QUANTUM COMPUTERS AND ATOMIC CLOCKS

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Recent developments in quantum information processing may be applicable to future atomic clocks. In this paper we discuss two potential applications to trapped-ion frequency standards. In the first, quantum-mechanical entanglement can provide a resource for increased measurement precision in spectroscopy. In the second, we indicate how a simultaneously trapped auxiliary ion species can be used to provide cooling and as a quantum measuring device; this could be used to increase the number of ion species than can be used as frequency standards.

1 Introduction

The subject of quantum information processing (QIP) has recently received attention because quantum computers could provide a substantial speedup in factoring numbers[1] and in searching databases.[2] In spite of considerable interest in these goals, it is generally agreed that a quantum computer capable of useful factorization or searching (beyond what is possible with classical computers) will, at best, be extremely difficult to achieve in any currently proposed implementation.[3,4] Nevertheless, it is highly likely that other, more tractable applications of QIP will be found and implemented. This paper cites two possible applications of QIP to frequency standards based on trapped atomic ions. Although the basic ideas for these applications emerged before the tidal wave of interest in QIP appeared (ca. 1995), these ideas have matured more rapidly due to advances in QIP. The first application, which we only summarize here, is to use entanglement to reduce frequency instability $\sigma_y(\tau)$ to the minimum level allowed by quantum mechanics. The second application uses the ideas of QIP to remove the functions of cooling and detection from the ion that is used for the frequency standard and place these functions on a second, simultaneously trapped ion species.

2 Entanglement-enhanced spectroscopy

A collection of atoms (neutral or charged) whose internal states are entangled in a specific way can improve the quantum-limited signal-to-noise ratio

in spectroscopy. In this case, the time to reach a certain measurement precision could be reduced by a factor equal to the number of atoms entangled (Eq. (2) below). Therefore, entanglement might be useful in atomic ion frequency standards where, to maintain high-accuracy, a relatively small number of trapped ions ($N \simeq 10-100$) appears optimum due to various experimental constraints, as in the $^{199}Hg^{+}$ microwave clock experiment at NIST.[5]

In spectroscopic experiments on N two-level atoms, we can view the problem in the following way using the spin-1/2 analog for two-level atoms. The total angular momentum of the system is given by $\mathbf{J} = \sum_{i=1}^{N} \mathbf{S}_i$, where $\mathbf{S}_i$ is the spin of the ith atom ($S_i = 1/2$). The task is to measure ω_0, the frequency of transitions between the $|\downarrow\rangle$ and $|\uparrow\rangle$ states, relative to the frequency ω_R of a reference oscillator. We first prepare an initial state for the spins. We assume spectroscopy is performed by applying (classical) fields of frequency ω_R according to the method of separated fields by Ramsey.[6] We also assume that the maximum value of T_R, the time between Ramsey pulses, is limited by other experimental constraints such as the desire to lock the local oscillator to the atoms in a practical length of time. After applying these fields, we measure the final state populations; for example, the number $N_\downarrow$ of atoms in the $|\downarrow\rangle$ state. This has been accomplished with nearly 100 % efficiency through detection of laser fluorescence.[7] In the spin-1/2 analog, measuring $N_\downarrow$ is equivalent to measuring the operator J_z, since $N_\downarrow = J\tilde{I} - J_z$ where $\tilde{I}$ is the identity operator. The signal-to-noise ratio (for repeated measurements) is fundamentally limited by the quantum fluctuations (projection noise[8]) in the number of atoms observed to be in the $|\downarrow\rangle$ state. Spectroscopy is typically performed on N atoms initially prepared in the (unentangled) coherent-spin state[9] $\Psi(t=0) = \prod_{i=1}^{N} |\downarrow\rangle_i$. For this case, the imprecision in a determination of the frequency of the transition is limited by projection noise to the "standard quantum limit" $\sigma_y(\tau) = 1/(\omega_0\sqrt{NT_R\tau})$, where $\tau > T_R$ is the total averaging time. This limit has been observed in spectroscopy of ions[8] and in Cs-fountain atomic clocks.[10] If the atoms can be prepared in particular entangled states, it is possible to achieve $\sigma_y(\tau) < 1/(\omega_0\sqrt{NT_R\tau})$.

The trick is how to actually generate the states useful for spectroscopy. One possibility is to entangle a phonon or electromagnetic field with the atoms; for example, coupling atoms to coherent or squeezed states can lead to "spin-squeezed" states.[11,12,13,14] Squeezed light was recently observed to impart spin squeezing to a collection of ($S_i = 5$) Cesium atoms,[15] and squeezing of an ($S_i = 4$) atomic sample through a quantum-nondemolition measurement of transmitted light has been observed,[16] although squeezing in these experiments was not applied to measurement. The quantum computation scheme of Cirac and Zoller[17] provides a way to realize any desired entangled state for

ions; however, Sørensen and Mølmer[18] have recently devised a more efficient way to create spin-squeezed states. Using this scheme, spin squeezing of two $^9Be^+$ ions was recently demonstrated.[19]

In the above, we have assumed that the operator J_z is the experimental observable. If we consider other observables, it is possible to reach the Heisenberg limit of uncertainty in spectroscopy. Bollinger *et al.*[20] investigated the use of states having the form (after the first Ramsey pulse)

$$\psi_{max} = \frac{1}{\sqrt{2}}\left(|\downarrow\rangle_1|\downarrow\rangle_2\cdots|\downarrow\rangle_N + e^{-iN\omega_0 t}|\uparrow\rangle_1|\uparrow\rangle_2\cdots|\uparrow\rangle_N\right). \quad (1)$$

For this state, the total angular momentum vanishes along any axis, thereby precluding the measurement of J_z for determining $\omega_0 - \omega_R$. However, if after application of the the second Ramsey pulse, we measure the operator $\tilde{O} \equiv \prod_{i=1}^{N} S_{zi}$, the resulting signal[a] gives the exact Heisenberg limit for spectroscopy.[20] Therefore, ideally

$$\sigma_y(\tau) = \frac{1}{\omega_0\sqrt{NT_R\tau}} \rightarrow \frac{1}{\omega_0 N\sqrt{T_R\tau}}. \quad (2)$$

Applying the method of Sørensen and Mølmer,[18] the state ψ_{max} was generated for two and four $^9Be^+$ ions[22] and used in a spectroscopic experiment on two ions.[19] Although the fidelity and number of ions must be increased significantly to be useful in clock applications, this experiment was able to demonstrate a frequency precision better than could possibly be obtained without the use of entanglement.

3 Quantum detectors for spectroscopy

A second potential application of QIP employs an auxiliary ion species that is simultaneously trapped with the "clock" ions. This second species could provide cooling and state detection of the clock ions, thereby widening the choice of potential clock ions. The basic idea[23,24] can be understood by considering two ions that are trapped simultaneously in the same trap. The "logic" ion is used to laser-cool all (harmonic) modes of both ions to their ground states.[b] We assume that the clock ion, having states $|\downarrow\rangle_C$ and $|\uparrow\rangle_C$, is optically pumped to its initial state $|\downarrow\rangle_C$. After the clock radiation is applied,

[a] In this scheme, the first Ramsey pulse could be replaced by the operation that creates the state given in Eq. (1). This might lead to unknown phase shifts relative to the second (conventional) Ramsey pulse that must be accounted for.[21]

[b] Typically cooling employs Doppler cooling followed by sideband cooling.[25]

the logic ion is used to detect whether or not the clock ion absorbed the clock radiation.

We will assume that the logic ion uses two states $|\downarrow\rangle_L$ and $|\uparrow\rangle_L$ for the detection process. We will also assume that we can spectrally resolve sideband transitions of a particular mode of motion that will be employed to perform quantum logic operations.[24] The Fock states of this motional mode are designated by $|n\rangle_M$ where n is an integer. After cooling and optical pumping, the initial state of the system is

$$\psi_0 = |\downarrow\rangle_L|\downarrow\rangle_C|0\rangle_M. \tag{3}$$

In Eq. (3) we have not written the states for the other modes of motion; for simplicity, we assume they are cooled to and remain in their ground states. We then apply (coherent) clock radiation to the ions, leading to

$$\psi_0 \rightarrow \psi_1 = |\downarrow\rangle_L\Big[\alpha|\downarrow\rangle_C + \beta|\uparrow\rangle_C\Big]|0\rangle_M = |\downarrow\rangle_L\Big[\alpha|\downarrow\rangle_C|0\rangle_M + \beta|\uparrow\rangle_C|0\rangle_M\Big]. \tag{4}$$

We now drive a blue-sideband π-pulse[26] on the clock ion so that

$$\psi_1 \rightarrow \psi_2 = |\downarrow\rangle_L\Big[\alpha|\uparrow\rangle_C|1\rangle_M + \beta|\uparrow\rangle_C|0\rangle_M\Big] = |\downarrow\rangle_L|\uparrow\rangle_C\Big[\alpha|1\rangle_M + \beta|0\rangle_M\Big], \tag{5}$$

thereby mapping the clock state to the motion.[c] This is followed by a red-sideband π-pulse on the logic ion yielding

$$\psi_2 \rightarrow \psi_{final} = \Big[\alpha|\uparrow\rangle_L + \beta|\downarrow\rangle_L\Big]|\uparrow\rangle_C|0\rangle_M, \tag{6}$$

and thereby mapping the clock state onto the logic ion. We now measure the state of the logic ion projecting to the states $|\uparrow\rangle_L$ or $|\downarrow\rangle_L$ with respective probabilities $|\alpha|^2$ and $|\beta|^2$. Finally, we re-initialize[d] the system to the state ψ_0 and repeat the experiment with, in general, a different value of the clock radiation frequency ω_R. Although this technique requires several steps, they can all be accomplished reliably and on a time scale of order 1 ms.[28] Therefore, the overhead in time will be negligible for precision clock experiments.

We recall that the group IIIA ions Tl^+ and In^+ have received considerable attention because, as Dehmelt pointed out,[29] the $^1S_0 \rightarrow {}^3P_0$ transitions are free from static quadrupole shifts. These ions also have transitions ($^1S_0 \rightarrow {}^3P_1$) that can be used for cooling and detection (see the contribution to this meeting on In^+ clocks by the Munich group and the work at Seattle[30]). If we

[c]Note that the state $|\uparrow\rangle_C|0\rangle_M$ is unaffected by this operation since the state $|\downarrow\rangle_C|-1\rangle$ does not exist;[24,26] this is the key element of quantum logic used here.

[d]If the lifetime of the clock transition is very long compared to the interrogation cycle, then it is not necessary to re-pump the clock ion since we only need to look for changes in the clock state.[27]

transfer these functions to the logic ion, B^+, Al^+, and other ions might also be considered as clocks. The logic ion might be $^9Be^+$ or Mg^+, but many other possibilities exist. Since the logic ion is assumed to be a different species, it is likely that laser cooling could be applied during application of the clock radiation without significantly affecting the clock levels. This would be important for very long clock interrogation times.

3.1 $^{10}B^+$ optical clock

The relevant energy levels of $^{10}B^+$ (20% natural abundance) are shown in the left-hand side of Fig. 1. As with Tl^+ and In^+, we consider the $^1S_0 \rightarrow {}^3P_0$ clock transitions. In $^{10}B^+$ the nuclear spin is $I = 3$ so that the $m_I = 0 \rightarrow m_I = 0$ transition is "field-independent" as the magnetic field B approaches 0.[e] Therefore we take $|\downarrow\rangle_C$ as the $^1S_0$ ($m_I = 0$) state and $|\uparrow\rangle_C$ as the $^3P_0$ ($m_I = 0$) state. To pump to the $|\downarrow\rangle_C$ state we could drive the $^1S_0 \rightarrow {}^3P_1(F = 3)$ transition with π-polarized light.[f] Since the lifetime of the $^3P_1$ state is around 0.1 s,[33] this process takes about 1 s. Although the pumping process is time-consuming, we note that once $^{10}B^+$ is in the manifold of the two clock states, decay out of these states is extremely unlikely. As a check of this, we could occasionally interrupt the clock interrogation cycles and rapidly drive resonant π-pulses on the clock transition, followed by logic-ion checking to test that the clock ion remained in the clock manifold of states. As an alternative approach, we might prepare the $|\downarrow\rangle_C$ state from a mixture of the seven ground-state levels by first driving the nuclear Zeeman transition in the ground state and then testing to see if the ion is in the clock manifold by the same checking procedure. Although the lifetime of the $^3P_0$ level is rather long ($\tau \simeq 4300\ s$[34]) and the dipole matrix element rather weak, we estimate Stark shifts from coupling to other levels to be less than 10^{-18} for Rabi rates less than 1 radian-s^{-1}.

3.2 $^{27}Al^+$ optical clock

The ion $^{27}Al^+$ is the only stable isotope of aluminum and has nuclear spin $I = 5/2$; we show some of its relevant energy levels in the right-hand-side of Fig. 1. The $^1S_0 \rightarrow {}^3P_0$ transitions in $^{27}Al^+$ ($\lambda \simeq 267.44$ nm, $\tau(^3P_0) = 284\ s$[35])

[e] Dehmelt *et al.*[31] and Yu *et al.*[32] have proposed taking advantage of similar field independent transitions in $^{204}Tl^+$ ($I = 2$) and $^{26}Al^+$ ($I = 5$); however, these ions have the complication of being radioactive ($\tau_{1/2}(^{204}Tl^+) \simeq 3.8$ years, $\tau_{1/2}(^{26}Al^+) \simeq 10^6$ years).

[f] We could also pump to the $^1S_0$ ($m_I = \pm 3$) states using circularly polarized light on the $^1S_0 \rightarrow {}^3P_1$ transition. We could then "walk" the ion to the $|\downarrow\rangle_C$ state with four coherent π-pulses between the $^1S_0$ and $^3P_1$ states (this should require negligible time).

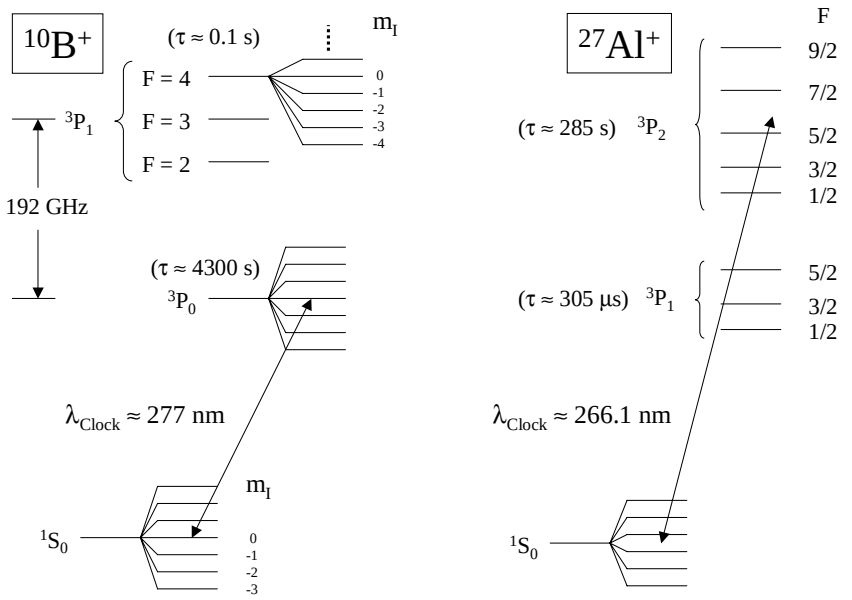

Figure 1. Partial energy-level diagrams for $^{10}B^+$ and $^{27}Al^+$ relevant for potential ion optical clocks. In $^{10}B^+$, the $^1S_0(m_I = 0) \rightarrow {}^3P_0(m_I = 0)$ transition is field-independent for $B \rightarrow 0$ and has no static electric quadrupole shift. For the $^1S_0 \rightarrow {}^3P_2$ transitions in $^{27}Al^+$, several are field-independent for $B \simeq 100$ G; however, it is necessary to average the clock frequency for three equal-magnitude mutually orthogonal directions of the B-field.

would have features similar to the corresponding transitions in In^+, including a linear magnetic field dependence because of the difference in nuclear g-factors in the ground and excited states.[29] Nevertheless, if one deals with this complication as in the Munich and Seattle experiments, $^{27}Al^+$ can be used in a similar fashion as $^{10}B^+$ above (with the advantage that the initial pumping of the clock states could be much faster because of the 305 μs lifetime[33] of the $^3P_1$ state). We also note that the magnetic-quadrupole-allowed $^1S_0 \rightarrow {}^3P_2$ transitions at $\lambda \simeq 266.1$ nm could be driven by the fourth harmonic of a Nd-YAG laser. The $^3P_2$ hyperfine constant has been calculated to be $A_{hfs}(^3P_2) = 1125$ MHz,[g] so that there are several field-independent transitions at magnetic fields around 100 G. Unfortunately, at the field-independent magnetic fields all $^3P_2$ states would have components with angular momentum $F > 1/2$,

[g] P. Jönsson, Lund Institute of Technology, private communication.

resulting in a static quadrupole shift as with D-state levels.[h] This would necessitate averaging the clock frequency for three equal-magnitude mutually-orthogonal B-field directions.[36] This might be easier than in the case of ions such as $^{199}Hg^{+}$ since the tolerances on setting the field are less stringent.[i]

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[h] At a minimum, we must account for the quadrupole shift of the clock ion from the Coulomb fields of the logic ion; this would be on the order of a few Hz for an ion-ion separation of 10 μm.

[i] Alternatively, one could drive a $^1S_0 \rightarrow {}^3P_2(F = 1/2)$ transition at $B \simeq 0$, for which the static quadrupole shift is absent. This, however, is not a very attractive alternative as there is a strong linear dependence on magnetic-field, approximately equal to the electron Zeeman effect.

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