ANALYSIS OF NOISE MECHANISMS LIMITING FREQUENCY STABILITY OF MICROWAVE SIGNALS GENERATED WITH A FEMTOSECOND LASER

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1. Introduction

Femtosecond laser is a key element of a coherent link between optical and microwave domains. Such a link has already enabled measurements of optical frequencies with an accuracy of a current time standard - Caesium beam microwave 'clock' [1, 2]. Making use of a femtosecond laser it is also possible to transfer the frequency stability of an optical 'clock' to radio-frequencies. This may result in redefinition of a 'second' when optical frequency standards supersede their microwave counterparts in terms of accuracy [3, 4].

Time transfer from optical to microwave frequencies is realised by referencing a femtosecond laser to an optical 'clock' and extracting a microwave signal at one of the harmonics of pulse repetition rate. Recent experiments at NIST indicate that fractional instability of pulse repetition rate of a phase-locked femtosecond laser can be as good as that of an optical 'clock' with uncertainties not exceeding $7 \cdot 10^{-16}$ over $1 s$ of averaging [5]. By referring to these results it should be noted that they were deduced from measurements of optical beat notes between two femtosecond combs referenced to the same optical frequency standards without a direct comparison of two microwave signals. So far, measurements of frequency stability of microwave signals synthesised with femtosecond lasers failed to confirm the above results indicating at least two orders of magnitude worse noise performance. The main reason for such a discrepancy was found to be related to an excess phase noise associated with a microwave signal extraction system of an optical frequency synthesiser.
In this work we discuss the basic noise mechanisms affecting the frequency stability of a microwave signal produced with a femtosecond laser, including power-to-phase conversion in the photodetector and laser beam-pointing fluctuations.

II. Synthesis of microwave signals with femtosecond lasers

As was experimentally established, spectrum of a femtosecond laser consists of equidistantly spaced spectral lines at frequencies

$$f_n = nf_R + f_o \quad , \qquad (1)$$

where n is a large integer, f_R and f_o are pulse repetition rate and offset frequency, respectively [6]. Both frequencies (f_R and f_o) must be controlled with high degree of precision to ensure a phase coherence across the entire optical comb. This is achieved by stabilising frequencies f_R and f_o either with respect to a microwave 'clock', for example, a hydrogen maser, or an optical frequency standard, such as a cold atom Ca-clock [5]. The latter approach was taken in the construction of NIST optical synthesiser (Fig. 1).

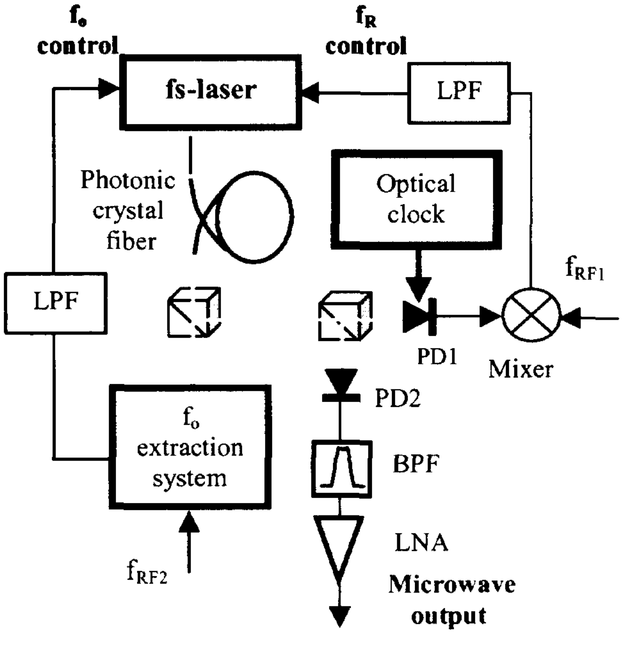

Fig. 1. NIST optical synthesiser

Here, the beat note between the optical 'clock' and one of the comb spectral lines is referenced to RF-synthesiser (f_{RF1}) by controlling the laser pulse repetition rate. Offset frequency f_0 is also stabilised by mixing a frequency doubled infrared part of the optical comb with the green part and phase-locking the resulting beat note to another stable RF-oscillator (f_{RF2}). With both frequencies (f_R and f_0) stabilised, the optical frequency synthesiser can serve as a source of a microwave signal with almost the same fractional frequency stability as that of an 'optical' clock, provided that the useful signal (at one of harmonics of pulse repetition rate) is extracted from the output of the 'in-loop' photodetector PD1 (Fig. 1). The disadvantage of this type of signal extraction is related to a small power of the useful signal as the optical comb is band-pass filtered before falling on the photodetector. This filtering was proposed to improve the shot noise floor of the phase-locked loop which controls the beat frequency. On the other hand, there are no experimental data regarding the value of such a noise floor. For this reason it is important to investigate to what extent the power of the useful signal can be increased before its frequency stability becomes compromised.

A relatively strong microwave signal can be extracted by diverting a significant fraction of the optical comb to the 'external' photodetector PD2 (Fig. 1). The main drawback of this scheme is a loss of coherence between the opposite ends of a frequency chain due to the noise sources associated with the operation of a microwave signal extraction system. Below we analyse some noise mechanisms affecting the frequency stability of the microwave signal extracted 'externally'.

III. Noise properties of microwave signal extraction system

First, we estimated the effect of laser shot noise on frequency stability of the microwave signal by utilising an analytical expression for Square Root Allan Variance of fractional fluctuations of pulse repetition rate from [4]:

$$\sigma_y^{shot}(\tau) \approx \frac{1}{2\pi \tau n f_R}\sqrt{\frac{6 e \bar{I} \Delta f R}{P_{signal}}}, \qquad (1)$$

where τ is an integration time, Δf - bandwidth of the filter used for extraction of the microwave signal at frequency $n f_R$, $\bar{I}$ - is an average photocurrent, e - charge of electron, R - load impedance of a photodetector and P_{signal} - power of the signal at frequency $n f_R$.

For a typical high-speed photodetector illuminated by a train of femtosecond light pulses with an average power of $5\,mW$, the signal power was measured to be $0.3\,mW$ at frequency $1\,GHz$. By assuming that $R \approx 50\,\Omega$ and $\Delta f \approx 100\,kHz$, it results in $\sigma_y^{shot}(1\,s) \approx 10^{-15}$ which can be further improved by increasing the harmonic number n.

Secondly, phase fluctuations of the bi-polar transistor amplifier in the path of the extracted microwave signal were measured. The results of these measurements performed at different levels of input power P_{inp} are shown in Fig. 2. At $P_{inp} \approx -15\,dBm$ the limit imposed by amplifier phase fluctuations on frequency stability of the extracted microwave signal frequency $f_R \approx 1\,GHz$ was evaluated as $\sigma_y(1\,s) \approx 3 \cdot 10^{-16}$.

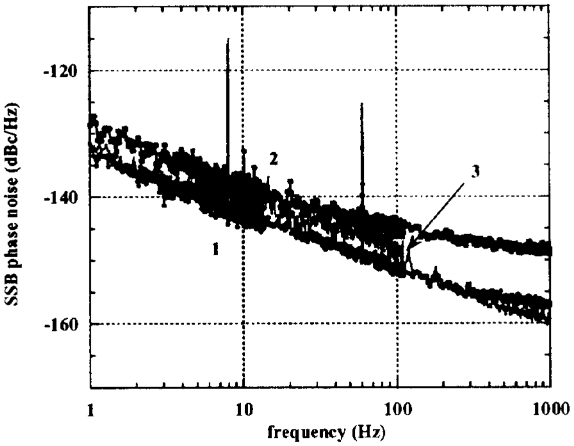

Fig. 2. Specra of phase fluctuations of $1\,GHz$ bi-polar transistor amplifier: $P_{inp} = -25\,dBm$ (curve 1), $P_{inp} = -15\,dBm$ (curve 2), $P_{inp} = -5\,dBm$ (curve 3).

The further study of noise properties of microwave signal extraction system was conducted with the experimental set-up shown in Fig. 3. Such a set-up permits extraction of a signal at frequency of pulse repetition rate and its phase locking to an external RF-synthesiser (master oscillator). Once the pulse repetition rate is stabilised, its residual fluctuations are measured. These measurements involve: (i) offsetting frequency of one of the extracted signal, (ii) recombining two signals at the non-linear element (mixer) and (iii) counting the beat note. In making these measurements one can ether choose to compare the signals extracted from PD1 and PD2 as shown in Fig. 3 or, alternatively, relate each extracted signal to that of a master oscillator. We've chosen the latter approach as allowing the source of the additional phase noise to be located unambiguously.

The fractional frequency resolution of the measurement system in Fig. 3 was determined to be of 10^{-15} over $1s$ of integration time. This resolution was achieved by utilising a digital signal generator referenced to the hydrogen maser for producing an offset frequency between two microwave signals. The offset frequency was chosen to be $10kHz$ to minimise the counter's trigger error.

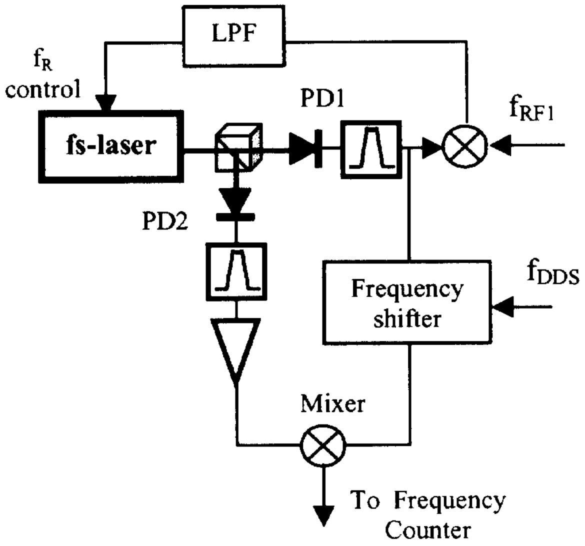

Fig. 3. Experimental set-up for measurement of differential frequency fluctuations in time domain.

Measurements of differential frequency fluctuations between signals extracted from the 'in-loop' photodetector and RF-synthesiser did not reveal any excess noise above the measurement system noise floor. On the other hand, differential frequency fluctuations between the signal extracted from the 'external' photodetector PD2 and master oscillator were measured to be $60\,\mu Hz$ at $\tau = 1s$, indicating an additional noise associated with the signal extraction system.

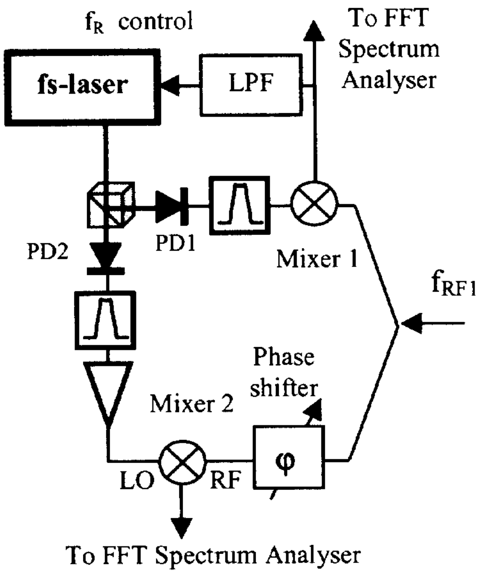

Fig. 4. Experimental set-up for measurement of phase fluctuations of extracted microwave signals

Measurements in time domain were complemented by those in frequency domain. The experimental set-up in Fig. 4 enabled simultaneous measurements of phase fluctuations of two extracted microwave signals relative to the master oscillator. The residual amplitude sensitivity of the external phase bridge based on mixer 2 was decreased by connecting the photodetector PD2 to the local oscillator port of the mixer 2. Fine tuning of the external phase bridge was accomplished by modulating the amplitude of the RF-synthesiser and minimising (with a phaseshifter φ) the amplitude of the output signal at frequency of modulation .

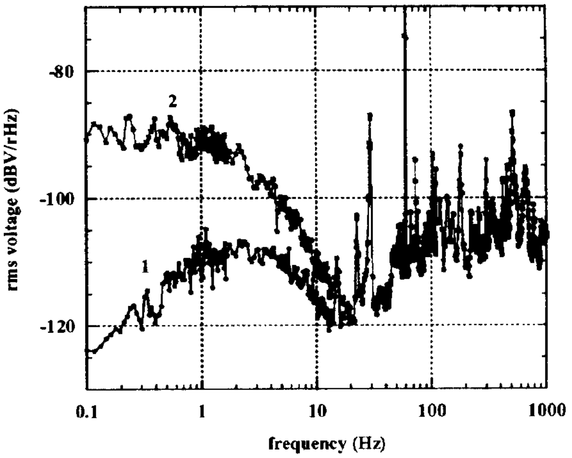

Fig. 5. Spectra of voltage fluctuations at the output of 'in-loop' (curve 1) and 'external' (curve 2) phase detectors.

Spectra of voltage fluctuations at the output of noise measurement system are given in Fig. 5. They show that the intensity of voltage fluctuations at the output of the 'external' mixer 2 (curve 2) is much higher than that at the output of the 'in-loop' mixer 1 (curve 1) which confirms the time domain observations regarding the increased frequency instability of the microwave signal extracted from the PD2.

Intensity of the additional phase noise was found to be independent on gain of the phase-locked loop. This is typical for noise mechanisms which originate inside the control loop. One of such intrinsic noise mechanisms is associated with a power-to-time delay conversion in a photodetector. Such a conversion was first discovered while studying the demodulation of femtosecond light pulses with an ultra-fast oscilloscope. It manifests itself as a power dependent time delay between optical and video pulses at the output of a photodetector. This effect can, therefore, lead to a timing jitter of the demodulated pulse train due to the laser light intensity fluctuations.

Introducing the spectral density of time delay fluctuations as S_τ, the spectrum of the additional phase noise can be found from the time shifting property of the Fourier transform:

$$S_\varphi^{add} = (2\pi n f_R)^2 \, S_\tau . \qquad (2)$$

Trying to understand the origin of the additional phase noise we measured the intensity of this noise at different levels of power fluctuations of the femtosecond laser. This was accomplished by stabilising the laser power with an acousto-optical modulator (AOM) as shown in Fig. 6. The amplitude detector (AD) in Fig. 6 acted as a sensor of the light intensity control system.

To our initial surprise, we did not see any noticeable effect of the power control system on the phase noise of the extracted microwave signal. We also found that stabilising light intensity in one arm of the optical readout system was accompanied by enhanced light intensity fluctuations in another arm, with the enhancement factor close to $3\,dB$.

IV. Discussion

1. Light intensity fluctuations

Even without specifying the noise mechanism responsible for the 'anomalous' behaviour of the amplitude control system, such a behaviour can be understood by assuming that fluctuations of the light intensity in the arms of the optical readout system (i) originate from the common source within the control system, (ii) have similar amplitudes and (iii) vary in opposite phases. Power fluctuations of the femtosecond laser do not satisfy the above criteria and, therefore, other noise mechanisms must be considered.

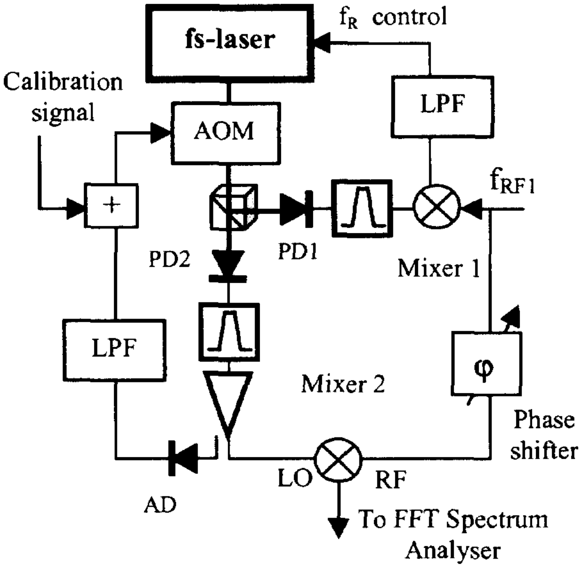

Fig. 6. Phase noise measurement system with light intensity stabilisation

Polarisation fluctuations of the laser light have been ruled out as a possible cause of the induced intensity noise, since insertion of a polariser between the acousto-optical modulator (AOM) and a beam-splitter did not affect the noise properties of the measurement system in Fig. 6.

Angular fluctuations of laser beam incident on the air-to-dielectric interface cause fluctuations of light intensity due to the angular dependence of the reflection coefficient as illustrated by Fig. 7. It is also worth noting, that power fluctuations induced in this case have opposite signs for reflected and transmitted beams.

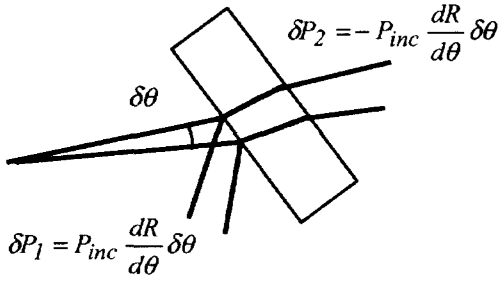

Fig. 7. Conversion of angle fluctuations into intensity fluctuations.

In general case, when both power and direction of the light beam incident on the beam-splitter fluctuate, power fluctuations of the reflected, δP_1, and transmitted, δP_2, beams are given by

$$\delta P_1 = R\ \delta P_{inc} + P_{inc}\ \dot{R}_\theta\ \delta\theta \qquad (3.1)$$

$$\delta P_2 = (1-R)\ \delta P_{inc} - P_{inc}\ \dot{R}_\theta\ \delta\theta\ , \qquad (3.2)$$

where δP_{inc} denotes intensity fluctuations of the laser light incident on a beam-splitter, $\delta\theta$ beam-pointing fluctuations, R and $\dot{R}_\theta$ are reflection coefficient and its angle derivative.

When power control loop is closed, light intensity fluctuations of the reflected and transmitted beams δP_1^{lock} and δP_2^{lock}, respectively, become:

$$\delta P_1^{lock} = \frac{R\ \delta P_{inc}}{1+\gamma} + P_{inc}\ \dot{R}_\theta\ \delta\theta\left(1+\frac{R}{1-R}\ \frac{\gamma}{1+\gamma}\right) \qquad (4.1)$$

$$\delta P_2^{lock} = \frac{(1-R)\ \delta P_{inc}}{1+\gamma} + \frac{P_{inc}\ \dot{R}_\theta\ \delta\theta}{1+\gamma}\ , \qquad (4.2)$$

where γ is a gain of the power control loop.

Assuming that $R \approx 1/2$ and $|\gamma| >> 1$, equations (4.1, 4.2) are simplified:

$$\delta P_1^{lock} \approx 2\, P_{inc}\ \dot{R}_\theta\ \delta\theta \qquad (5.1)$$

$$\delta P_2^{lock} \approx 0\ . \qquad (5.2)$$

On the other hand, from experimental observations it follows

$$\delta P_1^{lock} \approx 2\, \delta P_1\ , \qquad (6)$$

which means that

$$P_{inc}\ \frac{dR}{d\theta}\ |\delta\theta| >> R\ \delta P_{inc}\ , \qquad (7)$$

and, therefore, equations (3.1-3.2) can be rewritten as

$$\delta P_1 = \ P_{inc}\ \dot{R}_\theta\ \delta\theta \qquad (8.1)$$

$$\delta P_2 = -P_{inc}\ \dot{R}_\theta\ \delta\theta\ . \qquad (8.2)$$

This result can be considered as an initial proof that intrinsic beam-pointing fluctuations of the femtosecond laser are responsible for the additional phase noise in the spectrum of the extracted microwave signal.

Since beam-pointing fluctuations dominate the total intensity noise, one can evaluate the standard deviation of the angle of incidence, $\delta\theta_\tau$, from the measured spectrum of light intensity fluctuations. Assuming a beam splitter with the refractive index $n \approx 5$ and angle of incidence $\theta = 45 deg$, this results in: $\delta\theta_\tau(1s) \approx 5\cdot 10^{-3}\ deg$.

2. Additional fluctuations of pulse repetition rate

Referring to Fig. 6 and introducing fluctuations of pulse repetition rate at outputs of photodetectors PD2 and PD1 as $\delta\varphi_2$ and $\delta\varphi_1$, respectively, the additional phase noise of a signal extracted from the 'external' photodetector can be expressed as

$$\delta\varphi_{add} = \delta\varphi_2 - \delta\varphi_1 = \frac{d\theta_2}{dP}\,\delta P_2 - \frac{d\theta_1}{dP}\,\delta P_1 \qquad (9)$$

where $d\theta_1/dP$ and $d\theta_2/dP$ are power-to-phase conversions of PD1 and PD2.

The latter equation provides an explanation for a relative independence of the additional phase noise on operation of the power stabilisation system. Indeed, assuming that power control is disabled and substituting (8.1, 8.2) into (9) yields

$$\delta\varphi_{add} = \left(\frac{d\theta_1}{dP} + \frac{d\theta_2}{dP}\right) P_{inc}\ \dot{R}_\theta\ \delta\theta \qquad (10)$$

When power control loop is closed, combination of (5.1, 5.2) and (9) results in

$$\delta\varphi_{add} = 2\frac{d\theta_1}{dP}\ P_{inc}\ \dot{R}_\theta\ \delta\theta\ , \qquad (11)$$

therefore, the spectral density of additional phase noise is not affected by the power control system provided that photodetectors PD1 and PD2 have similar power-to-phase conversion efficiencies.

Making use of the experimental set-up in Fig. 6 the power-to-phase conversion coefficients of various photodetectors were measured.

This was accomplished by modulating intensity of the laser beam (with the AOM) and extracting a signal at frequency of modulation from the output of either 'in-loop' or 'external' phase detector. The frequency of modulation was chosen to be higher than the bandwidth of the phase-locked loop controlling the pulse repetition rate in order to avoid the cancellation of the useful signal. Power-to-phase conversion was found to be a diminishing function of optical power with the typical values of $35\,rad/W$ and $15\,rad/W$ at $P_{inc} = 1mW$ and $P_{inc} = 3mW$, respectively

To verify the above analysis one can infer the spectrum of additional phase noise from the light intensity fluctuations and compare it with the directly measured phase noise spectrum. In conducting such a verification, first, the experimental set-up in Fig. 6 was tuned to be phase sensitive and spectrum of output voltage fluctuations was measured (curve 2 in Fig. 5). Referring to this spectrum, it should be noted, that it truly represents the additional phase noise only at Fourier frequencies well within the bandwidth of the pulse repetition rate control servo. At Fourier frequencies above the servo bandwidth ($> 10\,Hz$) the main contribution to the output voltage noise comes from intrinsic fluctuations of pulse repetition rate of a femtosecond laser.

At the next stage of experiments, the measurement system was tuned to be amplitude sensitive by swapping mixer ports (RF port of the mixer 2 was coupled to the output of the PD2) and adjusting phaseshift φ to maximise the mixer DC voltage. Having measured the voltage noise in this regime, the spectrum of equivalent phase fluctuations was calculated by making use of power-to-phase conversion coefficient of a given photodetector.

The spectrum of the additional phase noise inferred from light intensity fluctuations is shown by curve 1 in Fig. 8. Total phase noise of a microwave signal extracted from the photodetector PD2 is given by curve 2. It is seen that two noise spectra are almost identical at Fourier frequencies below a few *Hz* which can be considered as a reasonable proof, that additional phase noise results from light intensity fluctuations which are, in turn, caused by the beam-pointing noise.

Divergence of two noise spectra at higher frequencies is caused by the diminishing gain of the PLL and, therefore, increased contribution of pulse repetition rate fluctuations of a free-running femtosecond laser to the total phase noise.

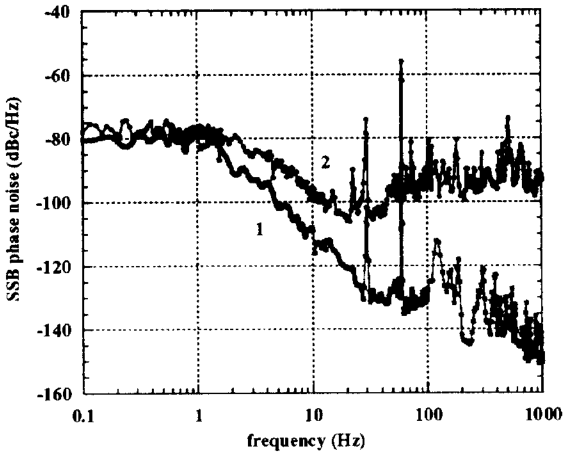

Fig. 8. Spectral density of additional phase fluctuations inferred from the light intensity noise (curve 1), phase noise of the microwave signal extracted from 'external' photodetector (curve 2).

Having identified the mechanism of the additional phase noise, one can think of various methods of its minimisation. We tried spatial filtering of laser light to reduce its angle fluctuations. For this purpose a piece of a single-mode fiber was introduced between the laser and acousto-optical modulator. Measurements of differential frequency fluctuations between the microwave signal extracted from the 'external' photodetector and RF-synthesiser revealed frequency errors of a few μHz at $\tau = 1s$ indicating at least an order of magnitude improvement in the stability of the extracted microwave signal.

V. Conclusion

Beam pointing fluctuations were found to be at the origin of the additional phase noise, observed when a microwave signal is extracted from a phase-locked femtosecond laser by utilising an 'external' photodetector. The effect of other noise mechanisms, such as laser shot noise and phase noise of a microwave amplifier, on frequency stability of the extracted microwave signal was also examined. It was confirmed that fractional frequency stability of an optical clock could be preserved at the level of 10^{-15} when linking optical and microwave domains. A further investigation is required to demonstrate the above level of frequency stability in experiments with two separate femtosecond lasers referenced to either the same or different optical 'clocks'. Taking into account the high potential of optical 'clocks' it is also important to find out if frequency errors associated with the time transfer could be reduced below the level of 10^{-15}.

VI. Acknowledgments

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References

1. S. A. Diddams, D. J. Jones, J. Ye, T. Cundiff, J. L. Hall, J. K. Ranka, R. S. Windeler, R. Holwarth, T. Udem and T. W. Hansch, "Direct link between microwave and optical frequencies with a 300 THz femosecond laser comb," *Phys. Rev. Lett.*, vol. 84, pp. 5102-5105, 2000.

2. T. Udem, S. A. Diddams, K. R. Vogel, C. W. Oates, E. A. Curtis, W. D. Lee, W. M. Itano, R. E. Drullinger, J. C. Bergquist and L. Hollberg, " Absolute frequency measurements of the Hg+ and Ca optical clock transitions with a femtosecond laser," *Phys. Rev. Letters*, vol. 86, p. 4996, 2001.

3. J. L. Hall, J. Ye, S. A. Diddams, L.-S. Ma, S. T. Cundiff and D. J. Jones,'Ultrasensitive spectroscopy, the ultrastable lasers, the ultrafast lasers, and the seriously nonlinear fiber: a new alliance for physics and metrology,' in *IEEE Journal of Quantum Electronics*, vol. 37, no. 12, pp. 1482-1492, December 2001.

4. L. Hollberg, C. W. Oates, E. A. Curtis, E. N. Ivanov, S. A. Diddams, T. Udem, H. G. Robinson, J. C. Bergquist, R. J. Rafac, W. M. Itano, R. E. Drullinger and D. J. Wineland, "Optical frequency standards and measurements," *IEEE Journal of Quantum Electronics*, vol. 37, no. 12, pp. 1502-1513, December 2001.

5. S. A. Diddams, L. Hollberg, L. S. Ma et al., "A Femtosecond-laser-based Clockwork with Instability < 6.3×10^{-16} in $1s$, "to be published in Optics Letters.

6. T. Udem, J. Reichert, R. Holzwarth and T. W. Hansch, "Accurate measurement of large optical frequency differences with a mode-locked laser," Opt. Letters, vol. 24, p. 881, 1999.