A FEMTOSECOND-LASER-BASED OPTICAL CLOCKWORK*

S. A. DIDDAMS, TH. UDEM[§], K. R. VOGEL, L.-S. MA[†],
L. ROBERTSSON[‡], C. W. OATES, E. A. CURTIS[♯], W. M. ITANO,
R. E. DRULLINGER, D. J. WINELAND, J. C. BERGQUIST, L. HOLLBERG

National Institute of Standards and Technology, 325 Broadway
Boulder, CO 80305
E-mail: sdiddams@boulder.nist.gov

In this article we summarize our progress in developing and testing a femtosecond-laser-based optical clockwork that provides a single-step phase-coherent connection between emerging optical frequency standards and the cesium microwave standard on which the SI second is based. This clockwork enabled absolute optical frequency measurements with statistical uncertainty $\sim 2 \times 10^{-15}$, and it was used in the demonstration of an optical clock with instability $\leq 7 \times 10^{-15}$ in 1 s. Recent experiments demonstrate the intrinsic fractional instability and inaccuracy of the clockwork are less than 6.3×10^{-16} in 1 s and 4×10^{-17}, respectively.

1 Introduction

Emerging optical frequency standards based on laser-cooled atoms and ions promise superior stability and accuracy over existing microwave standards[1,2,3,4]. However, until recently an appropriate clockwork for dividing down the very fast optical oscillations to a countable frequency had been missing. As demonstrated in the past two years, a clockwork based on mode-locked femtosecond lasers provides a convenient, robust, and accurate means of phase-coherently linking optical frequencies to standards in the microwave domain[5,6,7,8,9]. This breakthrough has revolutionized optical frequency metrology and has enabled a new generation of atomic clocks based on optical transitions. Here we summarize our recent progress in using the femtosecond-laser-based clockwork in conjunction with the Ca and Hg^+ optical frequency standards[10] to make a functioning optical clock.

2 Frequency measurements and an optical clock

A mechanical analog of the femtosecond-laser-based clockwork is that of a large reduction/multiplication gear that provides a single-step connection between the optical and microwave domains of the electromagnetic spectrum [Fig. 1(a)]. As illustrated by this analogy, the clockwork can be engaged in the microwave domain, where it functions to "multiply up" a Cs-referenced oscillator to the optical domain. Or alternatively, the clockwork can be phase-

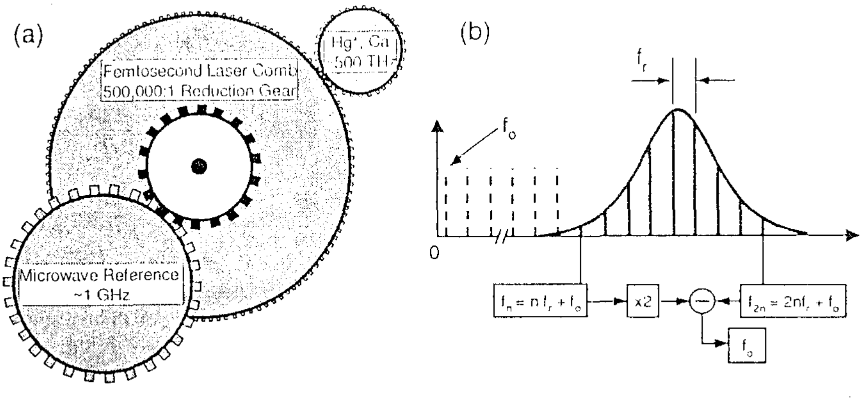

Figure 1. (a) Using a mechanical analogy the femtosecond-laser-based clockwork functions as a $\sim 500,000 : 1$ reduction gear (clearly not to scale here) that enables a single step connection between the optical and microwave domains. (b) In reality, the clockwork consists of an octave-spanning comb of phase-locked CW oscillators generated with a femtosecond laser and nonlinear microstructure fiber.

locked to an optical frequency standard and thereby "divide down" to a countable frequency in the RF/microwave domain. As shown in Fig. 1(b), the femtosecond-laser-based clockwork actually consists of an octave-spanning comb of phase-locked continuous-wave (CW) oscillators that are generated when the spectral output of a femtosecond laser is broadened in nonlinear microstructure fiber. The spectrum obtained directly from a mode-locked laser consists of a comb of regularly spaced continuous waves that are separated by the pulse repetition rate f_r. The frequency of the n^{th} mode of the comb is given by $f_n = nf_r + f_o$, where f_o is the frequency offset common to all modes that is caused by the difference between the group and the phase velocity inside the laser cavity[5,11,12]. The easily measured f_r is directly linked to an optical frequency (f_n) provided that f_o is known. If the frequency comb of the laser covers an entire octave, f_o can be readily measured by frequency-doubling many of the infrared modes and heterodyning them with existing modes in the visible portion of the comb[8]. The heterodyne signal yields the difference $2(nf_r + f_o) - (2nf_r + f_o) = f_o$. With the arrival of novel microstructure silica fibers[13,14] the typical femtosecond laser bandwidth of 15 THz can be broadened to the required octave (> 300 THz) via self-phase modulation. This is possible even with the relatively low energy pulses (< 1 nJ) obtained from high-repetition-rate femtosecond lasers. In contrast to the work of at the PTB[15], the approach we have taken is to fully control both f_o and f_r for

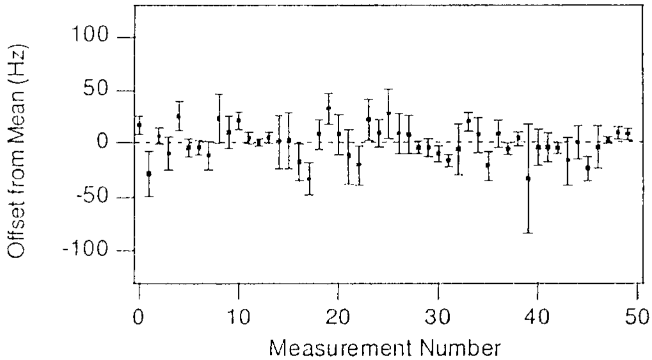

Figure 2. Record of 50 measurements of the stabilized laser that has its harmonic locked to the Hg^+ clock transition. All measurements were taken over a two week period in August of 2000, and the weighted mean is 532 360 804 949 571.3 Hz. Only statistical error bars are shown, and they are representative of the counter gate time and number of samples acquired for each measurement. The statistical uncertainty in the mean of these data is 1.2 Hz.

the octave spanning comb such that every tooth of the comb will ultimately bear the same properties as the optical or microwave oscillator that serves as the reference.

For our experiments we employ a compact Ti:sapphire femtosecond laser with $f_r = 1$ GHz[16]. A detailed description of the operation and servo-control of this laser has been provided elsewhere[17]. Control of f_o is achieved by varying the power of the laser that optically pumps the femtosecond laser. Once f_o is phase-locked to a stable RF source[18], the remaining free parameter of the comb, f_r, can be phase-locked to either a microwave or an optical standard. When the reference is a microwave standard, f_r is detected by illuminating a fast photodiode with a portion of the continuum output from the microstructure fiber. A suitable error signal used to control the femtosecond cavity length is generated by mixing the 1 GHz harmonic of the photocurrent with a signal synthesized from a stable RF oscillator. We have verified[17] that the instability of each optical comb element is then limited by the instability of the reference oscillator—in our case the hydrogen maser with Allan deviation $\sigma_y(1s) \sim 2 \times 10^{-13}$. Furthermore, a measure of the ability of the clockwork to accurately compare a Cs primary standard (the NIST-F1 Cs fountain) to optical standards has been achieved in the measurement of the clock transitions of the Hg^+ and Ca standards at NIST[19]. Figure 2 shows a record of 50 different measurements of the frequency of the local oscillator locked to the Hg^+ frequency standard taken over a two-week period in August 2000. The weighted mean of these data have a statistical uncertainty of only 1.2 Hz, or fractionally 2.3×10^{-15}. For the total integration time of approximately 20 000 s, this uncertainty is near that of the hydrogen maser that provided the RF reference and only a factor of ~ 1.5 above the uncertainty of NIST-F1 (which calibrates the frequency of the hydrogen maser). The conclusion from this result is that the clockwork can accurately transfer the stability and accuracy of the best microwave standards to the optical domain.

With f_o again fixed as already described, one can alternatively control f_r by phase-locking one mode of the comb to the laser oscillator of the Hg^+ or Ca standard. If the modes of the comb are phase-coherent, this approach provides the tremendous leverage of the mode number $m \sim 500\ 000$ in controlling f_r. Indeed, if $f_o = 0$, then f_r is the m^{th} sub-harmonic of the optical standard, with the fractional frequency fluctuations of the optical standard being transferred to f_r. This scheme contains all essential elements of an optical clock: a stabilized laser oscillator locked to an atomic transition and a clockwork capable of counting a certain number of optical cycles to generate an interval of one second. We have recently demonstrated such an optical clock with the Hg^+ standard serving as the reference[20]. Initially, we were able to verify that the instability in f_r was comparable to that of the hydrogen maser, but lacking a more stable microwave source, we were unable to determine whether the instability of f_r approached that of the stabilized laser of the Hg^+ standard [$\sigma_y(\tau) < 2 \times 10^{-15}$ for $1\text{s} < \tau < 100$ s]. However, we could test the stability of the comb in the optical domain by measuring the fluctuations of the beat between one tooth of the Hg^+-stabilized comb and the Ca standard. In such an experiment, we measured an Allan deviation as low as 7×10^{-15} in 1 s of averaging—limited by the present instability of the Ca standard. This provided the first indication that it was possible to transfer the stability of the Hg^+ standard to the entire comb. Furthermore, it demonstrated the ability to directly compare the frequency ratio and relative stability of widely separated optical standards (76 THz in this case) without the use of a RF/microwave reference oscillator.

3 Comparisons between two clocks

A much more rigorous test of the clockwork involves checking the stability and accuracy of the nonlinearly generated comb across its entire octave span. To accomplish this we have operated two clockworks that are both phase-coherently locked to the same CW reference laser (at 456 THz). Both combs were controlled to have the same value of f_r but different values of f_o. Sub-

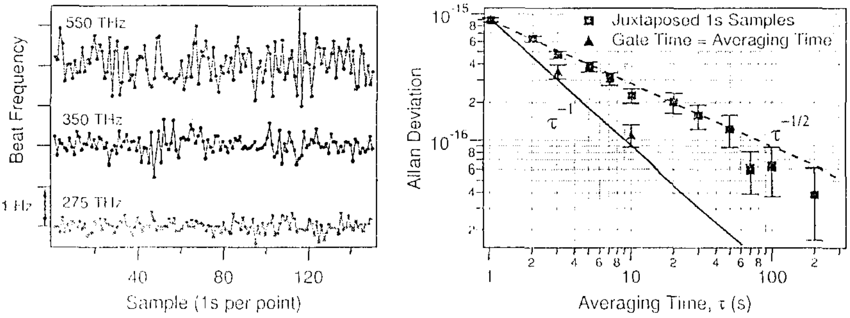

Figure 3. (left) Time records of the measured optical beat between two combs at the frequencies of 275, 350 and 550 THz. The three traces are offset for display purposes. The vertical scale is 1 Hz per division. (right) Allan deviation computed from the data at 550 THz. The $\tau^{-1/2}$ dependence is the result of juxtaposing 1 s samples to compute the Allan deviation at longer gate times. When we compute the Allan deviation from data acquired with counter gate times equal the averaging time, we see a dependence closer to τ^{-1}. This is expected for white phase noise with an rms phase fluctuation constant in time.

sequently, we measured and analyzed the heterodyne beats between the two combs in different spectral regions to determine how precisely they track the reference laser[21]. Since the noise of the CW laser was common to both combs, we could evaluate the precision of the various phase-lock loops, as well as the stability and frequency accuracy of the comb elements that are nonlinearly generated in the microstructure optical fiber. Using optical filters in conjunction with different detectors, we measured the absolute value of the offset between the two combs and its instability at 550 THz, 350 THz, and 275 THz. Sample results are shown in Fig. 3. While the magnitude of the fluctuations is seen to increase with frequency, the fractional fluctuations are approximately constant across the spectrum. In fact, an average of all data acquired shows that the 1 s Allan deviation differs by no more than 30 % for the three measurement frequencies across the optical octave. Since both combs contribute to the measured instability, we can assume that the fluctuations of a single comb are smaller by a factor of $\sqrt{2}$. Taking the slightly larger Allan deviation at 550 THz, we then establish an upper limit of 6.3×10^{-16} for the one-second instability of the femtosecond-laser-based clockwork. Furthermore, the offset of the measured beat between the two combs from the expected phase-locked value provides information about possible frequency errors that might occur in the nonlinear generation of the octave-spanning comb. The best previous test[9] of the actual frequencies of the elements of an octave-spanning comb

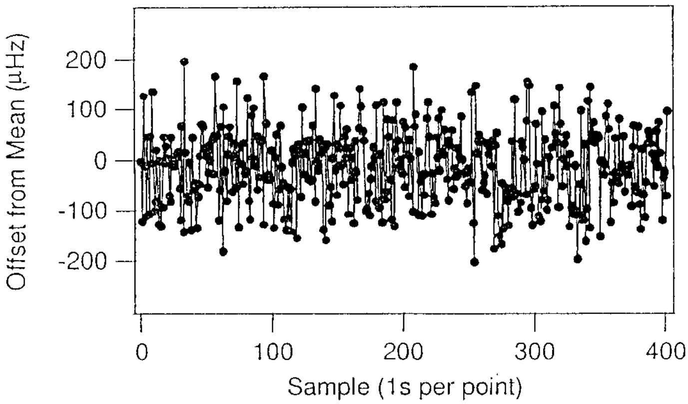

Figure 4. Time record of the difference frequency between the ~1 GHz output of two optical clocks. The mean difference frequency was approximately 10 kHz. The one-second Allan deviation of this data is 76 μHz, which implies a fractional instability of 5.3×10^{-14} for the 1 GHz output of one of the clocks.

demonstrated an upper limit uncertainty of 5×10^{-16}. In our case, we find the scatter of the measured offsets to be only 14 mHz, which yields an average uncertainty in the frequencies of the comb lines of about 4×10^{-17}. Both of these results are at the limit imposed by Doppler shifts, and active control of all optical paths will be necessary in the future to reach higher stability and accuracy.

The experiments just described prove that the stability of the best current optical standards can be transferred to the octave-spanning comb. However, the extraction of a microwave signal at frequency $f_r \approx 1$ GHz with instability equaling that of the optical comb has thus far proven difficult. In order to remove any uncertainty introduced by the optical standard itself, we again operated two optical clocks referenced to the same stabilized diode laser at 456 THz. In this case we set the mode spacings of the two combs to differ by about 10 kHz (i.e., $\Delta f_r = |f_{r1} - f_{r2}| \approx 10$ kHz). A fraction of the light from each comb was focused onto its own fast photodiode such that microwave signals at f_{r1} and f_{r2} (and their higher harmonics) were generated. These signals are band-pass filtered, amplified, and then mixed using low-noise components. The resulting beat at 10 kHz was further amplified and low-pass filtered before being sent to a counter. The typical 1 second instability of this counted beat was about 150 μHz, which implies a fractional frequency instability of 1×10^{-13} on the 1 GHz carrier for one of the clocks. However,

Figure 4 shows data for which the instability was decreased by about a factor of 2. This yields a fractional instability of 5.3×10^{-14} at 1 s for one of the clocks. The Allan deviation of the 1 GHz output was also observed to average down as τ^{-1}. Nonetheless, this short-term stability is nearly two orders of magnitude above what we would expect based on optical comparison of the two combs, and at least an order of magnitude above the more fundamental limitation imposed by shot noise[22]. While the reasons for this surprisingly high short-term instability are not yet fully understood, the photodetection process itself along with excess noise generated in the microstructure fiber[22] and the femtosecond laser continue to be areas of investigation.

Acknowledgments

The authors acknowledge the thoughtful comments of J. Ye and J. L. Hall. We are endebted to R. S. Windeler of Lucent Technologies for providing the microstructure fiber, and are grateful to A. Bartels of GigaOptics GmbH for his assistance with the femtosecond laser.

* Contribution of NIST, and therefore not subject to U.S. copyright.

§Permanent address: Max-Planck-Institut für Quantenoptik, Hans Kopfermann Str.1, 85748 Garching, Germany.
†Also with: JILA, Univ. of Colorado, and BIPM. Permanent address: Laboratory for Quantum Optics, East China Normal University, Shanghai, China.
‡Permanent address: BIPM, Pavillon de Breteuil, 92312 Sevres, France
‖ Also with: Dept. of Physics, Univ. of Colorado, Boulder, CO 80309.

References

1. C. W. Oates, E. A. Curtis, L. Hollberg, Opt. Lett. **25**, 1603 (2000).
2. B. C. Young, F. C. Cruz, W. M. Itano, J. C. Bergquist, Phys. Rev. Lett. **82**, 3799 (1999).
3. R. Rafac, B. C. Young, J. A. Beall, W. M. Itano, D. J. Wineland, J. C. Bergquist, Phys. Rev. Lett. **85**, 2462 (2000).
4. A. A. Madej, J. E. Bernard, in *Frequency Measurement and Control*, A. N. Luiten, ed. (Springer-Verlag, Berlin, 2001), pp. 153-194.
5. Th. Udem, J. Reichert, R. Holzwarth, T. W. Hänsch, Phys. Rev. Lett. **82**, 3568 (1999).
6. Th. Udem, J. Reichert, R. Holzwarth, T. W. Hänsch, Opt. Lett. **24** , 881 (1999).
7. S. A. Diddams, D. J. Jones, J. Ye, S. T. Cundiff, J. L. Hall, J. K. Ranka,

· R. S. Windeler, R. Holzwarth, Th. Udem, T. W. Hänsch, Phys. Rev. Lett. **84**, 5102 (2000).
8. D. J. Jones, S. A. Diddams, J. K. Ranka, R. S. Windeler, A. J. Stentz, J. L. Hall, S. T. Cundiff, Science **228**, 635 (2000).
9. R. Holzwarth, Th. Udem, T. W. Hänsch, J. C. Knight, W. J. Wadsworth, P. St. J. Russell, Phys. Rev. Lett. **85**, 2264 (2000).
10. See contributions by E. A. Curtis *et al.* and J. C. Bergquist *et al.* also in these Proceedings.
11. J. Reichert, R. Holzwarth, Th. Udem, T. W. Hänsch, Opt. Comm. **172**, 59 (1999).
12. H. R. Telle, G. Steinmeyer, A. E. Dunlop, J. Stenger, D. H. Sutter, U. Keller, Appl. Phys. B **69**, 327 (1999).
13. J. K. Ranka, R. S. Windeler, A. J. Stentz, Opt. Lett. **25**, 25 (2000).
14. W. J. Wadsworth *et al.*, Electron. Lett. **36**, 53 (2000).
15. See the contribution of H. Telle in these proceedings.
16. A. Bartels, T. Dekorsy, H. Kurz, Opt. Lett. **24**, 996 (1999).
17. S. A. Diddams, Th. Udem, K. R. Vogel, C. W. Oates, E. A. Curtis, R. S. Windeler, A. Bartels, J. C. Bergquist, L. Hollberg, in Laser Frequency Stabilization, Standards, Measurement, and Applications, J. L. Hall, J. Ye, eds., Proc. of SPIE Vol. 4269, 77-83.
18. As originally demonstrated in Ref. [8], f_o (and any other beat for that matter) can be phase-locked to an RF signal derived from f_r itself, thereby eliminating its dependence on an external oscillator.
19. Th. Udem, S. A. Diddams, K. R. Vogel, C. W. Oates, E. A. Curtis, W. D. Lee, W. M. Itano, R. E. Drullinger, J. C. Bergquist, L. Hollberg, Phys. Rev. Lett **86**, 4996 (2001).
20. S. A. Diddams, Th. Udem, J. C. Bergquist, E. A. Curtis, R. E. Drullinger, L. Hollberg, W. M. Itano, W. D. Lee, C. W. Oates, K. R. Vogel, D. J. Wineland, Science **293**, 825 (2001).
21. S. A. Diddams, L.-S. Ma, L. Robertsson, L. Hollberg, to appear in Opt. Lett.
22. L. Hollberg, C. W. Oates, E. A.Curtis, E. N. Ivanov, S. A. Diddams, Th. Udem, H. G. Robinson, J. C. Bergquist, R. J. Rafac, W. M. Itano, R. E. Drullinger, D.J. Wineland, to appear in IEEE J. Quant. Electron.