Optical phase locking in the microradian domain: potential applications to NASA spaceborne optical measurements

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We have constructed and demonstrated a high-performance optical phase-locked loop between two cw Nd:YAG lasers. The feedback system is relatively simple, employing only the laser's internal piezoelectric transducer and an external acousto-optic modulator. A residual phase noise of the order of 1 μrad between two optical fields is achieved in the locked condition, as is verified by time- and frequency-domain analyses by the two-oscillator approach that is conventionally used in rf measurement. Potential applications to NASA space-based optical measurements are discussed.

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The ability to control precisely the frequency and phase of a coherent optical wave has a wide variety of applications in scientific research and explorations. In some demanding cases, an extremely accurate phase relationship between two optical fields is required, with a stable rf oscillator providing the appropriate precision frequency offset. The advantage of using a coherent, phase-stable source for high-precision measurement is quite clear, as such a source offers a potential measurement uncertainty of the order of $1/N^{3/2}$, where N is the total number of elapsed cycles in the measurement time period.[1] This uncertainty is in contrast to that of frequency-based experiments, for which the measurement imprecision scales as $1/N$.

In space-based measurements using coherent sources, optical approaches provide a clear advantage over microwave techniques in terms of resolution. For example, in a proposed[2] 5×10^6-km baseline laser gravitational wave interferometer, some 1.5×10^{16} optical cycles are involved in a round trip between two phase-locked lasers on their corresponding satellites; phase changes of the order of a few microradians are to be detected, leading to the desired measurement resolution of a few picometers. With three satellites, an interesting Sagnac rotation sensitivity would be obtainable. Assuming that the three satellites are placed 5×10^6 km apart, phase differences of a few microradians accumulated over the 50-s round-trip time would imply a frequency shift of 0.2 μHz, which in turn would permit the detection of Sagnac rotation of the order of 5×10^{-23} Hz, with the area and perimeter of the planar surface defined by the satellites. Representative applications include several relativity frame-dragging experiments and steering of large-baseline optical stellar interferometers. A tight new limit of the preferred-frame parameter could certainly be reached.[3]

The search for microradian phase sensitivity clearly presents a challenge to the laser stabilization technology. We are working actively toward achieving a high-performance (both long-term and short-term) single-laser frequency/phase stabilization system (with frequency instability now reaching 5×10^{-15}); here we report our recent progress on phase tracking between two independent optical oscillators. This phase-tracked master–slave laser system represents an attractive approach to development of high-power, phase-stable, tunable light sources. Such systems should also facilitate research involving Raman transitions and further investigations of quantum effects such as extreme spectral narrowing and cancellation of fluorescence decay in atoms.[4]

Rapid time scales for frequency fluctuations of optical sources present a challenge for any controller and especially for an optical phase-locked loop (PLL). The linewidth of a particular laser is usually indicative of the ease or difficulty of the phase-locking process. A narrow linewidth corresponds to a relatively slow phase diffusion rate, so a somewhat leisurely phase-correction rate can maintain phase lock. However, achieving a tighter lock requires acquisition of additional loop gain through extension of loop bandwidth. For diode-pumped solid-state Nd:YAG lasers, their free-running intrinsic linewidths are of the order of 10 kHz, leading to a few radians' phase excursion over a time scale of 0.1–1 ms. Hence a servo unity-gain bandwidth of 150 kHz is sufficient for microradian-level phase tracking between two lasers.

Figure 1(a) shows the schematic of our experiment. The optical phase difference between the two incident optical waves at 1.064 μm is detected with a signal-to-noise ratio of 55 dB at 300-kHz resolution bandwidth, with a 5-GHz frequency offset between the two lasers. For a precise frequency-tuning offset between the two phase-locked lasers, a low-noise radio frequency (rf) oscillator (reference 1; single-sideband phase noise < -110 dBc at 1-kHz offset) is used as an offset stage to downconvert the fast beat signal to a 4-MHz waveform. Low-noise double-balanced mixers (3×10^{-14} rad^2/Hz) are used as phase comparators and frequency-shifting converters.[5] The 4-MHz signal is then compared with the phase of a second stable oscillator (reference 2) to provide the error signal for the PLL. According to the Nyquist criterion, the carrier frequency of 4 MHz is fast enough to provide a sufficient sampling rate for a servo loop bandwidth of a few hundred kilohertz. This arrangement permits independent consideration of the loop design and the tuning range of the slave laser. An auxiliary downconversion stage, fed from a separate

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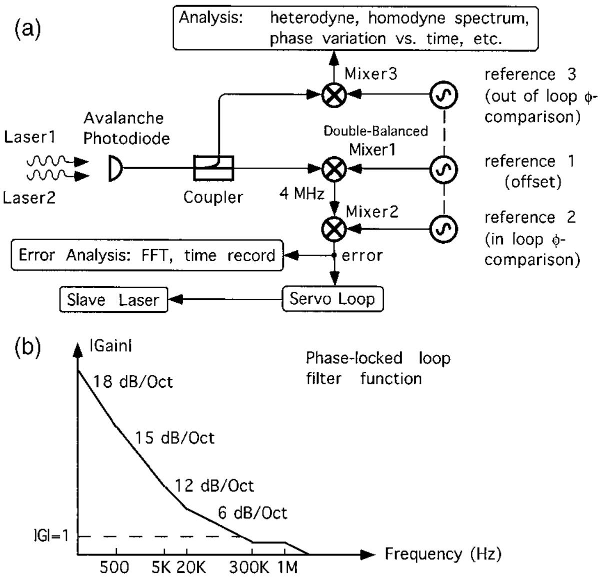

Fig. 1. (a) Experimental setup for the optical PLL between two Nd:YAG lasers. All rf reference oscillators share a common time base. (b) Filter transfer function of the PPL. Break points of the gain curve are indicated by frequency marks on the horizontal axis. A 6-dB/octave roll-off corresponds to a first-order integrator.

frequency source (reference 3), is used to monitor the optical phase variations outside the servo loop. All three rf oscillators share a common time source.

The optical frequency/phase-correction elements consist of a piezoelectric transducer (PZT) mounted upon the laser crystal and an acousto-optic modulator (AOM) placed at the output port of the slave laser. The PZT corrects any slow but potentially large laser frequency drifts, and the AOM eliminates fast frequency fluctuations. The use of an external component for laser frequency stabilization was developed by Hall and Hänsch[6] in connection with a cw dye laser, for which a fast electro-optic modulator was used in addition to the AOM to extend the servo bandwidth beyond 1 MHz. In the present context, the use of AOM alone is sufficient. The AOM provides an ~200-kHz servo bandwidth, limited by the propagation time delay of the acoustic wave inside the AOM crystal.

The crossover between the PZT and the AOM is ~20 kHz. The two servo elements are combined to form a smooth loop filter transfer function, as shown in Fig. 1(b). The unity gain bandwidth is ~150 kHz. The use of a higher-order filter function permits a more aggressive attack on the increasingly larger laser phase excursions at the low frequency while still maintaining a relatively robust time response to step changes of the optical frequency/phase. This feedback loop can also be used to stabilize a Nd:YAG laser onto a high-finesse cavity, permitting laser/cavity locking of the order of 1 mHz.[7] At time scales short compared with the cavity storage time, the cavity effectively becomes an optical phase detector, sensitive to the phase difference between the incident field and the cavity-stored-and-averaged field.

To measure and analyze the performance of the optical PLL we examined the residual phase noise at the output of the in-loop phase detector [Mixer 2, Fig. 1(a)] under the locked conditions. It is important to note that this measurement indicates only the degree of tightness of the servo loop. For example, when a noisy mixer is used as the phase detector of the locking loop, its output may still indicate a tight lock, while in actuality the mixer noise has been written back onto the slave laser.

This consideration has led us to the use of an independent mixer (Mixer 3) outside the PLL to measure the phase difference between the optical beat and the rf reference. Although we have also tried to measure the phase jitter in the time domain between the two locked rf waveforms, we found that a frequency counter's internal jitter prevents precision phase measurement below 100 μrad. A low-noise mixer becomes a natural choice for making such demanding measurements, as adopted routinely in the rf and microwave community.[8] Besides such direct phase comparison between the requested and obtained laser beat waveforms, we can also perform a heterodyne measurement between the optical beat and reference 3 [Fig. 1(a)] with an imposed frequency offset. This result offers a direct frequency spectral analysis of the phase-locked beat waveform. The heterodyne carrier frequency can be shifted conveniently to a low value in the audio range where a high-resolution, fast-Fourier-transform spectrum analyzer suits the application.

Figure 2 shows the downconverted heterodyne beat spectrum from the output of Mixer 3. Here reference 3 was tuned 1 kHz away from the phase-locked optical beat, and the optical phase spectrum was analyzed about this 1-kHz downconverted carrier. Inasmuch as the ratio between the single-sideband noise power density and the carrier power is directly related to the phase spectral density $S_\phi(f)$, the beat signal indicates that $S_\phi(f)$ approaches -110 dB rad/$\sqrt{\mathrm{Hz}}$ (~3 μrad/$\sqrt{\mathrm{Hz}}$). However, estimation of the quantity $S_\phi(f)$ based on such a heterodyne method is necessarily conservative, as AM noise is included. Furthermore, this approach requires a huge dynamic range for resolution of the extremely small phase-noise modulations on the carrier.

To close in on the phase-noise sidebands near the carrier we progressively reduced the resolution bandwidth on the fast-Fourier-transform analyzer to smaller values. Three representative data curves are shown in Fig. 2. As the resolution bandwidth is reduced from 31 Hz to 0.95 mHz, the noise floor is not reduced by the corresponding bandwidth ratio. This is so because

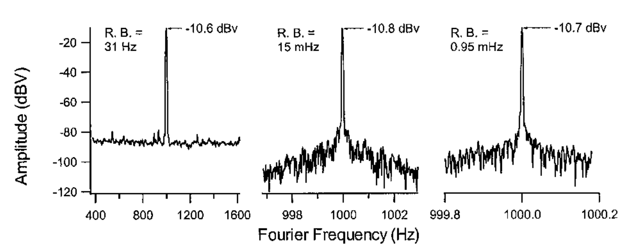

Fig. 2. Fourier spectra of the optical heterodyne beat signal after it was downconverted to a 1-kHz carrier frequency. R.B's, resolution bandwidths of the analyzer.

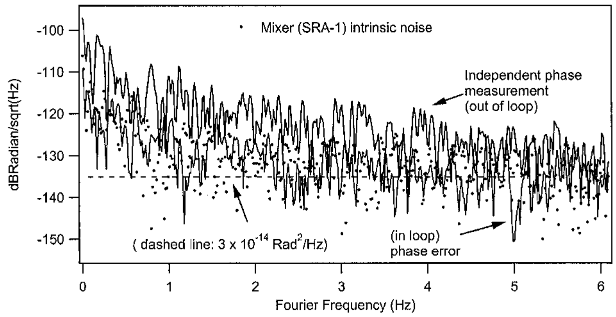

Fig. 3. Frequency spectrum of the mixers' output, indicating the phase-noise spectral density inside (lower solid curve) and outside (upper solid curve) the servo loop. The dotted curve is calibration of the noise floor for the mixer.

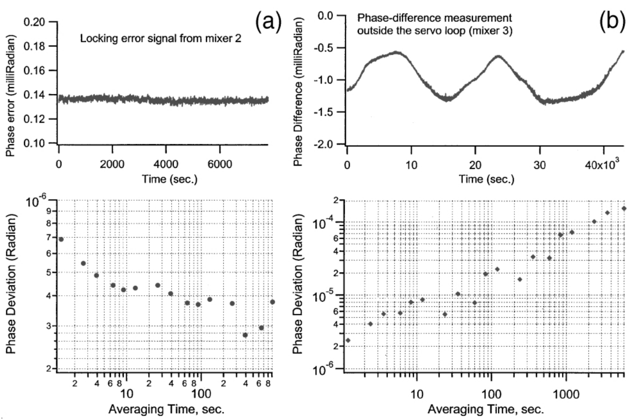

Fig. 4. Time records of the phase measurement inside (a) and (b) outside the servo loop. Phase Allan variances are calculated from these time records.

the noise power previously unresolved from the carrier starts to emerge from the carrier. The reference oscillators also contribute to the phase noise of the final downconverted signal. On the other hand, the carrier amplitude remains a constant, within the measurement uncertainty of the analyzer. At the resolution bandwidth of 0.95 mHz, the optical beat linewidth is still unresolved. Such behavior is characteristic of a tightly locked PLL system.

The direct phase-noise measurement was carried out with baseband (homodyne) detection from the mixers. (The rf reference 3 was now tuned onto the beat signal itself.) We used two identical mixers for the PLL (Mixer 2) and for the out-of-loop phase detection (Mixer 3). Mixer 3 dc output was maintained at 0 V by use of an appropriate phase shifter. Figure 3 shows the frequency spectrum of the mixers' output, in units of dB rad/$\sqrt{\text{Hz}}$. First we calibrated the intrinsic noise floor of the mixers. A single low-phase-noise oscillator provided both inputs to the mixer with an appropriate phase shift ($\sim\pi/2$) between the two paths such that the mixer's dc output was zero. The calibration result is shown by the dotted curve. Calibrated baseband amplifiers ($\times 500$) were used at the mixers' output to reduce the influence of the external noise pickup after the mixing process. The bottom solid curve in Fig. 3 represents the phase-noise spectral density produced by the in-loop phase detector. This curve roughly overlaps the mixer noise floor. We thus confirm the existence of an extremely tight feedback loop that essentially tracks out any frequency/phase difference between the optical beat and the rf reference down to the mixer noise limit, which is approximately 3×10^{-14} rad^2/Hz at a Fourier frequency of 1 Hz and above. The upper solid curve in the figure is a measurement of the out-of-loop phase spectral density at Mixer 3. The result is approximately 5–10 dB worse than the in-loop measurement, of the order of 3×10^{-13} rad^2/Hz. If one chooses a measurement bandwidth of 10 Hz, the residual rms phase fluctuation is ~ 1.7 μrad, which implies that the loss of the carrier power to the phase-noise modulation is less than 2×10^{-12}.

From the time records of the mixers' output, we used an algorithm similar to that which is employed in frequency Allan variance calculations to obtain phase variance results,[9] as shown in Fig. 4. A long-time ($\sim$12 h) record of the phase- measurement result from Mixer 3 was made, as it reflected the out-of-loop evolution of the phase difference between the optical beat and the rf reference. (A temperature severo on the laser crystal prevented saturation of the PZT voltage.) The data show increased phase fluctuation, owing in main part to the instability of lengths of electric cables. However, it is satisfactory to find that the peak-to-peak phase variation was below 1 mrad.

A more rigorous test of the optical PLL will use a second photodiode to measure an independent optical beat and then analyze its phase-noise spectrum. However, this procedure will have to be carried out inside an evacuated and vibration-free environment, because any small fluctuations of the air's refractive index and mirror positions will cause a significant amount of optical phase shift at the microradian level. We believe that a space-based environment should be ideal for testing and application of this experiment.

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