Perspectives on the quantum Zeno paradox[1]

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Abstract. As of October 2006, there were approximately 535 citations to the seminal 1977
paper of Misra and Sudarshan that pointed out the quantum Zeno paradox (more often called
the quantum Zeno effect). In simple terms, the quantum Zeno effect refers to a slowing down of
the evolution of a quantum state in the limit that the state is observed continuously. There has
been much disagreement as to how the quantum Zeno effect should be defined and as to whether
it is really a paradox, requiring new physics, or merely a consequence of "ordinary" quantum
mechanics. The experiment of Itano, Heinzen, Bollinger, and Wineland, published in 1990, has
been cited around 347 times and seems to be the one most often called a demonstration of the
quantum Zeno effect. Given that there is disagreement as to what the quantum Zeno effect
is, there naturally is disagreement as to whether that experiment demonstrated the quantum
Zeno effect. Some differing perspectives regarding the quantum Zeno effect and what would
constitute an experimental demonstration are discussed.

1. Introduction
A recent entry in Wikipedia, an Internet-based encyclopedia, defines the quantum Zeno effect
as follows:

> The quantum Zeno effect is a quantum mechanical phenomenon first described by
> George Sudarshan and Baidyanaith Misra of the University of Texas in 1977. It
> describes the situation that an unstable particle, if observed continuously, will never
> decay. This occurs because every measurement causes the wavefunction to "collapse"
> to a pure eigenstate of the measurement basis [1].

This definition is close to the original language of Misra and Sudarshan [2], but is not
sufficiently general to describe the many situations that are considered to be examples of the
quantum Zeno effect. It is true that the quantum Zeno effect describes the situation in which the
decay of a particle can be prevented by observations on a sufficiently short time scale. However,
the quantum Zeno effect is much more general, since it describes the situation in which the
time evolution of *any* quantum system can be slowed by sufficiently frequent "observations."
The references to observations and to wavefunction collapse tend to raise unnecessary questions
related to the interpretation of quantum mechanics. Actually, all that is required is that some
interaction with an external system disturb the unitary evolution of the quantum system in a
way that is effectively like a projection operator. Finally, the word "never" describes a limiting
case. A slowing of the time evolution, as opposed to a complete freezing, is generally regarded
as a demonstration of the quantum Zeno effect.

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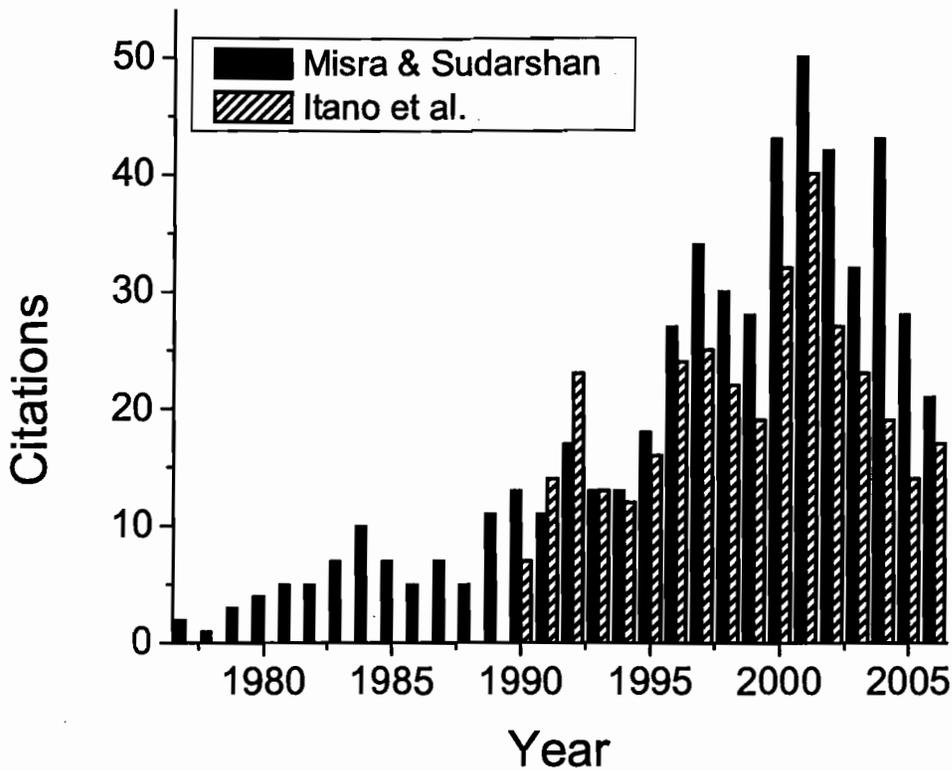
**Figure 1.** Graph of the number of citations per year to Misra and Sudarshan [2] and to Itano *et al* [3].

2. The Misra and Sudarshan Paper

The 1977 article "The Zeno's paradox in quantum theory" by Misra and Sudarshan [2] studied the evolution of a quantum system subjected to frequent ideal measurements. They showed that, in the limit of infinitely frequent measurements, a quantum system would remain in its initial state. Applied to the case of an unstable particle whose trajectory is observed in a bubble chamber or film emulsion, this result seemed to imply that such a particle would not decay, in contradiction to experiment. In this case, the resolution to the apparent paradox lies in the fact that the interactions between the particle and its environment that lead to the observed track are not sufficiently frequent to modify the particle's lifetime.

The time distribution of literature citations to Misra and Sudarshan [2] is shown in Fig. 1. The total number of citations listed in the Web of Science database in October 2006 was 535. The graph shows a relatively low but steady number of citations per year for about a decade, followed by a large increase that continues for over a decade, possibly peaking about 25 years after the original publication date. The great increase in the rate of citations in recent years is partially due to the increased interest in quantum information processing, where the quantum Zeno effect may find practical applications.

3. Simple derivation of the quantum Zeno effect

The quantum Zeno effect can be derived in an elementary way by considering the short-time behavior of the state vector [4]. (The treatment of Misra and Sudarshan [2] is more general since it involves the density matrix.) Let $|\phi\rangle$ be the state vector at time $t = 0$. If H is the Hamiltonian, in units where $\hbar = 1$, then the state vector at time t is $e^{-iHt}|\phi\rangle$, and the survival
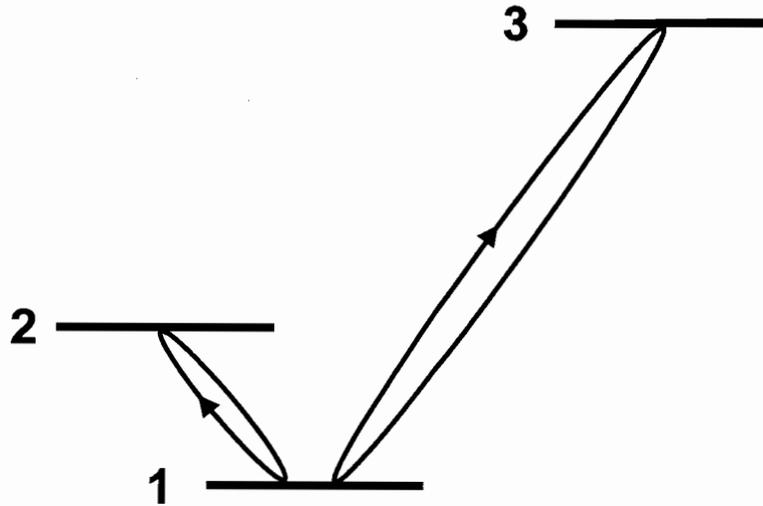
Figure 2. Energy-level diagram for the IHBW demonstration of the quantum Zeno effect.

probability is $S(t) = |\langle\phi|e^{-iHt}|\phi\rangle|^2$. If t is small enough, it should be possible to make a power series expansion:

$$e^{-iHt} \approx I - iHt - \frac{1}{2}H^2t^2\ldots, \tag{1}$$

so that the survival probability is

$$S(t) = |\langle\phi|e^{-iHt}|\phi\rangle|^2 \approx [1 - (\Delta H)^2 t^2], \tag{2}$$

where

$$(\Delta H)^2 \equiv \langle\phi|H^2|\phi\rangle - \langle\phi|H|\phi\rangle^2. \tag{3}$$

Many quantum systems have states whose survival probability appears on ordinary time scales to be a decreasing exponential in time. This is inconsistent with the quadratic time dependence of Eq. (1) and implies that in such cases Eq. (1) holds only for very short times. Consider the survival probability $S(T)$, where the interval $[0, T]$ is interrupted by n measurements at times $T/n, 2T/n, \ldots, T$. Ideally, these measurements are instantaneous projections and the initial state $|\phi\rangle$ is an eigenstate of the measurement operator. In that case, the survival probability is

$$S(T) \approx [1 - (\Delta H)^2 (T/n)^2]^n, \tag{4}$$

which approaches 1 as $n \to \infty$.

It is important to note that at this level there should be nothing controversial or problematic about the existence of the quantum Zeno effect. The quantum Zeno effect should be observed as long as the physical system can be made to display the behavior shown in Eq. (4). For a given system, it may be difficult or impossible to make measurements quickly enough for the quadratic time dependence of the survival probability to be observed, so that, as a practical matter, the quantum Zeno effect cannot be observed. It should be noted that the semantic arguments over terms such as "measurement" or "observation" can be avoided if we accept that a "measurement" is an operation that interrupts the unitary time evolution governed by H in such a way as to yield Eq. (4) as a good approximation. That is, the "measurement" should effectively act as a projection operator. According to this view, it is not necessary that the "measurements" be recorded by a macroscopic apparatus or that they be instantaneous.

n measurement pulses
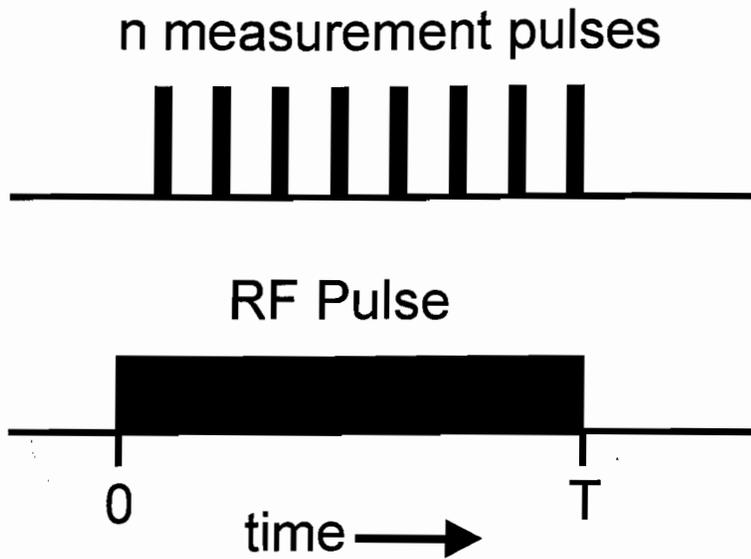
Figure 3. Timing of the radio frequency and optical fields applied to the beryllium ions in the IHBW experiment

4. The IHBW Experiment

The experiment of Itano, Heinzen, Bollinger, and Wineland (IHBW) [3] was based on a proposal of Cook [5] for observing the quantum Zeno effect in a three-level atom (see Fig. 2). Levels 1 and 2 are stable on the time scale of the experiment. Level 3 decays to level 1 with the emission of a photon. In the experiment of IHBW, levels 1 and 2 were two of the hyperfine sublevels of the ground $^2S_{1/2}$ state of the Be^+ ion. Level 3 was a sublevel of the $^2P_{3/2}$ excited state that decayed only to level 1.

The experiment was carried out with a sample of about 5000 Be^+ ions confined by electric and magnetic fields in a Penning trap. The steps in the experiment were as follows:

(i) The ions were prepared in level 1 by optical pumping with the laser beam.

(ii) A resonant radio frequency (RF) magnetic field was applied for the interval required to drive the ions to level 2.

(iii) During the time that the RF pulse was applied, a variable number n of equally spaced short laser pulses was applied to the ions (see Fig. 3).

(iv) The laser (resonant with the 1-to-3 transition) was turned on, and the induced fluorescence was recorded.

The intensity of the laser-induced fluorescence at the end of the experiment was proportional to the population of level 1. If there are no optical pulses during the long RF pulse, the population of level 2 as a function of the time t that the RF pulse is applied is

$$P_2(t) = \sin^2(\Omega t/2) = \frac{1}{2}[1 - \cos(\Omega t)], \qquad (5)$$

where Ω is proportional to the amplitude of the RF field. If the duration of the RF pulse is chosen to be $T = \pi/\Omega$ (a pi-pulse), then all of the population is transferred from level 1 to level 2. If n equally-spaced laser pulses of negligible duration are applied during the RF pi-pulse, the population of level 2 at time T is

$$P_2(T) = \frac{1}{2}\left[1 - \cos^n(\pi/n)\right], \qquad (6)$$
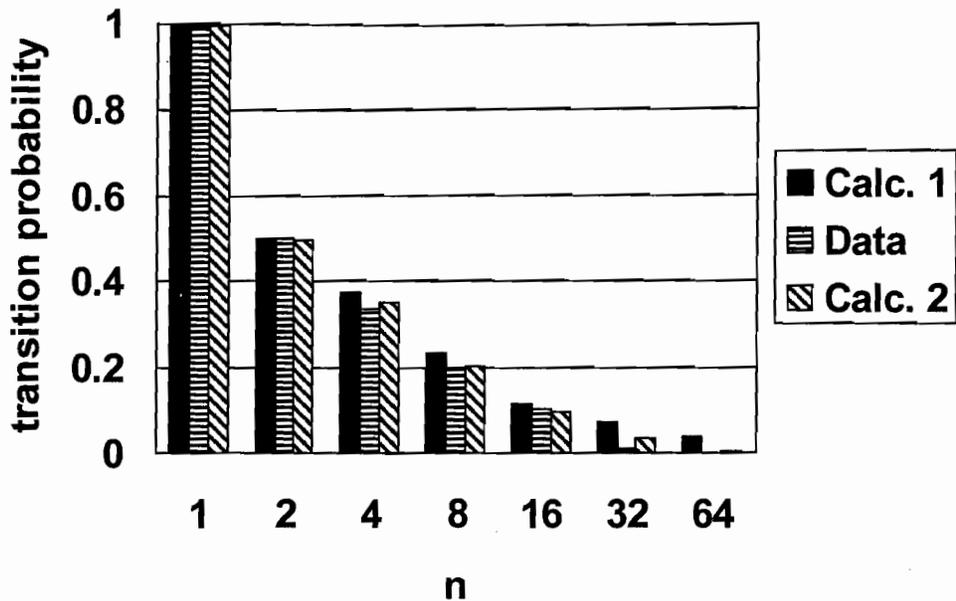
Figure 4. Probability of making the 1-to-2 transition as a function of the number n of optical "measurement" pulses.

which approaches 0 as n goes to infinity.

Figure 4 compares the data to theory. The solid bars represent the transition probability as a function of n according to the simplified calculation of Eq. (6). The bars with horizontal stripes represent the data. The bars with diagonal stripes represent a calculation that takes into account the finite duration of the laser pulses and optical pumping effects. The data are in reasonably good agreement with the simplified calculation and in better agreement with the improved calculation. The decrease in $P_2(T)$ as n increases demonstrates the quantum Zeno effect.

A variation of the experiment was carried out by initializing the ion in level 2 and then applying the RF field and the laser pulses. In this case, the transition from level 2 to level 1 was inhibited as n increased. This is another example of the quantum Zeno effect. In this case, the inhibition of the transition is accompanied by the *absence* of laser-induced fluorescence.

Recently, the quantum Zeno effect was observed for an unstable quantum system by Fischer *et al* [6]. The quantum Zeno effect for induced transitions and for unstable systems are not fundamentally different, since they both follow from the general arguments of Misra and Sudarshan [2], but it has been difficult to observe in the latter case, because of the short times over which the decay is nonexponential. Fischer *et al* were able to create an artificial system (atoms tunneling from a standing-wave light field) in which the interactions could be controlled so as to observe the desired effects.

5. Responses to the IHBW Experiment
As can be seen by the history of citations (Fig. 1), the publication of the IHBW experiment [3] generated considerable interest. Initially, some of the responses were critical in one way or another. Some (e. g., Ref. [7]) objected to the use of the term "wavefunction collapse" in describing the experiment. The authors responded that the concept of wavefunction collapse was not essential, and that any interpretation of quantum mechanics that yielded the same prediction of the experimental results should be regarded as valid [8]. Some objected to the

fact that photons were not actually observed during the intermediate "measurements," in the sense of having the scattered photons registered by a detector, so that the experiment did not actually demonstrate the quantum Zeno effect [7, 9, 10, 11]. However, the results are predicted to be the same whether or not the intermediate measurements are made. It is enough that the measurements *could* have been made. As long as the laser interactions act effectively as projection operators, so that the algebra of Eqs. (1)–(4) is followed, the experiment should be regarded as a demonstration of the quantum Zeno effect. It should be noted that none of the criticisms were directed at the execution of the experiment itself, only at the interpretation. For the most part, the citations to Ref. [3] simply accept it as a demonstration of the quantum Zeno effect. In fact, it is cited in quantum mechanics textbooks [12, 13, 14, 15] and popular science books [16, 17, 18].

6. Distinctions

While Misra and Sudarshan originally used the term "quantum Zeno paradox," as did Peres [4] and others, the more recent work usually uses the term "quantum Zeno effect," perhaps because the effect no longer seems paradoxical. Some authors distinguish between the quantum Zeno *paradox* and the quantum Zeno *effect*, but they do so in differing ways. Pascazio and Namiki [19] call the situation in which the frequency of measurements is finite and the evolution is slowed the quantum Zeno *effect,* and the limiting case in which the frequency of measurements is infinite and the evolution is frozen the quantum Zeno *paradox.* Block and Berman [20] call the inhibition of spontaneous decay the quantum Zeno *paradox* and the inhibition of induced transitions (as in the IHBW experiment) the quantum Zeno *effect.* In Ref. [21], Home and Whitaker reserve the term quantum Zeno *paradox* for a negative-result experiment involving observations with a macroscopic apparatus. This definition of the quantum Zeno paradox seems to exclude most, if not all, feasible experiments. In this context, the IHBW experiment is not regarded as an example of the quantum Zeno paradox because a local interaction is present between the laser field and the atoms, and also because the electromagnetic field, containing zero or a few scattered photons, is not regarded as a macroscopic observation apparatus. They regard the type of experiments where the time evolution of a quantum system is affected by a direct interaction, for example with an external field, as examples of the quantum Zeno *effect.* However, in a later publication [22] the same authors treat the terms quantum Zeno *paradox* and quantum Zeno *effect* as synonymous and restrict both to nonlocal negative-result experiments involving a macroscopic observation apparatus. Experiments that do not meet these criteria would not be examples of *either* the quantum Zeno paradox *or* the quantum Zeno effect, according to their later definition.

7. Extensions

Several variations on the general theme of quantum Zeno effects have been described. Soon after the IHBW experiment was carried out, Peres and Ron [23] showed that a partial quantum Zeno effect results if the measurements are too weak to completely destroy the coherence of the state of the measured system. A modification of the IHBW experiment was proposed in which the measurement laser pulses are weakened. Jordan *et al* [24] showed that a related effect, damped oscillations of the state populations, can occur if the duration of the experiment is extended, while weak measurements are made.

   Some, including Kofman and Kurizki [25] and Facchi *et al* [26] have shown that the decay of an unstable quantum system can be accelerated by frequent observations. This is called the quantum anti-Zeno effect or the inverse quantum Zeno effect. As is the case for the quantum Zeno effect, the observations must take place before the decay becomes exponential. Unlike the quantum Zeno effect, which follows from rather general arguments, e. g. Eqs. (1)–(4), the possibility of observing a quantum anti-Zeno effect depends on the details of the system.

The experiment of Fischer *et al* [6] demonstrated the quantum anti-Zeno effect as well as the quantum Zeno effect.

An interesting generalization of the quantum Zeno effect is the concept of quantum Zeno dynamics [27, 28]. Frequent measurements can confine the evolution of a quantum system to a subspace of the Hilbert space rather than simply to the initial state. Compared to the ordinary quantum Zeno effect, the difference is that the measurements distinguish not between the initial *state* and all other states but between a *subspace* and the rest of the Hilbert space. This form of quantum Zeno effect may find application in quantum information processing.

8. Applications
As already noted, the recent increase in the rate of citations to the articles of Misra and Sudarshan [2] and IHBW [3] is partially related to increased interest in quantum information processing. In this context, there have been various proposals to use the quantum Zeno effect to preserve quantum systems from decoherence.

Beige *et al* [29] have proposed an arrangement of atoms inside an optical cavity capable of carrying out quantum logic operations with low error rates within a decoherence-free subspace of the Hilbert space. States outside the decoherence-free subspace are coupled strongly to the environment. The quantum Zeno effect then leads to effective confinement of the system to the decoherence-free subspace, which is an example of a quantum Zeno subspace.

Franson *et al* [30] have proposed use of the quantum Zeno effect to suppress errors in a linear optics implementation of quantum computation. In this implementation, the presence of two photons in the same mode indicates an error. The presence of a strong two-photon absorber in an optical fiber takes the role of the "observer" and suppresses the errors. Other proposed applications of the quantum Zeno effect to error prevention in quantum computation are discussed in Refs. [31, 32, 33, 34].

Quantum "bang-bang" control and related dynamical decoupling techniques [35, 36] utilize frequent, pulsed interactions to effectively prevent decoherence of a quantum system by confining the dynamics to a subspace. This is not exactly the quantum Zeno effect, since the interactions are unitary, but the results are mathematically similar to those for the quantum Zeno effect.

Dhar *et al* [37] have discussed the "super-Zeno effect," which preserves a state (or more generally, keeps a quantum system within a subspace of the Hilbert space) with a set of pulsed interactions unequally spaced in time. The timing of these interactions can be arranged so as to be more efficient than can be done with the same number of equally spaced interactions (ordinary quantum Zeno effect). Also, it should be noted that the pulsed interactions are unitary kicks, as in the so-called "bang-bang control" [35], and not observations in the usual sense.

9. Conclusion
The 1977 publication of Misra and Sudarshan stimulated a great deal of theoretical and experimental work that has enhanced our understanding of the time development of quantum systems, such as the short-time nonexponential decay of unstable quantum systems. The results of the IHBW experiment, published in 1990, was a clear confirmation of the existence of the quantum Zeno effect for the case of the inhibition of an induced transition. Interest in the quantum Zeno effect continues to be high, partially due to the possibility of practical applications in quantum information processing.

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