Study of the Excess Noise Associated with Demodulation of Ultra-Short Infrared Pulses

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***Abstract*—The demodulation of ultra-short light pulses with photodetectors is accompanied by excess phase noise at the pulse repetition rate and harmonics in the spectrum of the photocurrent. The major contribution to this noise is power fluctuations of the detected pulse train that, if not compensated for, can seriously limit the stability of frequency transfer from optical to microwave domain.**

By making use of an infrared femtosecond laser, we measured the spectral density of the excess phase noise, as well as power-to-phase conversion for different types of InGaAs photodetectors. Noise measurements were performed with a novel type of dual-channel readout system using a fiber coupled beam splitter. Strong suppression of the excess phase noise was observed in both channels of the measurement system when the average power of the femtosecond pulse train was stabilized.

The results of this study are important for the development of low-noise microwave sources derived from optical "clocks" and optical frequency synthesis.

I. Introduction

Kerr-lens femtosecond lasers have revolutionized the entire field of optical frequency metrology by allowing quick and precise measurements of optical frequencies with the resolution close to the limit of the best microwave frequency standards based on the laser-cooled Cs atoms [1]–[3]. By making use of femtosecond lasers, it also is possible to convert the lightwave output of an optical frequency standard (optical "clock") into an ultra-stable microwave signal [4], [5]. The frequency/time transfer from optical to microwave domain is realized by synchronizing the pulse repetition rate of a femtosecond laser with a beat rate of an optical "clock" and detecting the optical pulse train with a high-speed photodetector. Demodulation of the ultra-short light pulses is not a noise-free process. It is accompanied by the excess phase noise in the spectra of the microwave signals extracted at pulse repetition frequency and its harmonics. The origin of the excess phase noise was studied in [6]. It was linked to the power-to-phase conversion phenomena in the photodetectors. Such phenomena manifests itself as a power broadening of the demodulated light pulses, as well as a power-dependent time delay between optical and electrical pulse trains. Power-to-phase conversion is a main mechanism by which fluctuations of the detected optical power affect the fidelity of phase/frequency transfer from optical to microwave domain. For example, in the initial experiments at the National Institute of Standards and Technology (NIST, Boulder, CO) the fractional frequency instability of the extracted microwave signals was limited by the excess phase noise at the level of $6 \cdot 10^{-14}$ over 1 s of integration [6]. Later, when the main causes of the excess phase noise were identified, the fractional uncertainty of the frequency/time transfer was reduced to $3 \cdot 10^{-15}$. This uncertainty is relatively small, but it is still more than two orders of magnitude higher than that associated with the intrinsic noise of the control electronics enabling the phase synchronization of a femtosecond laser [7].

Improving the accuracy of frequency/phase transfer is especially important from the viewpoint of the recent developments of optical frequency standards based on laser-cooled atoms that potentially could achieve fractional frequency instability less than $10^{-18}/\sqrt{\tau}$, where τ is an observation time [5], [8], [9].

The previous study of the excess noise associated with the photodetection process was conducted at frequencies of visible light generated by Ti:sapphire femtosecond laser ($f \approx 375$ THz). Present work extends this study to the range of infrared frequencies accessible with a Cr: forsterite femtosecond laser ($f \approx 230$ THz). Apart from investigating the noise properties of different types of the photodetectors, all the experiments were performed with an improved precision, relative to [6], due to the use of the novel measurement techniques described in details below.

II. Noise Properties of Cr: Forsterite Femtosecond Laser

We first studied the noise properties of the infrared Cr: forsterite ($Cr:Mg_2SiO_4$) femtosecond laser [10]. The laser operated as a ring laser in the double bow-tie configuration using six chirped mirrors for dispersion compensation. It was pumped at 1.07 μm with a 10 W Ytterbium fiber laser. When operating in a mode-locked regime, the average power of the emitted pulse train was 0.6 W, at a repetition rate $f_R \approx 433$ MHz.

Fluctuations of the pulse repetition rate were measured with the experimental setup shown in Fig. 1(a). Here, a pulsed laser light illuminates a photodetector, and signals at harmonics of pulse repetition rate are extracted from the spectrum of the photocurrent. The phase of the extracted signal is compared to that of a low-noise radio frequency

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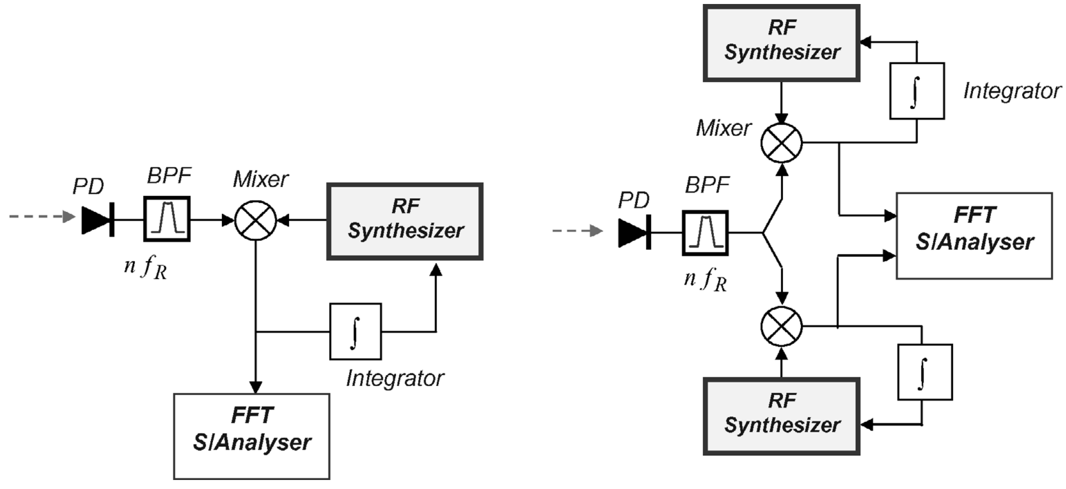

Fig. 1. Single-channel (a) and dual-channel (b) two oscillator phase noise measurement systems.

(RF) synthesizer with a double-balanced mixer. The latter serves as a phase detector of the phase-locked loop (PLL) controlling the mean frequency of the RF synthesizer.

The spectral density of voltage fluctuations at the output of the phase detector is given by:

$$S_u(f) \approx \frac{S_{PD}^2}{|1+\gamma|^2}\left\{n^2 S_{rep}(f) + S_{synth}(f)\right\}, \qquad (1)$$

where S_{PD} is the efficiency of phase detector in V/rad, f is a Fourier frequency, γ is a gain of the PLL, $S_{rep}(f)$ is a spectral density of pulse repetition rate fluctuations (to be measured) and $S_{synth}(f)$ is the spectral density of phase fluctuations of the RF-synthesizer.

The intrinsic noise of the double-balanced mixer is not present in (1) as its contribution to the output voltage noise is negligible relative to that of the RF synthesizer.

As follows from (1), the resolution with which fluctuations of pulse repetition rate are measured is limited by phase fluctuations of the RF synthesizer. Such limitation is less severe in a dual-channel readout system with two RF synthesizers [Fig. 1(b)]. This is because frequency fluctuations of RF synthesizers are uncorrelated, and their effect on the measurement system noise floor decreases with the number of averages taken by the fast Fourier transform (FFT) spectrum analyzer when a cross-correlation function between the voltage fluctuations of the two channels is calculated [11].

Curve 1 in Fig. 2 shows the spectrum of phase fluctuations of a signal at frequency $2f_R$ at the output of the photodetector. At low Fourier frequencies ($f \leq 10$ Hz) the power spectral density of phase fluctuations varies at a rate close to -40 dB/decade, which is likely due to environmental (temperature, vibration, etc.) fluctuations affecting the travel time of an optical pulse in the laser resonator. Curve 2 shows the spectrum of relative intensity fluctuations of the Cr: forsterite laser. This spectrum was inferred from the voltage noise across the bias resistor of the photodetector. As in the case of Ti: sapphire laser, the relative

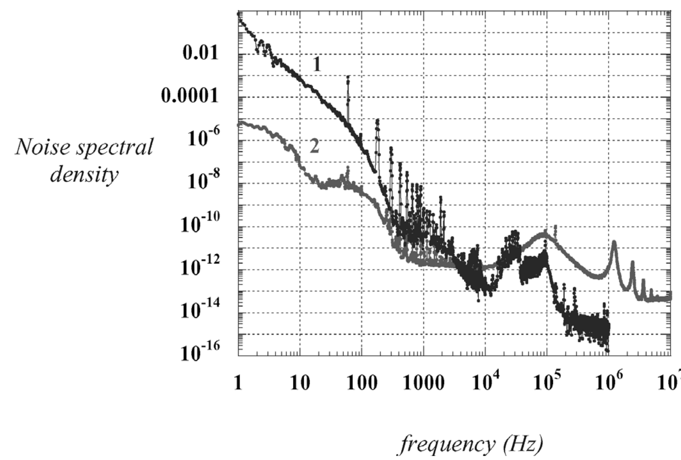

Fig. 2. Spectra of phase (curve 1) and amplitude (curve 2) fluctuations of a signal at second harmonic of pulse repetition rate at the output of a photodetector illuminated with a Cr: forsterite femtosecond laser. The units of the vertical axis are rad^2/Hz for the phase noise and 1/Hz for the amplitude noise, respectively.

intensity fluctuations of the Cr: forsterite laser were found to be strongly correlated with those of a pump laser. For example, a sequence of peaks at multiples of 1.2 MHz is due to the relaxation oscillations of the pump laser.

III. Power-to-Phase Conversion in Photodetectors

Photodetectors are known to exhibit a power-to-phase conversion [12]. This is a main mechanism due to which fluctuations of the detected optical power give rise to the excess phase noise in the spectrum of the extracted microwave signal [6].

The experimental setup for measurement of the power-to-phase conversion in photodetectors is shown in Fig. 3. Here, the average power of the optical frequency comb is modulated with an acousto optical modulator (AOM).

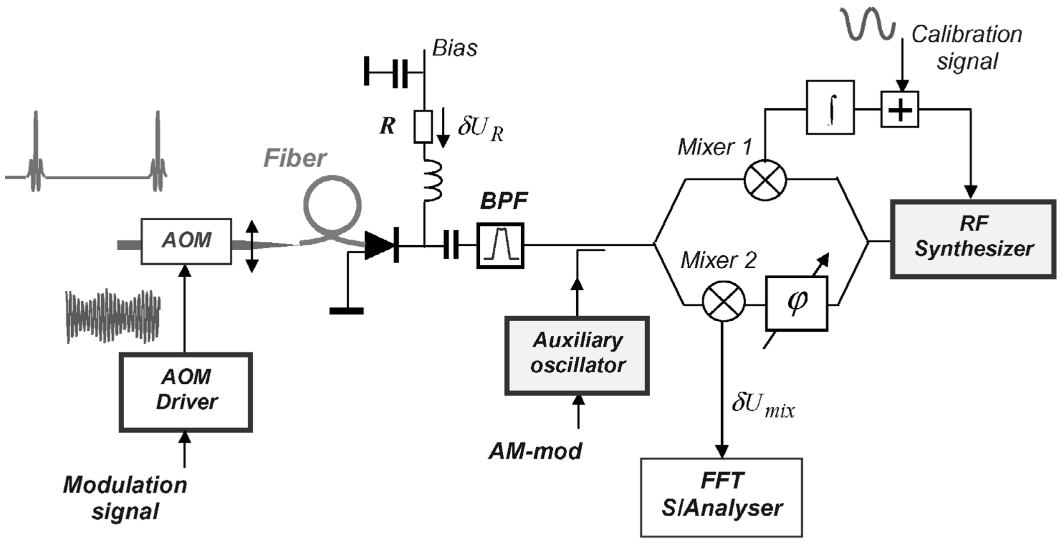

Fig. 3. Experimental setup for measurement of power-to-phase conversion in photodetectors.

This induces both AM and PM modulation sidebands around the harmonics of pulse repetition rate in the spectrum of the photocurrent. The signal at the n^{th} harmonic of pulse repetition rate is extracted from the photodetector with a bandpass filter (BPF), and the phase of the extracted signal is compared to that of a reference oscillator (RF synthesizer). Such a comparison is accomplished with a PLL controlling the frequency of the reference oscillator. With the reference oscillator phase locked, index of the induced phase modulation could be determined from the amplitude of the alternating current (AC) signal at frequency of modulation at the output of mixer 1. The main drawback of this approach, which makes interpretation of the measurement results difficult, is related to the amplitude sensitivity of the PLL. To overcome this problem, two conditions must be met. First, the modulation frequency must be chosen to be outside the PLL bandwidth in order to prevent the PM-modulation of the RF-synthesizer by power variations of the extracted microwave signal. Second, the PLL must be complemented with a second (external) channel containing a variable phaseshifter φ (see Fig. 3). The latter enables amplitude insensitive tuning of the external channel. Such a tuning is achieved by blocking the laser light and substituting the photodetector signal with that from an auxiliary oscillator, the frequency and power of which are closely matched to those of the extracted signal. By modulating the amplitude of the auxiliary oscillator, the phase shift φ is adjusted until the AC response of the external channel is cancelled.

Assuming that the amplitude sensitivity of the measurement system is suppressed, the power-to-phase conversion of the photodetector can be calculated from:

$$\frac{d\varphi}{dP} = \frac{\delta U_{ext}}{S_{PD}\delta P_{opt}}, \tag{2}$$

where δU_{ext} is the amplitude of the AC signal at the output of the external channel at the frequency of modulation f_{mod} and S_{PD} is the phase-to-voltage conversion of the external channel. It is measured by injecting a sine wave calibration signal at the FM-input of the RF synthesizer. Parameter δP_{opt} is the magnitude of optical power variations. It is calculated from: $\delta P_{opt} = \delta U_R/(\eta R)$, where δU_R amplitude of the AC signal across the photodetector bias resistor R and η is the responsivity of the photodetector.

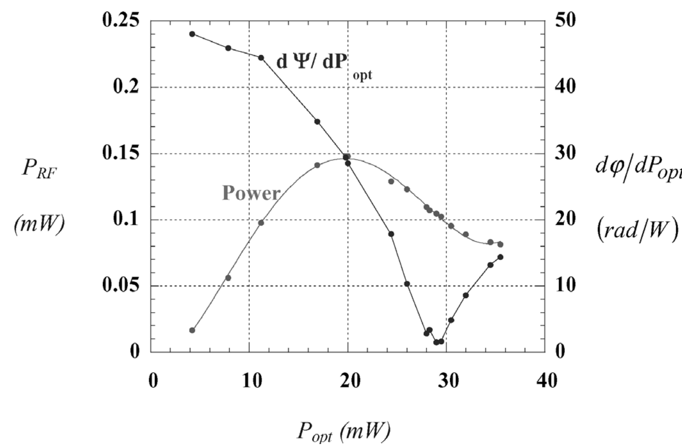

Fig. 4. Power of second harmonic of pulse repetition rate (866 MHz) and power-to-phase conversion at the same frequency as a function of average power of the optical comb. "Discovery" photodetector, load resistor 100 Ω, bias voltage 6 V.

Photodetectors (DSC30S from Discovery Semiconductors, Inc., Ewing, NJ)[1] were used and had a bandwidth ~18 GHz. Maximum power at the repetition rate of 433 MHz was approximately 0.7 mW with the average optical power 24 mW.

The results of the measurements performed with the DSC30S photodetectors at the second harmonic of pulse repetition rate are summarized in Fig. 4. Here, optical

[1]Mention of a specific product is for technical clarity only and does not represent an endorsement by the National Institute of Standards and Technology.

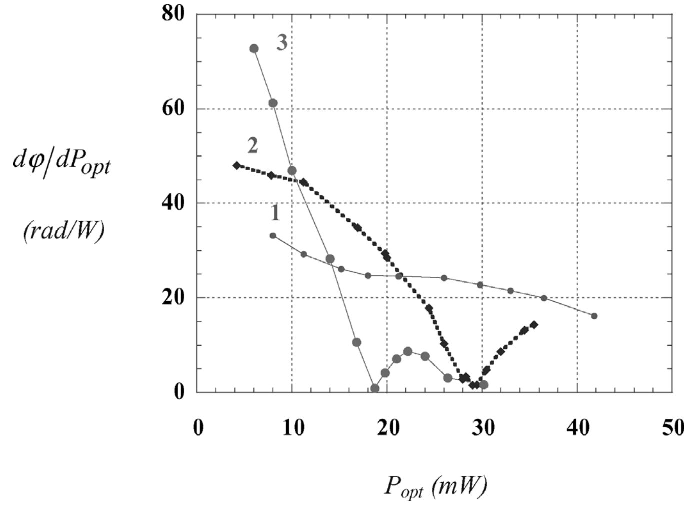

Fig. 5. Power-to-phase conversion at different harmonics of pulse repetition rate as a function of average power of the optical comb: curve 1 is for 433 MHz, curve 2 is for 866 MHz, curve 3 is for 1299 MHz.

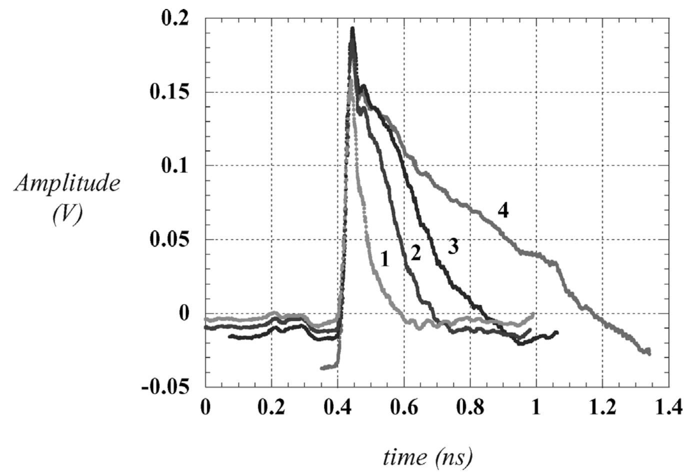

Fig. 6. Demodulated light pulses at the output of "Discovery" photodetector at different levels of optical power (photocurrent): curve 1, photocurrent $I_{DC} = 1.1$ mA; curve 2, photocurrent $I_{DC} = 3$ mA; curve 3, photocurrent $I_{DC} = 5$ mA; curve 4, photocurrent $I_{DC} = 11$ mA. Photodetector is directly connected to a matched input of a high-speed oscilloscope.

power-to-microwave phase conversion $d\varphi/dP$ and power of the extracted RF signal P_{RF} are shown as a function of average optical power P_{avg}. For example, we measured $d\varphi/dP \approx 30$ rad/W and P_{RF} (866 MHz) ≈ 0.15 mW at $P_{avg} \approx 20$ mW.

Fig. 5 shows the power dependence of $d\varphi/dP$ for the first three harmonics of the pulse repetition rate. All the dependencies are nonmonotonic with deep minima for the signals at the second and third harmonics. This is not something one may expect from the Fourier analysis of the demodulated pulse train. Indeed, a pulse from such a train is shown in Fig. 6. It was recorded with the high-speed oscilloscope directly coupled to the photodetector. The shape of the pulses is close to triangular and, therefore, the phase of the n^{th} harmonic of the pulse repetition rate is approximately:

$$\varphi_n \approx -2n\,\tau\, f_R, \tag{3}$$

where τ is a duration of the pulse measured at its base. In the above example, pulse duration increases with optical power almost linearly at the rate $d\tau/dP \approx 0.06$ ns/mW. Knowing this coefficient, the power-to-phase conversion of a photodetector can be estimated as:

$$\left|\frac{d\varphi_n}{dP}\right| = 2n\, f_R \frac{d\tau}{dP}. \tag{4}$$

Assuming that $n = 1$ and $f_R \approx 433$ MHz, one can obtain: $d\varphi_n/dP \approx 53$ rad/W. This is not very different from the measured small-signal value of $d\varphi/dP$. However, (4) does not explain the power dependence of $d\varphi/dP$. The main reason for such dependence may be related to the way the signal at the harmonics of pulse repetition rate are extracted from the spectrum of the photocurrent. The BPF used for signal extraction is only "transparent" in a narrow window around the frequency of the particular harmonic of pulse repetition rate. Other harmonics in the spectrum of the photocurrent are reflected from the filter creating standing waves in the transmission line between the filter and photodetector. This means that the regime of the photodetector may be affected by the complex impedances at frequencies of all harmonics within its bandwidth. Interaction between multiple harmonics on the photodetector could be responsible for the cancellation of power-to-phase sensitivity at certain levels of optical power. It is also worth noting that not only the magnitude of the power-to-phase conversion, but also its phase, varies as a function of optical power in the vicinity of these minima. Interestingly enough, the optical power at which the power-to-phase sensitivity vanishes, also depends on the length of a transmission line between the photodetector and BPF. This may be considered as an additional argument in favor of the above phenomenological explanation for the power dependence of $d\varphi/dP$.

IV. Excess Phase Noise Associated with Detection of Ultra-Short Light Pulses

The study of the excess phase noise was performed with a dual-channel measurement system shown in Fig. 7. Here, one channel (internal) is used for phase locking the "slave" oscillator (RF synthesizer) to a chosen harmonic of pulse repetition rate. Another channel (external) enables measurements of the differential phase fluctuations induced in the spectra of demodulated signals.

Assuming that both channels of the read-out system are phase sensitive and considering only slowly varying phase fluctuations, the root mean square voltage noise at the output of the external channel can be expressed as:

$$\delta u_{ext} \approx S_{PD}\left(\delta\varphi_{add\ 2} - \delta\varphi_{add\ 1}\right) + \delta u_{int}, \tag{5}$$

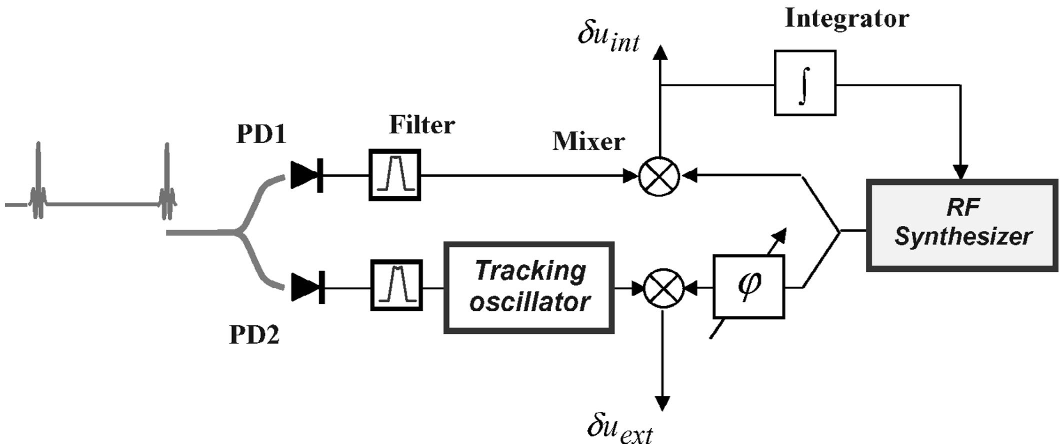

Fig. 7. Schematic of a dual-channel read-out system for measuring the excess phase noise associated with demodulation of ultra-short light pulses.

where S_{PD} is phase sensitivity of the external channel and $\delta\varphi_{add\ 1}$ and $\delta\varphi_{add\ 2}$ characterize the excess phase noise in the spectra of the signals extracted from the first and second photodetectors, respectively. The last term in (5) is approximately equal to the voltage noise at the output of the internal channel. It stems from fluctuations of the pulse repetition rate, as well as the phase noise of RF synthesizer. The effect of this noise on the resolution of spectral measurements is reduced by increasing the gain of the PLL locking the RF synthesizer to the extracted microwave signal.

The tracking oscillator and phase shifter φ in Fig. 7 are required to ensure an AM-insensitive tuning of the external channel. Such a tuning is achieved in a manner very similar to that described earlier for the optimal tuning of the measurement system in Fig. 3.

To confirm that introduction of the tracking oscillator was not followed by any additional phase noise, voltage fluctuations at the output of the external channel (δu_{ext}) were measured at different gains of the PLL referencing the tracking oscillator to incoming signal. No effect on the intensity of voltage noise was observed at Fourier frequencies of interest (below 30 Hz) when changing the loop gain by almost 40 dB.

By making use of the measurement system in Fig. 7, we measured the voltage noise spectra at the output of the external channel at different levels of optical power. The results of these measurements are shown in Fig. 8. Taking into account the differential nature of the measurement process (5), the large variations in the intensity of voltage noise with power can be attributed to a strong correlation of the induced phase noise in the spectra of the extracted microwave signals.

In addition to the noise measurements described above, we also measured the power-to-phase conversion of both photodetectors. This enabled us to determine the regimes at which the output voltage noise δu_{ext} was dominated by one particular photodetector. For example, at $I_{PD1} \approx$ 11 mA (photocurrent of the first detector) output voltage noise was due to the first photodetector only, as the

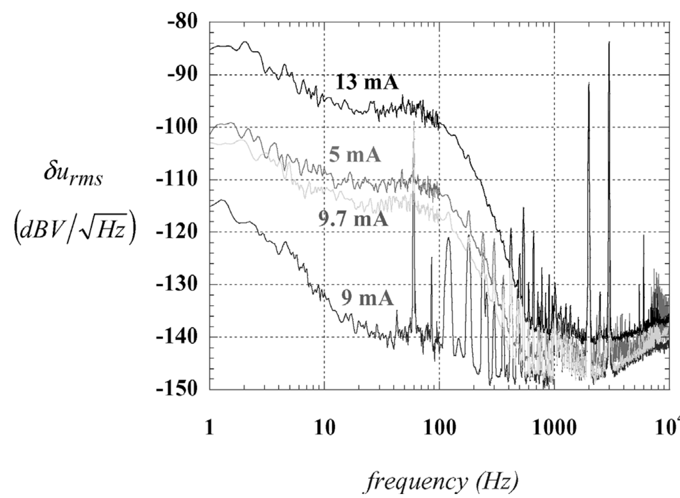

Fig. 8. Spectra of voltage fluctuations at the output of the external channel of the dual-channel phase sensitive read-out system at different levels of photocurrent. Measurements were carried out with two "Discovery" photodetectors at the frequency 866 MHz which is the second harmonic of pulse repetition rate. Bias voltages were set to 6 V, the load resistances were 100 ohm.

power-to-phase conversion of the second photodetector at this power was negligible relative to that of the first photodetector.

Interestingly, the power-to-phase conversions of two photodetectors were not exactly equal, when the strongest suppression of the output voltage noise was observed ($I_{PD1} \approx$ 9 mA, see Fig. 8). This could be explained by unequal splitting of the laser power between two photodetectors.

At the next stage of experiments, we studied the effect of optical power stabilization on the intensity of the excess phase noise. The diagram of the laser power stabilization system is shown in Fig. 9. The average power of the optical comb was stabilized by comparing the direct current (DC) voltage at the output of the first photodetector with that of a stable voltage reference and applying the differential signal (after appropriate filtering) to the AM-modulated input of the RF synthesizer driving the AOM.

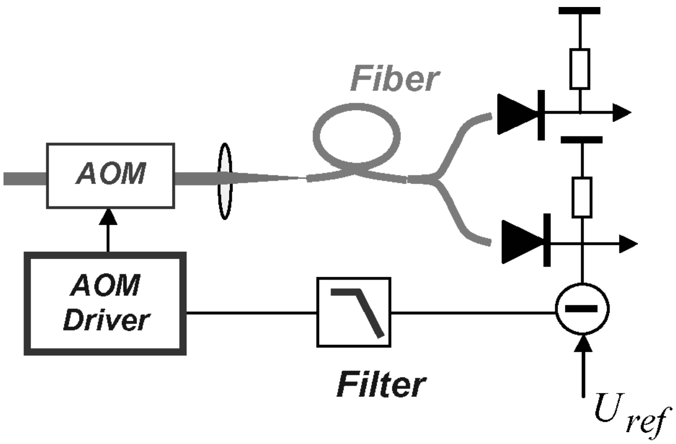

Fig. 9. Schematic of laser power stabilization system.

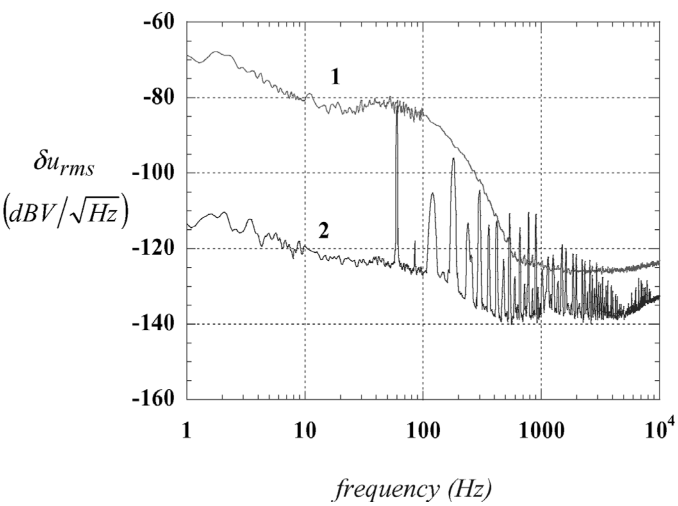

Fig. 10. Spectra of voltage fluctuations across the load resistor of a photodetector used as sensor for the control system stabilizing average power of the femtosecond pulse train: feedback loop is open (curve 1), feedback loop is closed (curve 2).

Good efficiency of the optical power stabilization system is evident from Fig. 10, which shows the spectra of voltage noise across the load resistor of the first photodetector measured with control loop activated (curve 1) and with the loop off (curve 2). These data show that activating the power control loop reduces the voltage noise by almost 40 dB within the loop bandwidth ($f \leq 300$ Hz). Voltage noise across the load resistor of the second photodetector also becomes suppressed by the power control system, but its suppression is ~20 dB. The different degree of voltage noise suppression indicates that fluctuations of optical power in two channels of the measurement system are not strictly synchronous.

It should be noted that the fiber-based measurement system in Fig. 7 appears to be immune to beam-pointing fluctuations of the femtosecond laser (such fluctuations were the main factor contributing to the output voltage noise in [6], when a glass plate beam-splitter was used for separating the channels).

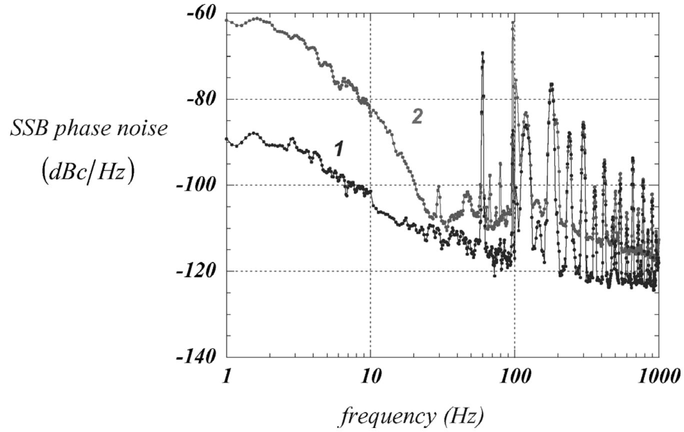

Fig. 11. Effect of power control system on the excess phase noise in the spectrum of the second harmonic of pulse repetition rate: control system is activated (curve 1), control system is off (curve 2). Photocurrent is 11.4 mA.

Having observed a strong suppression of optical power fluctuations in both arms of the read-out system, one would expect to see a similar suppression of the excess phase noise. Such suppression is clearly visible in Fig. 11, which shows the effect of the power control system on the excess phase noise in the spectrum of the second harmonic of the pulse repetition rate. To improve confidence in these measurements and extend their dynamic range, the average power of the optical pulse train was adjusted until the power-to-phase conversion of one of the photodetectors was suppressed. In such a case, with only one channel contributing to the output voltage noise, more than 30 dB of phase noise suppression was observed at $f \approx 1$ Hz regardless of which photodetector was used as a sensor of the power control system.

V. Conclusions

By using the Cr: forsterite laser as a source of infrared femtosecond pulses, we measured optical power-to-microwave phase conversion of different types of InGaAs photodetectors. The measurements were performed at the harmonics of the pulse repetition rate with a readout system immune, to the first order, to fluctuations of the average power of the optical pulse train.

At low optical power, the power-to-phase conversion was measured to be consistent with the analytical estimate based on the power broadening of the demodulated light pulses. In the large signal regime, the power-to-phase conversion varied nonmonotonically vanishing at a specific level of optical power. Some of this dependence may be explained by the resonant load of the photodetector that distorts the shape of the demodulated light pulses.

We also measured the spectral density of the excess phase noise associated with the demodulation of ultrashort light pulses. Measurements were conducted with a novel dual-channel readout system featuring a fiber-coupled beam splitter and a tracking oscillator. The former was used to improve the immunity of the measurements to the laser beam-pointing fluctuations. The latter enabled an optimal tuning of the measurement system with a strongly reduced residual AM-sensitivity.

We observed a strong suppression of the excess phase noise followed the stabilization of the average power of the optical pulse train. We attributed this effect to a strong correlation between optical power fluctuations in both channels of the measurement system. The experimental evidence of such correlation was obtained in a series of independent measurements.

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