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# A Quantum-Based Microwave Power Measurement Performed With a Miniature Atomic Fountain

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*Abstract* - **A microwave power standard directly traceable to a fundamental atomic process could potentially reduce the uncertainties associated with microwave power measurement. We report on a proof-of-principle experiment to measure microwave power by observing Rabi oscillations in the clock transition of $^{133}$Cs atoms. The measurements were performed using a miniature atomic fountain.**

***Keywords*** **- Atomic fountain, microwave power**

## I. Introduction

The uncertainties associated with standard techniques of microwave power measurement are large by modern standards. Typical values for the uncertainties are 0.24 % at 50 MHz and 2.5 % at 50 GHz (2σ values). The most accurate method currently used for measuring microwave power is based on the principle of DC substitution [1].

This study employs a miniature atomic fountain designed as a frequency standard [2] to measure microwave power. $^{133}$Cs atoms in the miniature fountain were exposed to a microwave pulse as they traveled upward through the fountain's Ramsey cavity. The microwave field caused the population to oscillate between the hyperfine ground states at the Rabi frequency, which is proportional to the amplitude of the microwave magnetic field in the cavity. The observed Rabi frequency was used to determine the microwave power applied to the cavity. The data show that the number of Rabi cycles scales as expected with cavity transit time and microwave power to within the statistical uncertainty of 0.4 %. The absolute values of the power applied to the Ramsey cavity as determined by the Rabi flopping measurement agree with values found using conventional techniques at the 5 % level.

Complementary experiments have also been performed at the National Research Council in Canada [3]. There are significant differences between the two experiments. In the Canadian experiment, rubidium atoms were dropped from a magneto-optical trap (MOT) and microwaves were applied to the atoms from an open-ended waveguide through the walls of the glass vacuum chamber. We launched cesium atoms in an atomic fountain and applied the microwaves within the fountain's Ramsey cavity.

## II. Experiment

A diagram of the miniature fountain apparatus is shown in Fig. 1. A collection of ~$10^7$ cesium atoms at a temperature of a few microkelvin is trapped and cooled using a vapor-cell MOT and an optical molasses. After collecting the atoms in the MOT for ~300 ms, the magnetic field is turned off and the atoms are further cooled in an optical molasses. The atoms are launched vertically in 1 ms by shifting the relative frequency between the upward- and downward-traveling laser beams. The atoms are then cooled in the moving frame for 2 ms before the beams are turned off, and they exhibit a fountain trajectory under the influence of gravity. The launched atoms are evenly spread among the nine magnetic sublevels ($m_F$ = -4 to +4) of the $|F{=}4\rangle$ hyperfine level of the $6^2S_{1/2}$ ground state. $F$ is the total angular momentum quantum number and $m_F$ is its projection along the DC magnetic field.

After being launched, the atoms first encounter a rectangular state-selection cavity operating in the $TE_{104}$ mode. Microwaves resonant with a transition between the $|F{=}4, m_F{=}0\rangle$ and $|F{=}3, m_F{=}0\rangle$ hyperfine ground states are fed into the cavity. The microwave amplitude is adjusted such that the atoms are exposed to a π pulse of microwave radiation as they traverse the cavity and the population in the $|F{=}3, m_F{=}0\rangle$ state is maximized. After the atoms leave the cavity, all residual atoms in the $|F{=}4, m_F \neq 0\rangle$ states are removed with a pulse of light along the axis of the toss tube. The remaining atoms are all in the $|F{=}3, m_F{=}0\rangle$ state.

The state-selected $|F{=}3, m_F{=}0\rangle$ atoms then pass through a Ramsey cavity, where the quantum-based measurements are performed. The Ramsey cavity is cylindrical with height 1.98 cm and radius 3.5 cm, and operates in the $TE_{011}$ mode. The mode is excited with one of two coupling loops (~3 mm diameter) located 180° apart in the cavity midplane. Large-diameter apertures (1.4 cm) in the center of the end caps are

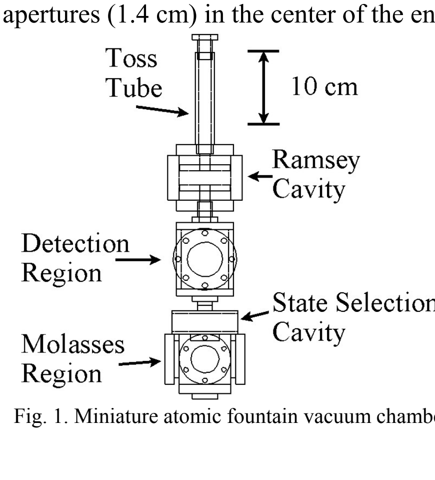


Fig. 1. Miniature atomic fountain vacuum chamber.

designed to allow a high atom flux through the cavity. This increases the detection signal level at the expense of perturbations to the field structure.

After the atoms leave the Ramsey cavity, the microwave power fed to the cavity is extinguished and the atoms fall back through the detection region. There, the fraction of atoms that have changed state is measured as a function of applied microwave power.

The detected fraction in the $|F=4\rangle$ state is

$$R = \frac{1-\cos(\Omega_{R,\mathrm{eff}} t_{\mathrm{cav}})}{2}, \tag{1}$$

where

$$\Omega_{R,\mathrm{eff}} = \frac{\mu_0 \mu_B g_J M H_{\mathrm{eff}}}{\hbar} \tag{2}$$

is the Rabi frequency, $t_{\mathrm{cav}}$ is the cavity transit time, $\mu_0$ is permeability of free space, $\mu_B$ is the Bohr magneton, $g_J$ is the electron $g$ factor, $M = \langle F,m_F | J_z | F+1,m_F\rangle$, $J_z$ is the component of the electron angular momentum parallel to the DC magnetic field, and $H_{\mathrm{eff}} = H_{\mathrm{peak}} \cdot 2/\pi$ is the effective microwave magnetic field amplitude. The factor of $2/\pi$ is a normalization factor that relates the average field seen by the atoms to the peak field at the cavity center.

The apparatus employed to measure the applied microwave power by conventional means is shown in Fig. 2 [4]. During the measurements of Rabi oscillations, a diode power sensor monitored $P_2$, the power delivered to port 2 of a 20 dB coupler, while the Ramsey cavity and its transmission line feeds were connected to port 3 of the coupler. The diode sensor was a continuous-wave sensor with wide dynamic range. Its nonlinearity was less than 3 % as used in this experiment. The power fed to the fountain apparatus was varied with an attenuator. The proportionality constant $c$ between $P_2$ and $P_{\mathrm{inc}}$, the incident power at port 3, was determined by attaching a thin-film bolometric standard to port 3 in place of the fountain apparatus. $P_{\mathrm{inc}}$ as determined by conventional techniques is given by

$$P_{\mathrm{inc},m} = c \cdot P_2 \frac{\left|1-\Gamma_g \Gamma_S\right|^2}{\left|1-\Gamma_g \Gamma_R\right|^2}. \tag{3}$$

The subscript $m$ indicates that the incident power was determined by conventional techniques. The ratio on the right-hand side of the equation is the mismatch factor correction. $\Gamma_g$ is the equivalent generator reflection coefficient of port 3 of the coupler, and $\Gamma_S$ and $\Gamma_R$ are, respectively, the reflection coefficients of the standard and of the combined Ramsey cavity and transmission lines. The reflection coefficients were measured with a vector network analyzer (VNA).

## III. RESULTS

Data were collected for toss heights of 32 and 45 cm. These heights represent the upper and lower limits for the apparatus, which are determined by the heights of the toss tube and the Ramsey cavity above the molasses region. In Fig. 3, we show the ratio $R = N_4/(N_3+N_4)$ plotted versus $\sqrt{P_{\mathrm{inc},m}} \cdot t_{\mathrm{cav}}$, where $N_4$ ($N_3$) is the number of atoms detected in the $F$=4 ($F$=3) state. $t_{\mathrm{cav}}$ was 18.1 and 10.3 ms for the 32 and 45 cm toss heights, respectively. $\sqrt{P_{\mathrm{inc},m}}$ is proportional to the microwave magnetic field amplitude. The data are presented in this way to demonstrate the relative agreement between the two data sets. The data were fit to damped cosines with the argument $f \cdot P_{\mathrm{inc},m}^{1/2} \cdot t_{\mathrm{cav}}$. The 0.1 % agreement for the fitted values of $f$ is within the statistical uncertainty of the measurements (0.4 %).

The damping of the Rabi oscillations results most likely from the combination of two factors. For one, the RF field amplitude over the aperture in the Ramsey cavity is not

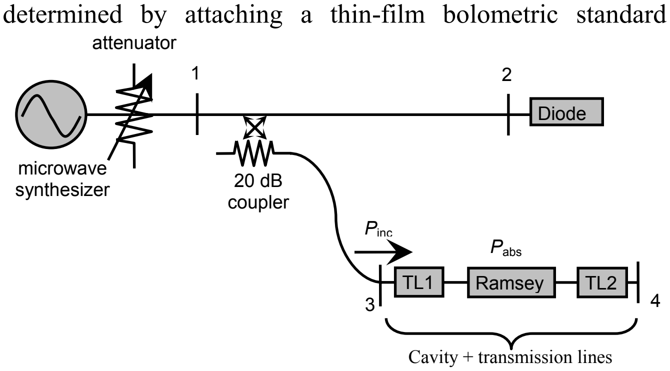


Fig. 2. The apparatus used to measure microwave power with conventional techniques.

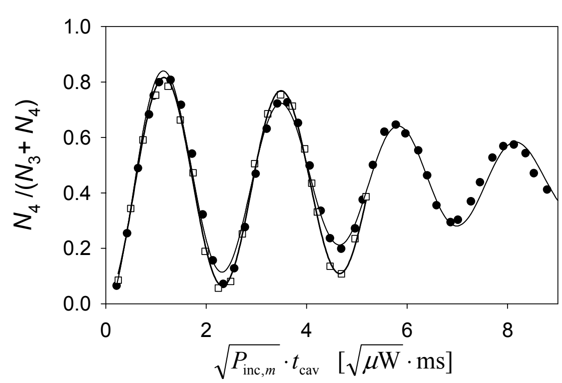


Fig. 3. Fraction of atoms in $F$=4 versus $P_{\mathrm{inc},m}^{1/2} \cdot t_{\mathrm{cav}}$ for two toss heights (● - 32 cm, ❒ - 45 cm).

constant, so different atoms experience slightly different field strengths depending on their trajectory through the cavity. Another cause of damping is the distribution of velocities in the atom cloud. To confirm that these factors are responsible for the damping would require numerical modeling to find the distribution of microwave exposure levels. This analysis has not been performed.

The absolute value of $H_{\mathrm{eff}}$ can be obtained if the phase of the Rabi oscillation signal is known. At the first maximum of the oscillation,

$$\Omega_{R,\mathrm{eff}} \cdot t_{\mathrm{cav}} = \pi. \tag{4}$$

This corresponds to

$$H_{\mathrm{eff}} = \frac{\pi \hbar}{\mu_0 \mu_B t_{\mathrm{cav}} g_J M}, \tag{5}$$

which yields $H_{\mathrm{eff}} = 1.57$ and $2.77 \times 10^{-3}$ A/m respectively for the 32 and 45 cm tosses.

## IV. Absolute comparison

To obtain the incident power based on the Rabi measurement we evaluated the expression

$$P_{\mathrm{inc},R} = \frac{P_{\mathrm{inc}}}{P_{\mathrm{abs}}} \frac{P_{\mathrm{abs}}}{W_{\mathrm{cav}}} \frac{W_{\mathrm{cav}}}{H_{\mathrm{eff}}^2} H_{\mathrm{eff}}^2, \tag{6}$$

where $P_{\mathrm{abs}}$ is the energy absorbed in the antenna coupling structure and the walls of the cavity, and $W_{\mathrm{cav}}$ is the stored energy in the cavity. The first ratio $P_{\mathrm{inc}} / P_{\mathrm{abs}}$ was obtained from the measured $S$ parameters of the combined Ramsey cavity and transmission lines (located between ports 3 and 4 in Fig. 2.) The $S$ parameters are the frequency-dependent complex reflection and transmission coefficients. We assumed that far off resonance the cavity reflects all power and thus used the off-resonant $S$ parameter measurements to determine the properties of the transmission lines. This allowed us to deduce the fraction of power absorbed in the cavity and coupling structure, which is the inverse of the first ratio. The second ratio can be rewritten as $P_{\mathrm{abs}} / W_{\mathrm{cav}} = \omega / Q_L$, where $\omega$ is the operating frequency and $Q_L$ is the loaded quality factor of the cavity. We assumed that the power stored in the antenna coupling structure is negligible. The third ratio, $W_{\mathrm{cav}} / H_{\mathrm{eff}}^2$, was estimated by means of analytical expressions for an ideal cavity without apertures. The results of this analysis for the first peak in the Rabi oscillations are presented in Table 1.

TABLE I
COMPARISON OF TRADITIONAL AND RABI POWER MEASUREMENTS OF POWER REQUIRED FOR A π PULSE

| | $P_{\mathrm{inc},m}$ | $P_{\mathrm{inc},R}$ |
|---|---|---|
| 32 cm toss | 4.16 nW | 4.35 nW |
| 45 cm toss | 12.94 nW | 13.56 nW |

The values of $P_{\mathrm{inc}}$ determined from conventional and Rabi oscillation measurements differ by less than 5 %.

The uncertainty in $P_{\mathrm{inc},R}$ is at least 25 %. The largest contribution is due to an uncertainty in the cavity temperature. The cavity was operated ~800 kHz away from resonance and was not temperature controlled. A change of 1 K in the cavity temperature between the periods when the fountain measurements and VNA measurements were performed corresponds to a change of 25 % in the first ratio in (6). The inherent inaccuracy of the $S$ parameter measurements also contributes an uncertainty of order 10 % in these measurements. These uncertainties can be substantially reduced with the same equipment by heating and temperature-controlling the Ramsey cavity to match the cavity resonance frequency to the clock transition frequency. The cavity's resonance frequency shifts at a rate of 155 kHz/K. Further reductions of the uncertainty can be obtained by building a new apparatus specifically designed for measurements of microwave power.

Future measurements will be conducted with the primary frequency standard NIST-F1. Although this apparatus was also not designed for microwave power measurements, it will likely lead to much more precise measurements because its Ramsey cavity is temperature controlled. NIST-F1 also has a larger range of available toss heights.

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