A Regenerative Frequency Comb

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Abstract--We describe a regenerative frequency comb generator (RCG) used to synthesize a signal coherent with the input signal with a fractional multiplication of m/n, where the frequency of n is proportional to 1/τ, where τ is the loop delay, and m is a positive integer less than n. We describe the RCG and compare its performance with traditional regenerative dividers, digital dividers and multipliers. Preliminary data for a divide by ten whose residual noise we measured at 100 MHz suggest superior performance to low noise digital dividers, with a SSB noise of -145 dBc/Hz at 100 Hz and 1/f characteristic. While we have not attained the broadband performance of the regenerative dividers and conjugate regenerative dividers studied in the past, we have attained -162 dBc/Hz at 100 kHz offset and expect to be able to improve the overall noise further by applying techniques investigated in the aforementioned devices.

I. Introduction

In the optical domain frequency division and synthesis can be achieved using frequency comb generators, more specifically femtosecond combs. There are a couple of common techniques that are currently used to achieve octave spanning combs and division ratios of 1000 [1]. The resulting comb spectrum is composed of a coherent sum of frequencies. Of equal importance is the use of such a device for synthesis. Consider an existing low noise source at f_c at the input of a comb generator with spacing frequency f_σ. The comb generator will yield an output signal with frequency $f = m f_\sigma$, where m is a positive integer. Driving a digital synthesizer with $2 f_\sigma$ and mixing the synthesized signal with different individual comb teeth can deliver a signal with frequency in the bandwidth of the comb generator.

This paper demonstrates a proof of concept comb generator, that operates in the microwave regime, using a regenerative technique, from a simple development of the concept to its implementation. The phase noise of the RCG was measured in the microwave and X bands of the frequency spectrum were conducting using analog and digital measurement techniques described in [2][3].

Much work has been done in microwave frequency synthesis using regenerative dividers. Most of the focus has been on regenerative divide by two circuits. There has also been some interest on circuits with higher division ratios. In general the regenerative divider relies on matching the phase and gain conditions around a loop in order to produce a self sustaining oscillation [4]. The input signal provides the additional necessary phase bias for these oscillations. For larger division factors the complementary pairs must be propagated in order sustain oscillations. Traditionally the different complementary frequencies were propagated through independent loops, thus allowing independent control of the phase condition [5]. The RCG is a single loop device that produces a coherent comb of frequencies that are integer related to the input signal and the delay around the loop. The loop resonance is satisfied at multiples of the frequency $1/\tau_{loop}$ and serves as the comb mode selector. As such it can operate in a wide variety of modes as determined by the loop delay, and various other loop parameters including loop gain, loop power etc. Comb spacing of $f_c/2$ can also be achieved. For higher frequency operation it is sometimes necessary to sacrifice broadband operation for performance and cost. This can be achieved by limiting the bandwidth and employing an external lock mechanism. The majority of our effort has focused on examining feasibility of and initial characterization of a band-limited RCG.

II. Simulation overview

Assuming ideal conditions, ignoring dispersion and other effects, a frequency comb in the microwave regime generated by an input signal at f_c would have a spectrum in the frequency domain related to the spacing frequency, f_σ, given by:

$$f = m f_\sigma ;\ m \text{ is a positive interger} \quad (1)$$

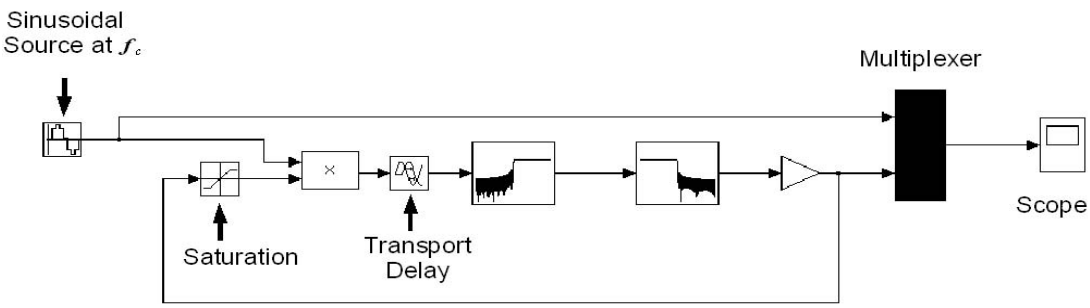

Figure 1:A diagram of the simulation circuit with lowpass cutoff frequency of $1/\tau_{loop}$ *and highpass cutoff frequency of* $4f_c$.

In coherent operation the spacing frequency is an integer factor of the input frequency, for a comb implementation n_σ is a fixed positive integer, the division factor.

$$f_c = n_\sigma f_\sigma ; n_\sigma \text{ is a fixed positive integer} . \quad (2)$$

The frequency of any comb tooth is a ratio of an arbitrary (positive) integer m, the spacing frequency and the the input frequency.

$$f_{tooth} = m/n_\sigma f_c ; m \text{ a positive integer} . \quad (4)$$

We can express the comb output as a sum of sinusoids centered about an input frequency fc. For simplicity we overlook the input signal having twice the amplitude.

$$\sum_{m=0}^{l} cos(2\pi f_c t(1+m/n_\sigma)) + cos(2\pi f_c t(1-m/n_\sigma)) \quad (5)$$

Rewriting this as a product we can factor out the source signal

$$\underbrace{(2 sin(2\pi f_c t))}_{(source\ signal)} \underbrace{(\sum_{m=0}^{l} cos(n_\sigma 2\pi f_c t))}_{(delay\ line\ resonator)} \quad (6)$$

The resulting equation resembles a product of a source and a delay line resonator. The circuit in Fig. 2 is the block diagram representation of the above.

A numerical simulation of the RCG circuit was performed (Fig. 1) and the input and output signals were plotted in Figures 3 and 4 for simulation results with different simulated loop delays. To reduce numerical errors a low pass with a $1/\tau_{loop}$ cutoff and a high pass with $4f_c$ with constant group day were also included in the simulation circuit.

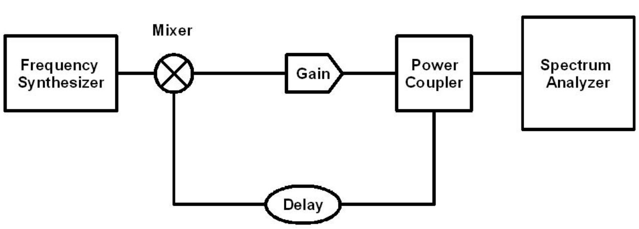

Figure 2: Regenerative comb generator test implementation; varying the input frequency excites a comb with when the loop is an integer factor of $1/f_c$.

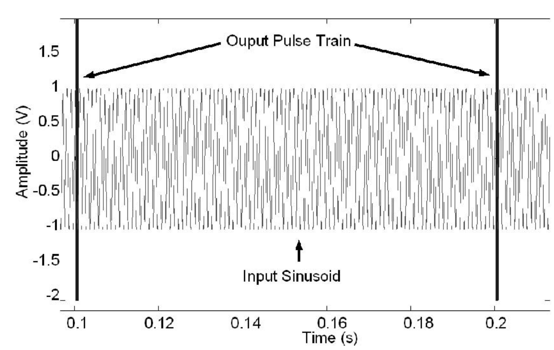

Figure 3: Time domain simulation input and output waveforms for $\tau_{loop} = 100/f_c$. *Simulation circuit generates an output pulse every 100 cycles of the input sinusoid.*

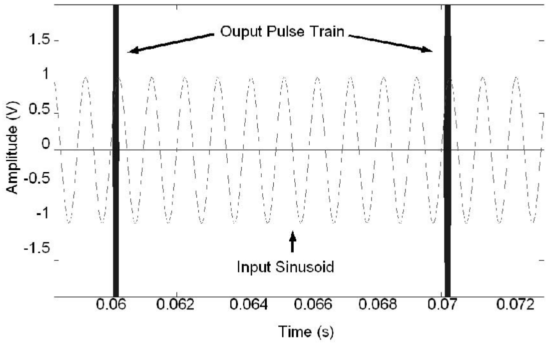

Figure 4: Time domain simulation input and output waveforms for $\tau_{loop} = 10 / f_c$. Simulation generates an output pulse every 10 cycles of the input sinusoid.

While the simulation was sensitive to the numerical parameters and required additional elements in the feedback path, the results were encouraging enough to pursue a hardware implementation (Fig. 2). Although further discussion is beyond the scope of this paper, it is important to mention the critical role of a nonlinear element in the loop. If we consider a pure multiplication without saturation, energy will continue to be translated up to higher frequencies and there may be no sustainable oscillations coherent with the input. The nonlinear element frequency multiplies individual sinusoidal components which are then modulated by the input. The inter-modulation terms above f_c support those below f_c and vice-versa.

The gain and saturation of the loop elements in conjunction with the bandwidth of the loop thus determine the shape of the spectrum; these relationships were observed empirically for bandwidth limited loops where all the comb teeth are not supported by inter-modulation products.

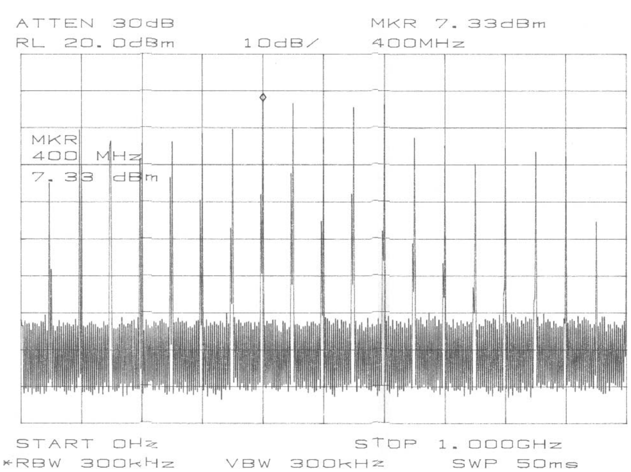

Figure 5: The measured output spectrum of an RCG with a 1 GHz input and 50 MHz comb spacing.

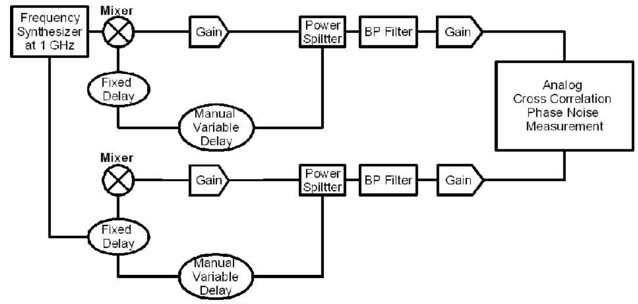

Figure 6: Two RCGs in a residual phase noise measurement setup.

III. Hardware configuration and results

The basic broadband circuit, with a fixed loop delay τ_{loop} where the loop components have a broadband frequency response extending from less than $1/\tau_{loop}$ to greater than twice the input frequency can be excited by input signals where $f_c \propto k / \tau_{loop}; k$ is a positive integer . For small changes in input frequency (a few kHz for an input of 1-2 GHz), individual comb teeth were observed to track proportionally. The output spectrum of a regenerative comb generator with an input frequency of 1 GHz and a comb spacing of 50 MHz is shown in Figure 5. The complete system with two RCGs and phase noise measurement system is shown in Fig. 6. Fig. 7 Plots the residual phase noise at 100 MHz for both devices. The level of performance was found to be as good as many digital dividers close to the carrier, with potentially better broadband performance. The bandwidth component requirements, however, can limit the use of this device at higher frequencies as very broadband devices are hard to build with suitable performance.

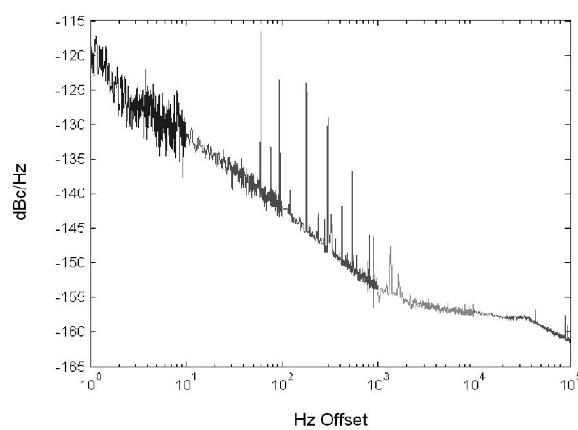

Figure 7: RCG residual phase noise at 100 MHz for two devices

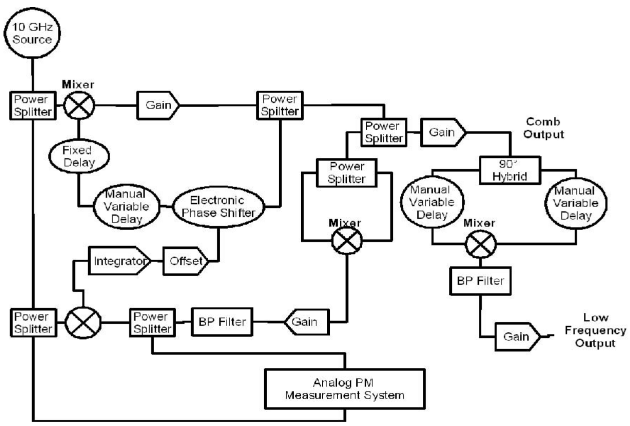

Figure 8: Block Diagram of a residual phase noise measurement setup at 10 GHz. Source is compared with the comb 5 and 15 GHz beat.

At high frequencies devices with operating bandwidths of one octave of the center frequency are more readily available. The drawback of not generating comb teeth all the way down to the spacing frequency is that an external locking mechanism is necessary for coherent operation. This was achieved by beating the comb with itself and filtering out the 10 GHz signal. The 10 GHz is the beat of the outermost teeth of the comb, the 5 and 15 GHz comb teeth. The beat signal is then compared to the source signal in a mixer and the integrated error signal servos an electronic phase shifter in the loop. An RCG at 10 GHz and the associated phase noise measurement setup to measure residual noise at 10 GHz is shown in Figure 8.

The residual phase noise is shown in Figure 9. This represents the phase fluctuations in the RCG loop; the residual phase noise at other comb frequencies should thus scale accordingly.

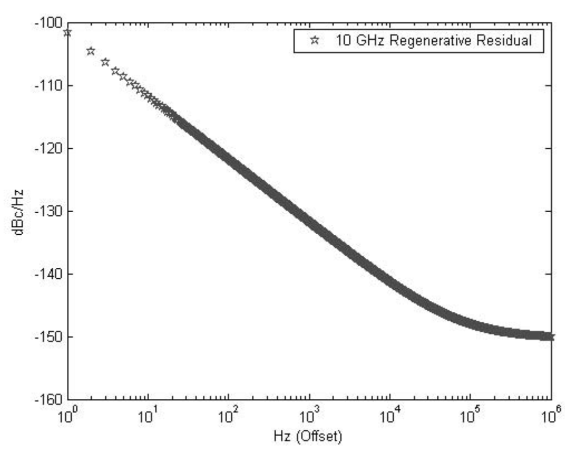

Figure 9: The fitted curve for a residual phase noise measurement at 10 GHz for an octave span RCG.

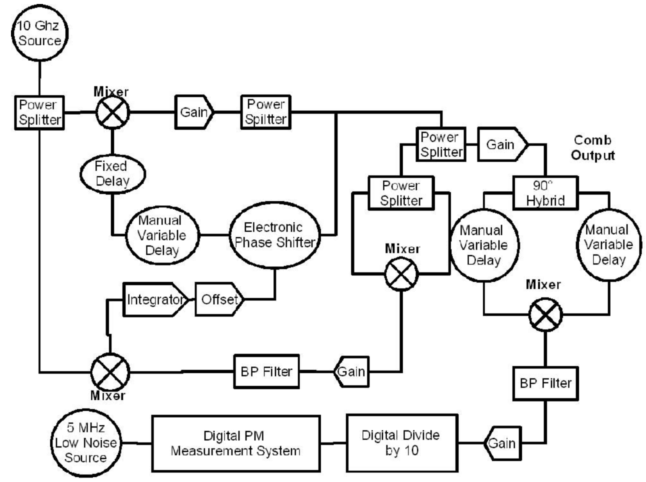

Figure 10: Diagram of an octave span RCG and measurement setup of the down converted spacing frequency.

The signal noise divided to 11.1 MHz and its associated block diagram are included for completeness. We expected the device's residual noise to scale down accordingly to the division ratio of the measured teeth. However the noise we measured did not correspond to the source noise plus residual noise at 10 GHz scaled down to 11.1 MHZ (≈ -59 dB).The 11.1 MHz noise reflects a variety of processes. The noise close to the carrier is the noise of the 10 GHz source minus 59 dB, as expected for a division ratio of 901. At and beyond 100 Hz,the system noise dominates. This system noise is a sum of the down-conversion noise and the residual noise of the device. Initial investigation suggests that some noise process in the down conversion dominates. We are optimistic that the noise performance at individual comb teeth can be achieved at levels reflecting the measured residual noise at 10GHz with the appropriate scaling factor.

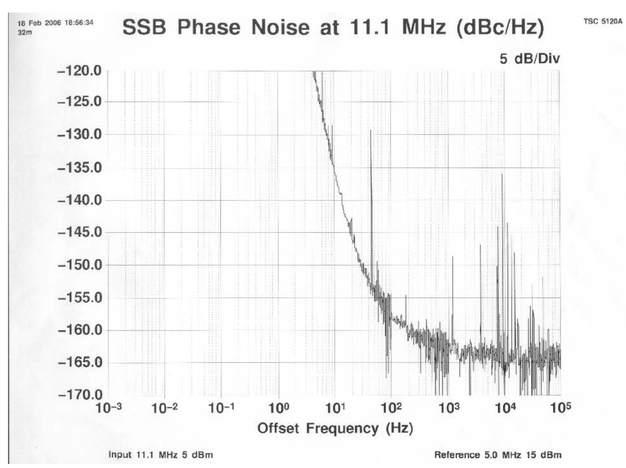

Figure 11: RCG spacing frequency phase noise measurement result.

Other hardware issues pending include issues with thermal stability and dispersion. For this paper, all hardware and experiments used commercial cables and components which were on the bench and not thermally controlled.

IV. Conclusion

The primary goal of this effort is to develop a regenerative comb generator that spans on the order of an octave of the input frequency. This facilitates the development of RCGs at higher frequencies with potentially better performance and lower cost. A more detailed analysis and evaluation of the RCG is the subject of future work. We hope to build a second octave spanning prototype to evaluate residual noise, implement thermal stabilization, optimize, and characterize the loop components and parameters. Overlapping modes were observed but not investigated and we also lump with future work. Integration of other nonlinear elements, such as step recovery diodes or nonlinear transmission lines with better broadband performance in the loop may mitigate dispersion effects in the mixer and yield improved performance.

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