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Five Years of VLF Worldwide Comparison of Atomic Frequency Standards

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The VLF radio broadcasts of GBR(16.0 kHz), NBA(18.0 or 24.0 kHz), and NSS(21.4 kHz) have enabled worldwide comparisons of atomic frequency standards to parts in 10^{10} when received over varied paths and at distances up to 9000 or more kilometers. This paper summarizes a statistical analysis of such comparison data from laboratories in England, France, Switzerland, Sweden, Russia, Japan, Canada, and the United States during the 5-year period 1961–1965. The basic data are differences in 24-hr average frequencies between the local atomic standard and the received VLF radio signal expressed as parts in 10^{10}. The analysis of the more recent data finds the receiving laboratory standard deviations, $\hat{\alpha}_i$, and the transmission standard deviation, $\hat{\tau}$, to be a few parts in 10^{11}. Averaging frequencies over an increasing number of days has the effect of reducing $\hat{\alpha}_i$ and $\hat{\tau}$ to some extent. The variation of the $\hat{\alpha}_i$ with propagation distance is studied. The VLF-LF long-term mean differences between standards are compared with the recent portable clock tests, and they agree to parts in 10^{11}.

1. Introduction

Six years ago in London, the XIIIth General Assembly of URSI adopted a resolution (No. 2) which strongly recommended continuous very-low-frequency (VLF) and low-frequency (LF) transmission monitoring throughout the world by atomic frequency standards (Decaux, 1961). URSI proposed such tests to obtain the day-to-day phase stabilities of VLF and LF radio signals and to determine their usefulness for precise time and standard frequency comparisons at distant points. Since that time, some 10 laboratories have collected and exchanged atomic standard frequency data, obtained through such transmissions as GBR (England, 16 kHz), NBA (Canal Zone, 18 or 24 kHz), WWVL (U.S.A., 20 and 19.9 kHz), NSS (U.S.A., 21.4 kHz), and WWVB (U.S.A., 60 kHz). Since the Twelfth General Conference of Weights and Measures authorized a temporary atomic designation for the physical measurement of time in October 1964 (General Conference of Weights and Measures XII, 1964), these comparisons of atomic frequency standards have taken on added significance.

This paper supplements a previous 18-month comparison of atomic frequency standards (Morgan, Crow, and Blair, 1965) and covers the period 1961 up through part of 1966. Many previous comparison studies through VLF and LF radio transmissions have been reported. In addition to those references given in our previous paper, see Nakajima, Suzuki, Azuma, Akatsuka, and Nakamura (1963); Reder, Åbom, and Winkler (1964); and Mungall, Bailey, and Daams (1966). These past 6 years have seen tremendous improvement in atomic frequency standards (McCoubrey, 1966); microsecond time synchronization at remote points via satellites (Steele, Markowitz, and Lidback, 1964; Markowitz, Lidback, Uyeda, and Muramatsu, 1966); improvements in the transmission of VLF and LF radio signals (Milton, Fey, and Morgan, 1962; Barnes, Andrews, and Allan, 1965; Bonanomi, 1966; U.S. Naval Observatory, 1966); and side-by-side worldwide comparisons of atomic frequency standards via portable cesium (Cs) standards to parts in 10^{12} (Bagley and Cutler, 1964; Bodily, 1965; Bodily, Hartke, and Hyatt, 1966).

In the light of such advances, it is the purpose of this paper to analyze the 5-year accumulation of daily observations of atomic frequency standards systematically made at certain laboratories by means of received VLF radio signals, to note the present-day status of such comparisons, and to contemplate further work and events which could eventually lead to a universal standard which might control an international atomic time-scale system (CCIR, 1966).

2. Characteristics of Atomic Frequency Standards

Characteristics of the atomic frequency standards located at the various receiving laboratories are listed in table 1. Figure 1 shows the locations of both the laboratories and the VLF and LF transmitters engaged in these studies, together with great circle distances between applicable transmitters and receivers. These propagation distances range from 50 to 9470 km, and all such paths lie in the northern hemisphere.

3. Summary of Statistical Analysis

The methods of comparison and statistical analysis have been described by Morgan, Crow, and Blair (1965). Each laboratory records the received VLF signals in

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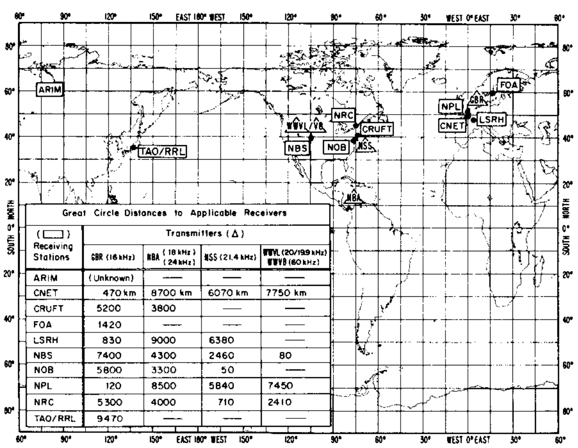
Table 1. Characteristics of Atomic Frequency Standards

Laboratory	Laboratory Abbreviation	Approximate Laboratory Geographic Location	Atomic Standard: Type	Atomic Standard: Period of Observation	Atomic Standard: Designation	Interaction Length-cm	Reported* Accuracy pts in 10^{11}	Reference
All-Union Research Institute of Physical Technical and Radio Technical Measurements Moscow, Russia	ARIM	UNKNOWN	--	Jan 1, 1964 to Dec 31, 1965	--	--	--	--
Centre National D'Études des Télécommunications Bagneux, France	CNET	48° 48' N 2° 19' E	Cs-Commercial	Jan 1, 1961 to Apr 30, 1965	A107	94	+22	Decaux (1963)
			Cs-Combination Commercial-Laboratory	Apr 30, 1965 to Present	Cs 2	25	± 2	Decaux (1967)
Cruft Laboratories Harvard University Cambridge, Mass., USA	CRUFT	42° 22' N 71° 6' W	Cs-Commercial (inactive)	Jan 1, 1961 to Dec 31, 1963	S 203	94	+50 (Mfr's guarantee)	Pierce (1963)
Research Institute of National Defence-FOA Stockholm, Sweden	FOA	59° 20' N 18° 3' E	Cs-Commercial	Mar 1, 1965 to Present	--	12.4	+ 2	Bodily (1966); Bodily, Hartke, and Hyatt (1966)
Laboratoire Suisse de Recherche Horlogère Neuchâtel, Switzerland	LSRH	47° 00' N 6° 57' E	Cs-Laboratory	Jan 1, 1961 to Dec 31, 1963	LSRH-II	409	± 3(1σ)	Kartaschoff (1962)
				Jan 1, 1964 to June 30, 1966			+ 1(1σ)	Kartaschoff (1964, 1967)
National Bureau of Standards Boulder, Colorado, USA	NBS	40° 00' N 105° 16' W	Cs-Laboratory	Jan 1, 1961 to Jun 30, 1962	NBS-I	55	± 1(3σ)	Beehler, Glaze (1966); Glaze (1967)
				Jul 1, 1962 to Aug 31, 1963	NBS-II	164	± 0.8(3σ)	
				Sept 1, 1963 to May 31, 1965				
				Jun 1, 1965 to present	NBS-III	366	+ 0.5(3σ)	
U. S. Naval Observatory Washington, D. C., USA	NOB	38° 55' N 77° 39' W	Cs-Beam (weighted average of 9 Cs stds)	Jan 1, 1961 to May 31, 1965	A.1	varied	+ 2	Markowitz (1962, 1964)
			Cs-Commercial	Jun 1, 1965 to present	Cs-60	12.4	+0.9(ref H_1)	Buchanan, Pritt, and Hastings (1965)
National Physical Laboratory Teddington, Middlesex, England	NPL	51° 25' N 0° 20' W	Cs-Laboratory	Jan 1961 to Nov 1963	NPL-II, MOD 1	265	± 3	Essen, Steele, and Sutcliffe (1964); Essen (1967)
				Dec 1963 to Jan 1964	NPL-II, MOD 2			
				Jan 1964 to Jan 1965	NPL-II, MOD 3			
				Jan 1965 to present	NPL-II, MOD 4			
National Research Council Ottawa, Ontario, Canada	NRC	45° 25' N 75° 43' W	Cs-Laboratory	Jan 1, 1961 to Nov 6, 1962	NRC-1	80	±30	Kalra (1961)
				Feb 17, 1965 to Present	NRC-2	210	+ 1.9	Mungall, Bailey, and Daams (1966)
Tokyo Astronomical Observatory Mitaka, Near Tokyo, Japan	TAO (RRL)	35° 40' N 139° 32' E	$N^{14}H_3$-Laboratory	Aug 14, 1964 to Present	--	--	+ 3 (resetability)	CCIR (1964)

*Methods of determining vary. Consult applicable reference for complete details for a particular standard. In most cases the precision of short-term measurements is reported also, but the methods of determining vary widely.



Great Circle Distances to Applicable Receivers

(▭) Receiving Stations	Transmitters (Δ): GBR (16 kHz)	NBA (18 kHz) (24 kHz)	NSS (21.4 kHz)	WWVL (20/19.9 kHz) WWVB (60 kHz)
ARIM	(Unknown)	—	—	—
CNET	470 km	8700 km	6070 km	7750 km
CRUFT	5200	3800	—	—
FOA	1420	—	—	—
LSRH	830	9000	6380	—
NBS	7400	4300	2460	80
NOB	5800	3300	50	—
NPL	120	8500	5840	7450
NRC	5300	4000	710	2410
TAO/RRL	9470	—	—	—

FIGURE 1. *Locations of receiving laboratories and VLF and LF transmitters.*

terms of a transfer oscillator, which is periodically calibrated in terms of the primary frequency standard. It should be emphasized that the data analyzed are thus only measurements at certain laboratories possessing atomic standards and systematically recording signals from VLF transmitters and do not include any measurements at the VLF transmitters. From the reduction of such phase recordings, daily (24-hr) averages of the received frequencies are determined and exchanged among the different laboratories.

Initial analysis of these data yields the long-term mean of the daily observations at each receiving laboratory and the variance s_i^2 (or standard deviation s_i) of these observations. The means give immediately the long-term mean differences between the atomic standard frequencies of the receiving laboratories. By averaging the daily observations from all receiving laboratories and a little further statistical analysis (see Morgan et al., 1965), the variance $\hat{\tau}^2$, common to all receiving laboratories, is obtained. Thus $\hat{\tau}$ includes variations of the transmitter oscillator, the transmitting system, and propagation variations common to all receiving stations. A decomposition of s_i^2 into two components can then be achieved, $\hat{\tau}^2$ being one component; the other component, $\hat{\alpha}_i^2$, may be defined simply as $\hat{\alpha}_i^2 = s_i^2 - \hat{\tau}^2$. It follows that $\hat{\alpha}_i$ consists of all variations of the measurements at the ith receiving laboratory *not* common to all laboratories, and thus includes variations of both primary and transfer oscillators, other parts of the receiving system, and of the propagating signal peculiar to the ith transmission path.

3.1. Long-Term Mean Differences Between Atomic Standards

It is of interest to test the mean frequencies measured simultaneously by receiving laboratories over periods as long as a year for the presence of systematic differences between atomic standards. Hence, the differences, Δ_i, of yearly means from the yearly grand mean for all laboratories are shown in tables 2 and 3 for GBR and NBA or NSS for the years 1961–1965. (NBA suspended operations for about a year in 1965, and NSS was then monitored instead.)

The mean differences of tables 2 and 3 are also combined and displayed in figure 2. We formulate the following generalizations from tables 2 and 3 and figure 2:

(a) The maximum of the yearly mean difference in table 2 between atomic frequency standards decreased from 39 parts in 10^{11} in 1961 to 8 parts in 10^{11} in 1965 (aside from one standard introduced in 1964); table 3 shows a similar decrease, from 42 to 5. Thus, as shown in figure 2, the mean difference of each standard from the grand mean has tended to decrease.

(b) Before 1965, the grand mean for each year and each transmitter differed up to 8 parts in 10^{11} from the prescribed fractional frequency offset of -1500 or -1300 parts in 10^{11} and up to 14 parts in 10^{11} from that of the other transmitter. However, these differences reduced to just 0.1 part in 10^{11} in 1965. (The prescribed fractional frequency offset, which is presently -3000 parts in 10^{11}, is an approximation, agreed upon internationally, to the difference in the rate of occurrence of time ticks on the universal time scale (UT2) and second pulses on the atomic or ephemeris time scales (Hudson, 1965).) However, among the differences between the two transmitters as measured by any given receiving laboratory, i.e., between corresponding means (Δ_i) of tables 2 and 3, the maximum absolute difference ranges only between 2 and 5 parts in 10^{11}.

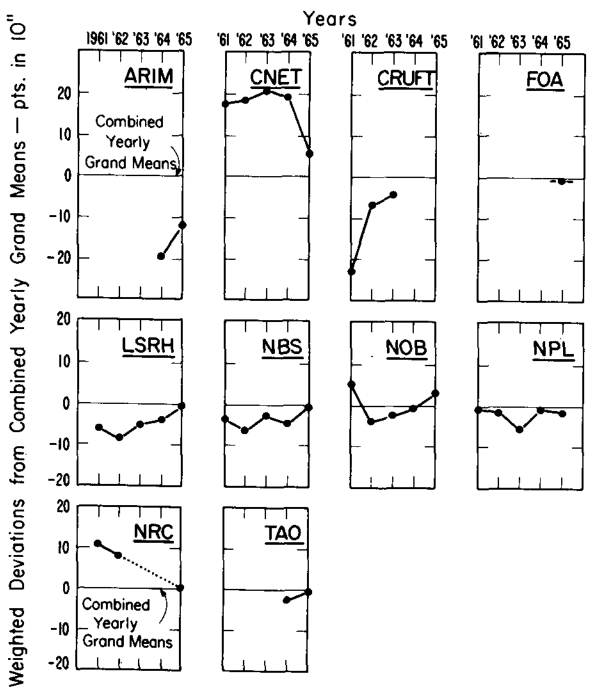


FIGURE 2. *Average agreement among atomic standards, 1961–1965 as indicated by transmissions of GBR and NBA (NSS, 1965) combined.*

(Some receiving stations have reported frequency values only in terms of GBR. Thus the deviations only from the GBR yearly grand means are shown for ARIM, FOA, TAO, and NRC (1965).)

3.2. Day-to-Day Fluctuations Associated with Receiving Laboratories

The part of the fluctuation from day to day of the daily frequency measurement (of a transmitted signal) by the ith receiving laboratory, which is associated uniquely with that laboratory, is characterized by a standard deviation α_i. This includes system errors and propagation effects peculiar to the ith path as well as receiver atomic standard variations. The estimate of α_i from any given quarter (of a year) is denoted by $\hat{\alpha}_i$. (Since the simultaneous statistical analysis for all $\hat{\alpha}_i$ requires complete sets of daily observations from all laboratories, using data in quarterly groups turned out to be desirable.) Yearly average estimates of α_i, denoted by $\bar{\hat{\alpha}}_i$, are presented in this paper, obtained as weighted root-mean-square combinations of the $\hat{\alpha}_i$ with weights equal to the number of days per quarter, minus 1.[3] Figures 3 and 4 give the $\bar{\hat{\alpha}}_i$ for the various laboratories reporting reception of GBR, NBA, and NSS during 1961–1965. We formulate the following generalizations from figures 3 and 4 and from approximate confidence intervals that are not shown:

(a) The standard deviation $\bar{\hat{\alpha}}_i$ for most laboratories tended to decrease from 1961 to 1965, whether derived from GBR transmissions or NBA and NSS transmissions. If this decrease were attributed to improvement in atomic standards, it would have to be assumed that the standards contribute a major portion of the α_i; however, it seems likely that the measuring systems have been improved too, and it is possible that propagation fluctuations decreased.

(b) The $\bar{\hat{\alpha}}_i$ derived from GBR transmissions and the corresponding $\bar{\hat{\alpha}}_i$ derived from NBA or NSS transmissions tend to be near each other. All of the values for 1965, with one exception, fall between two and six parts in 10^{11}. The recently reported within-laboratory standard deviations of atomic standard average frequencies for 1-day periods are at least an order of magnitude less than these values; that is, a few parts in 10^{12} (McCoubrey, 1966).

(c) Three of the $\bar{\hat{\alpha}}_i$ from GBR and NBA transmissions are zero (LSRH and NBS from GBR in 1962 and NBS from NBA in 1964). These and other substantial variations are easily accounted for by the uncertainties in the estimates. Even 95 percent confidence intervals based on independence of daily observations place upper limits for these two values of α_i at five parts in 10^{11}, and intervals taking account of dependence would

[3] See Morgan et al. (1965) for a more complete explanation of the theory and notation. In brief, α_i and τ are standard deviations of theoretical distributions that could be known only if infinite observations were made; $\hat{\alpha}_i$ and $\hat{\tau}$ are estimates of α_i and τ made from a finite number of observations, here generally restricted to 1 per day for a quarter of a year; $\bar{\hat{\alpha}}_i$ and $\bar{\hat{\tau}}$ are rms averages of the four $\hat{\alpha}_i$ and four $\hat{\tau}$ for each year.

Table 2. Yearly Means Δ_i and Standard Deviations s_i of 24-hour Average Frequencies Transmitted by GBR and Measured by Various Standards.

The Δ_i are deviations from the yearly grand mean of all standards (positive if ith standard is low).
n_i = number of 24-hour average frequencies
Frequency unit - 1 part in 10^{11}

Atomic Frequency Standard	1961			1962			1963			1964			1965		
	n_i	Δ_i	s_i	n_i	Δ_i	s_i	n_i	Δ_i	s_i	n_i	Δ_i	s_i	n_i	Δ_i	s_i
ARIM	-	-	-	-	-	-	-	-	-	238	-19.5	17.4	144	-12.8	10.3
CNET	313	+16.4	31.9	323	+17.8	17.9	345	+21.2	13.2	350	+20.1	10.4	310 * (230)	+6.4 (+0.4)*	14.3 (8.0)*
CRUFT	246	-22.9	32.0	282	-7.3	14.9	304	-5.0	10.9	-	-	-	-	-	-
FOA	-	-	-	-	-	-	-	-	-	-	-	-	258	-1.0	7.8
LSRH	329	-6.1	33.4	268	-7.8	14.9	330	-5.0	8.2	355	-2.7	9.9	342	-0.1	8.5
NBS	326	-4.2	32.4	302	-6.9	14.1	347	-3.1	8.8	350	-3.9	9.5	328	-0.9	7.8
NOB	321	+2.9	29.3	317	-4.6	13.1	319	-3.0	9.1	352	+0.7	10.4	350	+2.9	9.3
NPL	317	-0.6	29.6	304	-0.1	15.2	331	-6.4	8.2	351	+0.2	7.2	341	-1.4	7.7
NRC	339	+9.3	39.8	286	+6.9	21.9	-	-	-	-	-	-	285	0.0	9.4
TAO	-	-	-	-	-	-	-	-	-	135	-2.7	13.2	339	-1.0	11.8
GRAND MEAN	-1492.3			-1299.3			-1295.8			-1504.5			-1499.9		

*New standard, Apr.-Dec. 1965. Mean of new standard not included in grand mean.

Table 3. Yearly Means Δ_i and Standard Deviations s_i of 24-hour Average Frequencies Transmitted by NBA (NSS in 1965) and Measured by Various Standards.

The Δ_i are deviations from the yearly grand mean of all standards (positive if ith standard is low).
n_i = number of 24-hour average frequencies
Frequency unit - 1 part in 10^{11}

Atomic Frequency Standard	1961			1962			1963			1964			1965		
	n_i	Δ_i	s_i	n_i	Δ_i	s_i	n_i	Δ_i	s_i	n_i	Δ_i	s_i	n_i	Δ_i	s_i
CNET	251	+18.4	14.6	299	+18.9	12.8	276	+19.2	9.0	240	+17.5	9.3	204 (174)*	+1.7 (-3.0)*	13.0 (6.2)*
CRUFT	238	-23.8	15.2	263	-7.1	9.1	249	-4.6	9.0	-	-	-	-	-	-
LSRH	322	-6.1	11.9	284	-9.1	10.2	296	-5.4	5.6	257	-5.7	6.4	224	-1.1	8.1
NBS	230	-3.3	12.7	257	-6.0	8.6	297	-3.1	4.2	236	-6.4	4.3	245	-0.9	5.5
NOB	294	+6.5	11.6	316	-3.8	9.2	273	-2.3	4.3	290	-2.6	5.0	270	+2.1	7.1
NPL	259	+0.1	14.6	241	-2.9	14.2	205	-5.0	6.3	154	-3.2	8.0	169	-2.7	7.2
NRC	133	+13.8	22.4	240	+9.2	18.0	-	-	-	-	-	-	-	-	-
GRAND MEAN	-1506.2			-1295.1			-1298.9			-1495.9			-1500.0		

*New standard, Apr.-Dec. 1965. Mean of new standard not included in grand mean.

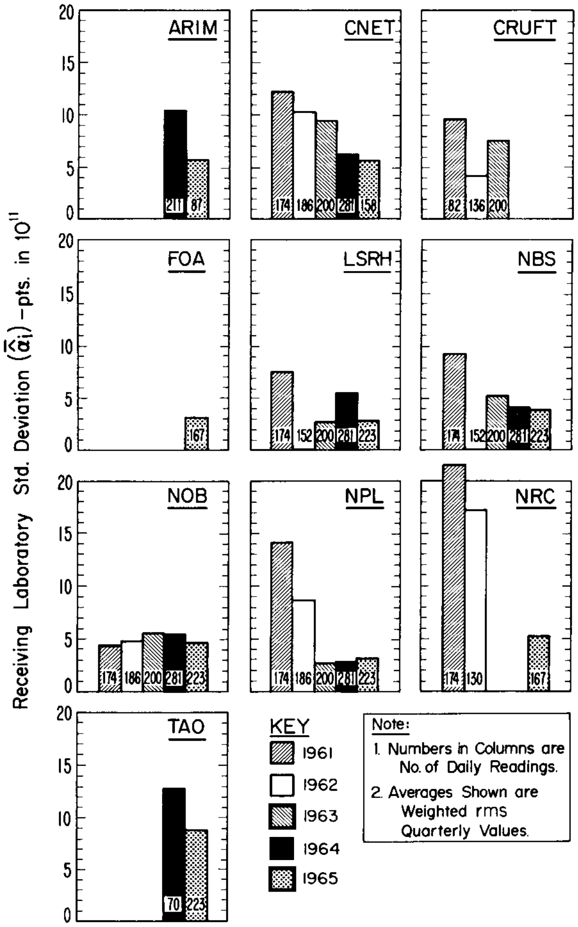


FIGURE 3. *Derived standard deviations, $\bar{\hat{\alpha}}_i$, (associated with receiving laboratories) of daily average frequency measurements of GBR transmissions.*

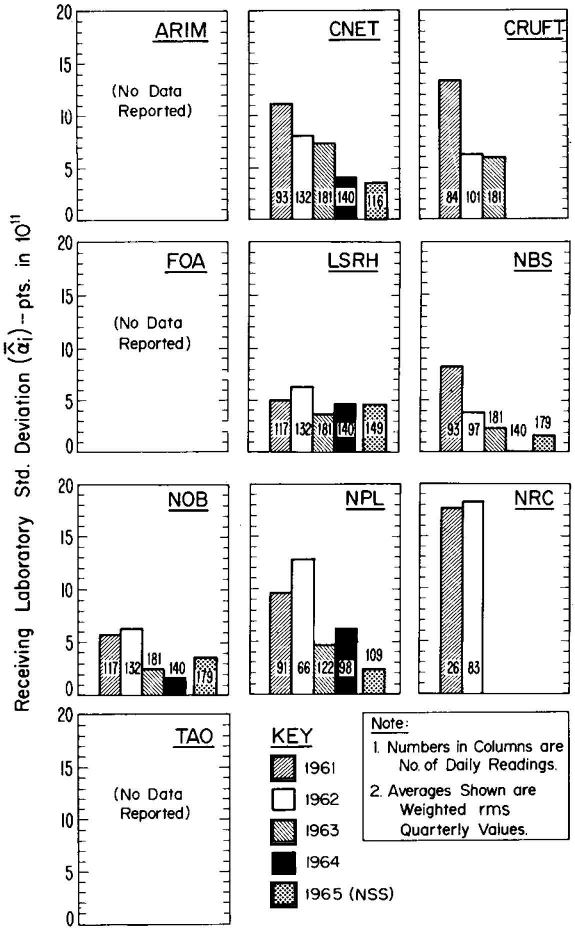


FIGURE 4. *Derived standard deviations, $\bar{\hat{\alpha}}_i$, (associated with receiving laboratories) of daily average frequency measurements of NBA (NSS in 1965) transmissions.*

place them even higher. (Uncertainties in estimates are discussed further in section 4.)

3.3. Day-to-Day Fluctuations of Transmissions

The day-to-day fluctuations in transmissions are characterized by a standard deviation τ, which includes transmitting system variability and propagation variability common to all receiving stations included in the analysis as well as transmitter oscillator variability. The data do not permit analysis into components arising from these three sources. Figure 5 shows the estimated standard deviations $\bar{\hat{\tau}}$ of transmission errors for GBR and NBA for each year 1961 through 1964, as well as 1965 values for GBR and NSS, and values for NSS and WWVL for parts of 1966. From 1961 through 1963 the $\bar{\hat{\tau}}$ decreased by 60 or 65 percent, after which they leveled off to values of less than 7 parts in 10^{11}. This improvement is believed to result from improved transmitter-oscillator control.

4. Effects of Averaging VLF Frequency Data

To show the effects of long-term averaging of standard frequency data, averages over consecutive nonoverlapping intervals of 7, 15, 30, 60, 120, and 240 days were computed for the 1963–64 GBR data and for similar intervals for the 1965 GBR data. Then the values of $s_i^2, \hat{\alpha}_i{}^2$, and $\hat{\tau}^2$ were calculated using these longer-term averages as the individual observations. An example of the effective reduction in the magnitude of s_i^2, $\hat{\alpha}_i^2$, and $\hat{\tau}^2$ is shown graphically in figure 6. The variances s_i^2 and variance components, $\hat{\alpha}_i^2$ and $\hat{\tau}^2$, are plotted with logarithmic scales because, in the case of independent observations from a stable distribution (as well as asymptotically in at least some cases of autocorrelated observations), they should vary inversely with the first power of the length of the averaging interval. (The variances s_i^2 of 1-day averages, as well as the $\hat{\alpha}_i^2$ and $\hat{\tau}^2$ derived from them, are determined from a large number of days, whereas the variances

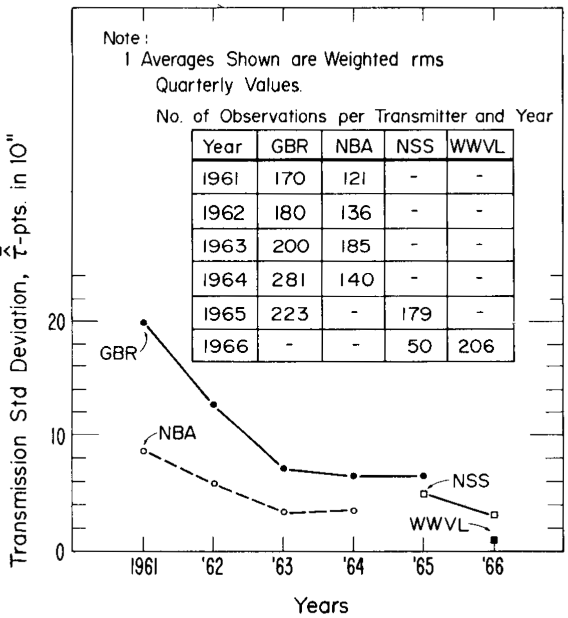


FIGURE 5. *Derived standard deviations, $\hat{\bar{\tau}}$, associated with transmitter (GBR, NBA, NSS, or WWVL), of daily average frequency measurements by receiving laboratories.*

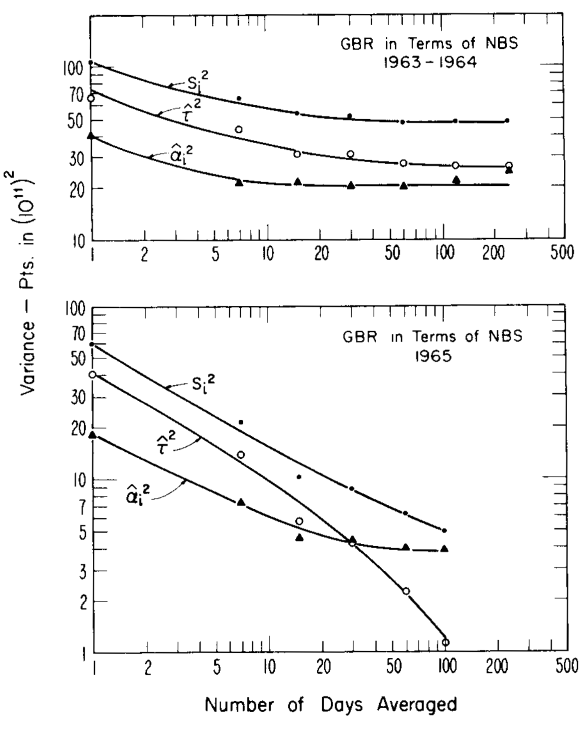


FIGURE 6. *Total variance $s_{\bar{i}}^2$, receiving laboratory variance $\hat{\alpha}_{\bar{i}}^2$, and transmission variance $\hat{\bar{\tau}}^2$ of frequencies averaged over the indicated number of days.*

$s_{\bar{i}}^2$ of 60-day averages are determined from 12 values for 1963–1964 and from 3 values for 1965. Other points are proportionally limited.)

We see from figure 6 that although there is at least an initial decrease as the averaging interval length increases, $s_{\bar{i}}^2$ levels off at about 1/2 of its value for 1-day averages in the 1963–1964 data, but decreases steadily as about the -0.6 power of the averaging interval length in the 1965 data. The leveling off in 1963–1964 is associated with substantial spectral components of periods longer than the averaging interval that are not included in values of $s_{\bar{i}}^2$, $\hat{\alpha}_{\bar{i}}^2$, and $\hat{\bar{\tau}}^2$ calculated from short-term (such as 1-day) averages. Figure 6 may be compared with Allan's figures 4–7 (1966), McCoubrey's figure 17 (1966), and Bodily's figure 5 (1966), which graph within-laboratory standard deviations of atomic standard frequencies against averaging intervals up to about 1 day. These graphs, showing the variation of measurement for averaging intervals of 1 day or less, indicate a slope near -0.5 for cesium standards, corresponding to a slope for variances near -1. Our figure 6 includes both propagation and measuring system variation, as well as atomic standard variation. Which components are substantial for periods beyond 1 day still appear to be incompletely known. The limited reduction in variances through averaging may result, at least in part, from long-term variations in the ionosphere such as solar-cycle, seasonal, and lunar-tide variations (Chilton, Crombie, and Jean, 1964). (An editorial reader, J. A. Barnes, notes that this limited reduction might be explained by the hypothesis that flicker noise frequency modulation is present, a fact recognized in several papers on crystal oscillators and in Vessot et al. (1966) for atomic devices.)

The relatively slow decrease of variability with increasing interval length (or number of observations) exemplified in figure 6 renders the classical confidence intervals based on independent observations too short and is the reason for not including them in this paper. The effect of averaging intervals, on $s_{\bar{i}}^2$, $\hat{\alpha}_{\bar{i}}^2$, and $\hat{\bar{\tau}}^2$ and their uncertainties is under further study.

5. Relation of Precision of Frequency Comparison With Distance

Since Pierce's pioneering work nearly a decade ago on the precision of short-term phase measurements of 16-kHz signals received over a 5200-km path (Pierce, 1957), there has been considerable question whether such precision depends upon distance. Pierce, Winkler, and Corke (1960) later extended the short-term results to observations over 24-hr periods. They found standard deviations of about 2 parts in 10^{11} for GBR frequency measurements over the same 5200-km path. The frequency measurements given in the present paper were made at consecutive 24-hr times, generally near the center of the all-daylight period, to minimize effects from the diurnal height changes of the ionosphere. The daily phase variations in such frequency data, attributable to ionospheric effects, are believed to result largely from day-to-day height changes of the reflecting layer of the ionosphere. Pierce et al. (1966) state that phase changes due to ionospheric height changes that are common to the whole path would be proportional to distance. Superimposed on these day-to-day variations are shorter-term phase fluctuations, whose contribution is difficult to assess at this point. Pierce et al. (1966) show some indication that these shorter-term variations in a received VLF signal increase with distance to the 1/4 power.

Our analysis of the observed variance s_i^2 of average frequency fluctuations into components $\hat{\tau}^2$ associated with the transmitter and $\hat{\alpha}_i^2$ associated with the receiver (including the path to the receiver) provides data for analyzing the variation with distance more specifically. Since the true variance $\hat{\alpha}_i^2$ for the ith receiver includes contributions from its atomic standard and measurement system that have nothing to do with distance from the transmitter, a simple reasonable model for it would be of the form

$$\alpha_i^2=\alpha_{0,\,i}^2+\gamma^2(D_i-L)^{2\beta}, \qquad L \leqslant D_i \leqslant U,$$

where D_i is the great-circle distance from the transmitter to the receiver over which the signal is received and $\alpha_{0,\,i}$, β, γ, L, and U are constants to be determined. A more refined model would be expected to have all of the constants depending on i, since path position and orientation, proportion of water below the path, and interference between short and long great-circle paths are factors. In addition, the model can be tested only by sample values $\hat{\alpha}_i^2$ (or averages thereof, $\bar{\hat{\alpha}}_i^2$), which are subject to considerable sampling error not included in the above equation; there is more relative error in $\hat{\alpha}_i^2$ than in the s_i^2 from which it is derived by subtracting $\hat{\tau}^2$.

However, it is possible to give outside bounds for the limiting distances L and U. Certainly U is no larger than the circumference of the earth, 40,000 km. A lower limit to L is provided by the distance beyond which first-order mode theory is satisfactory; the value of L so indicated is somewhat arbitrary, but the work of Wait (1957, 1962) suggests 2000 to 3500 km for daytime propagation. For shorter distances, higher-order modes interfere with no simple dependence on distance. In figure 7 we have plotted the yearly average receiver standard deviations, $\bar{\hat{\alpha}}_i$, against distance for the nine receiving stations recording at least one of GBR, NBA, and NSS during 1963–65. (The earlier values were not included because they tend to be larger than the values for the period 1963–65 which seems characterized by approximate stability.) Although the carrier frequencies of the transmitters vary from 16 to 24 kHz, and Pierce et al. (1966) normalize phase variations by division by the carrier frequency, we have not done this because $\hat{\alpha}_i$ includes atomic standard and system measurement variations as well as propagation variations and the effect is relatively small here anyway.

We observe from figure 7 that distance D does not appear to explain a major portion of the variation of $\bar{\hat{\alpha}}_i$. However, if we exclude the data for $D < 2000$ km on the basis of interfering modes as indicated above, there is a substantial correlation with distance, consistent with the model $\alpha^2=2^2+(D-2.5)^2$ for $2.5 \leqslant D \leqslant 10$ with D in megameters.

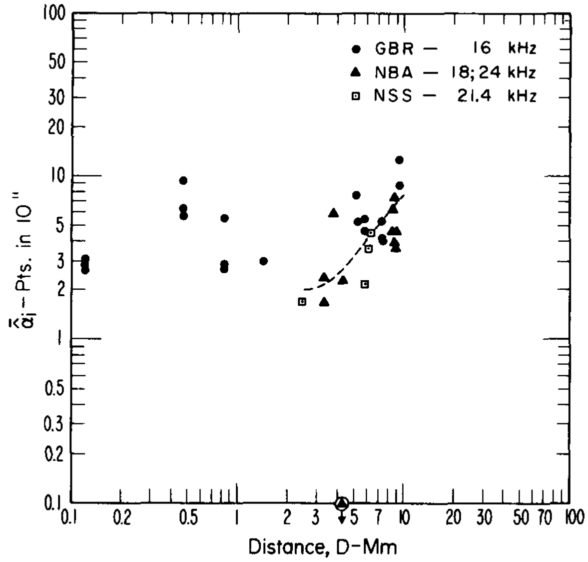


FIGURE 7. *Receiving laboratory standard deviation,* $\bar{\hat{\alpha}}_i$, *plotted against VLF propagation distance,* D.

6. Relation of Long Term VLF-LF Measurements to Direct Measurements With Portable Cesium Standards

Recently there have been several "flying clock" experiments in which portable atomic frequency standards have been intercompared side by side with atomic standards located in worldwide laboratories for short observation periods (Bagley and Cutler, 1964; Bodily, 1965; Bodily, Hartke, and Hyatt, 1966). Comparison between such measurements and the long-term VLF-LF data is shown in table 4 for the 1965 and 1966 data. (These data are in terms of deviations from NBS measurements to facilitate comparisons between the portable Cs standards and the VLF-LF measurements.) In most cases the direct measurements with portable Cs standards are within parts in 10^{12} of the VLF-LF mean values. Figure 8 gives the distribution of the differences of the daily observations of three receiving laboratories from those of NBS. (Superimposed on these distributions are fitted normal curves.) Also shown are several of the portable clock direct measurements.

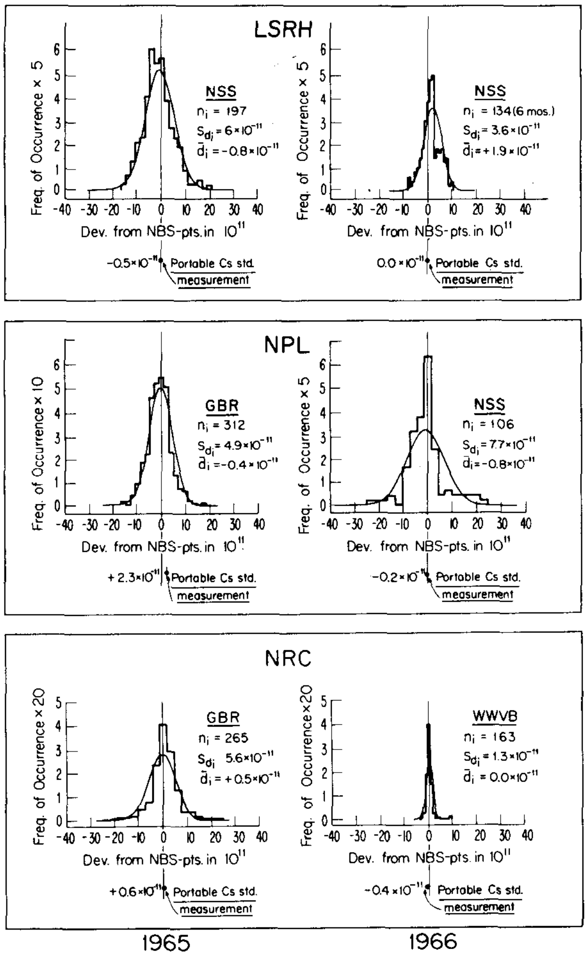


FIGURE 8. *Relation of portable Cs standard data to frequency distributions of differences of VLF–LF daily measurements by NBS and those by three other laboratories.*
(The portable Cs standard data are also shown as differences from NBS. Basic data are from Bodily (1965) and Bodily, Hartke, and Hyatt (1966).)

If one assumes that all the receiving laboratory standards are randomly selected members of a population of atomic standards and that the portable reference standards do not change systematically, a standard error of the mean difference between the receiving laboratories and the portable standards, $s/\sqrt{n}$ (where s is the standard deviation of the observed differences of the frequencies of n laboratories from those of the portable standards), can be computed for both 1965 and 1966, and confidence limits for the mean differences constructed. The 1965 and 1966 95 percent confidence limits are as follows:

$$\left.\begin{array}{l}(1965) -5.1 < +8.2 < +21.5 \ (n=7) \\ (1966) -3.2 < +1.1 < +5.4 \ (n=13)\end{array}\right\} \text{pts. in } 10^{12}.$$

Thus, there is no statistically significant mean difference between the assumed population of receiving laboratory atomic standards and the population of portable Cs standards. This latter population has been studied by Bodily (1966), whose histogram shows that 95 out of a population of 100 portable cesium standards fall within about ± 5 parts in 10^{12} of the reference cesium frequency.

7. Future Remote Comparisons of Atomic Frequency Standards

With the increased gains in the stability of atomic frequency standards, one can foresee the eventual need of comparing such standards at remote points with errors less than parts in 10^{12}. Longer period measurements may permit this to some extent, but improvements in transmitter control and long-term comparison studies using portable clocks as indicated below should obtain fuller realization of the maximum possible precision.

Transmitter improvements: Improved transmitter control has resulted in a reduction by a factor of three or more of the transmission standard deviation associated with VLF broadcasts (fig. 5). Thus, the WWVL transmission standard deviation, $\bar{\bar{\tau}}$, for 9 months in 1966, is less than 1 part in 10^{11}. The GBR transmitter has been modified recently (British Post Office, 1964) to radiate with increased power under control of an atomic frequency standard (Essen, 1965). The phase stability of the NSS, VLF transmitter has been improved within the past year by means of automatic antenna tuning (Williams, 1966), and the U.S. Navy Omega stations are using Cs standards for primary oscillator control (U.S. Navy, 1966). Since a goodly part of the standard deviation of reported daily frequency values at each receiving laboratory is attributable to transmitter fluctuation, one can expect less variation in the received daily frequencies in the future.

More detailed experiments: As Mitchell (1963) pointed out, the present precision of comparing atomic frequency standards via VLF signals is influenced also to a large extent by system measurement errors at the receiving station. It is apparent that further analysis of such receiving errors is essential to added gains in the precision of VLF measurements. Because of the success of the direct comparisons using portable Cs standards, we recommend long-term portable Cs standard experiments of a statistical design to (a) determine the propagation effects limiting the transfer and comparison of standard frequencies via VLF/LF at various distances and paths, (b) analyze and assess the receiving station error into components of primary and/or transfer oscillator instabilities, receiving equipment variations from the receiving antenna to the comparison instrumentation, and internal measurement errors in comparing, recording, and reducing the daily frequency observations, and (c) determine whether the ultimate precision of frequency standards measurements via long-distance radio paths can be improved through allowance for or prediction

Table 4. Comparison of Frequencies Derived from Radio Propagation and Portable Clock Data*

Frequency unit - 1 part in 10^{12}

Laboratory	VLF-LF DATA												Portable Clock Data[a]	
	1965						1966 (5-6 mos.)						1965	1966
	GBR			NSS			WWVL/VB			NSS				
	n_i	$\bar{d}_i$	s_{d_i}	n_i	$\bar{d}_i$	s_{d_i}	n_i	$\bar{d}_i$	s_{d_i}	n_i	$\bar{d}_i$	s_{d_i}	p_i	p_i
CNET	208	3	55	154	-16	34	129	-21 (WWVL)	31	114	-17	30	--	-12
FOA	233	+ 4	41	--	--	--	--	--	--	--	--	--	+6	+ 2
LSRH	308	+ 9	76	197	- 8	60	--	--	--	134	+19	36	-5	0
NOB	320	+40[b]	55	243	+33[b]	48	--	--	--	143	0	12	+6	- 2
NPL	312	- 4	49	154	-20	46	134	- 3 (WWVL)	40	106	- 8	77	+23	- 2
NRC	265	+ 5	56	--	--	--	163	0 (WWVB)	13	--	--	--	+ 6	- 4
Paris Observatory	--	--	--	--	--	--	--	--	--	65	+22	64	--	+16
RRL (TAO)	310	- 2	89	--	--	--	--	--	--	--	--	--	+ 1	+ 2

*Frequency data are deviations from NBS, i.e., NBS - Station standard (positive if NBS has the higher frequency)

a - (Bodily, 1965; Bodily et al., 1966)

b - weighted mean standard (Markowitz, 1962)

The column notation is as follows:

n_i = number of daily frequency deviations of ith receiving laboratory from NBS

$\bar{d}_i$ = mean daily frequency deviation of ith receiving laboratory from NBS

s_{d_i} = standard deviation of daily deviations

p_i = mean difference between NBS and ith receiving laboratory as measured by portable clocks

of propagation influences as alluded to by Chilton, Crombie, and Jean (1964).

With the improving state of the art in atomic frequency standards, the possibility of establishing an international atomic time scale as contemplated by the CCIR (1966) takes on new significance.

8. Conclusion

The worldwide comparison of atomic frequency standards over the past half decade has improved several fold in precision. The standard deviations characteristic of the receiving stations, $\bar{\alpha}_i$, are of the order of a few parts in 10^{11}. The average agreement between the VLF-LF long-term measurements and the portable Cs standards measurements is to parts in 10^{11}, or less, as good as the average agreement within the VLF-LF measurements. Averaging of daily frequency observations over long periods of time provides some improvement in the standard deviations associated with the transmitter and the receiving laboratories. There is some evidence that for those path distances which correspond to single mode dominance of VLF transmission, a linear dependence of precision on distance may apply; however, further work is necessary to substantiate this. Various VLF transmissions are being improved, and the standard deviations of some VLF transmissions, $\bar{\tau}$, have quite steadily decreased to parts in 10^{11}, or less. We recommend long-term round-robin experiments with portable cesium standards that are designed specifically to delineate the propagation limitations and to analyze the receiving station errors.

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