz sinusoid is simultaneously fed to two channels of the measurement system, in practice comparing the signal to itself, so that the residual phase differences that are measured are attributable to the non-ideality of the measurement system, thereby attesting its resolution. The Time variance, a two-sample based estimator of the true variance, is the parameter of choice for characterizing this kind of measurements.

![Figure 4](image)

**Figure 4.** Graphical illustration of the flow of measurement instances using the novel all-digital measurement system. The result of each sampling and fitting process is the phase of the sampled sinusoid at the time \( t_i \).

Although until now, for sake of simplicity, we have been talking about comparing the phase of the clocks’ signals, the real quantity of interest in the computation of time scales is the *time* difference between clocks, represented by a quantity called “time-phase”, defined as:

\[
x(t) = \frac{\varphi(t)}{2\pi \nu}
\]

where \( \nu \) is the frequency of the clock signal. This quantity is expressed in seconds and allows for clock differences larger than 2\( \pi \) radians.

In Figure 5 the Time deviation (squared-root of the Time variance) of the measured time-phase differences between two copies of the same signal is shown as a function of measurement time. It represents the ultimate sensitivity (resolution) of the measurement system, also referred to as the “noise floor” of the system, and allows a comparison of our novel, all-digital system with the latest state-of-the-art implementation of the DMTD approach.
The data shown refer to the novel all-digital system developed at NIST (red diamonds) and to the commercially available state-of-the-art systems based on the traditional DMTD approach (blue triangles).

The digital measurement system is well below the minimum required sensitivity of 1 ps for all measurement times displayed, and doesn’t show drifting behavior up to two weeks of continuous operation. Moreover, its sensitivity is at least a factor of two better than the current state-of-the-art, traditional DMTD system, for measurement times up to $10^6$ seconds (approximately two weeks’ time). Overall, the sensitivity of this novel measurement system is better than 0.5 ps, validating the all-digital as viable to be implemented into an all-digital time scale.

3. Conclusions and future work

The novel all-digital approach to the measurement of time-phase differences between atomic standards described in this paper has been completely validated by the comparison of its performance with the established traditional DMTD approach, presently implemented in the computation of the NIST active time scale. A first-generation multi-channel measurement system is now running at NIST and is providing data to be used in the computation of an all-digital time scale to be compared with the existing NIST time scale.

4. References


