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1/f PM and AM Noise in Amplifiers and Oscillators♦

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Abstract

In this paper we show that the 1/f AM and PM noise added to a carrier signal by a two-port device can be expressed in terms of its transfer function. If the transfer function is expressed in terms of the operating point, the current and voltage at various nodes, then we can compute the dependence of the added noise on these parameters. We illustrate this approach for a common emitter amplifier using a simple linear model of a Si bipolar junction transistor. We find that the added 1/f AM and PM noise is primarily the result of 1/f fluctuations in transistor current, transistor capacitance, circuit supply voltages, impedance levels, and circuit configuration. We then show how the PM noise of an oscillator depends on the PM noise of the resonator and the sustaining amplifier.

1. Introduction

The focus of this paper is twofold. First, to present a general two-port theory which explains the up-conversion of the 1/f baseband (near dc) noise of amplifier circuits into 1/f phase modulation (PM) and amplitude modulation (AM) noise about a coherent high frequency signal in linear amplifiers. Second, to show how amplifier PM noise affects the PM noise of oscillators.

The two-port theory is quite general in nature and can be formally applied to a wide variety of circuits. To provide a concrete example, we apply the formalism to a common-emitter (CE) amplifier with a Si bipolar transistor (BJT) gain element. To obtain insight into the upconversion processes we use a first order linear hybrid-π model for the BJT. A more complete analysis is possible using computer models but they often do not provide the insight obtained from a first order analytical model [1-2].

We then use Leeson's model to obtain the PM noise of an oscillator from the PM noise of the resonator and the amplifier. From this model we see that 1/f PM noise in the amplifier leads to $1/f^3$ PM noise inside the bandwidth of the resonator and 1/f PM noise outside the bandwidth of the resonator.

The work presented here provides a starting point for designing amplifiers and oscillators with low PM and AM noise and for estimating how the noise scales with operating point and signal frequency. In many cases the 1/f noise can be reduced to such an extent that it is much lower than the thermal noise for frequency offsets from the signal larger than a few Hz [3].

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2. Basic Definitions

Equation 1 shows the model of a noisy signal source with average frequency ν_o and amplitude V_o. The amplitude fluctuations are characterized by $\varepsilon(t)$ and the phase fluctuations are characterized by ϕ (t).

$$V(t) = \left[V_o + \varepsilon(t)\right]\cos\left(2\pi\upsilon_o t + \phi(t)\right). \qquad (1)$$

PM noise is specified as the power spectral density of phase fluctuations given by

$$S_\phi(f) = \left[\Delta\phi(f)\right]^2 \frac{1}{BW}, \qquad 0 < f < \infty, \qquad (2)$$

where BW is the bandwidth of the measurement, and f is the Fourier frequency offset from ν_o.

AM noise is specified as the power spectral density of fractional amplitude fluctuations given by

$$S_a(f) = \frac{\left[\Delta\varepsilon(f)\right]^2}{V_o^2} \frac{1}{BW}, \qquad 0 < f < \infty, \qquad (3)$$

where BW is the bandwidth of the measurement.

These definitions show that one cannot define the added PM noise or AM noise of an amplifier except in the presence of a carrier signal. In much of the literature the single sideband AM noise, 1/2 S_a(f), and PM noise, *L*(f) = 1/2 S(f), are used and we will also fo the remainder of the paper. The most common units for *L*(f) are dB below the carrier in a 1 Hz bandwidth (dBc/Hz)

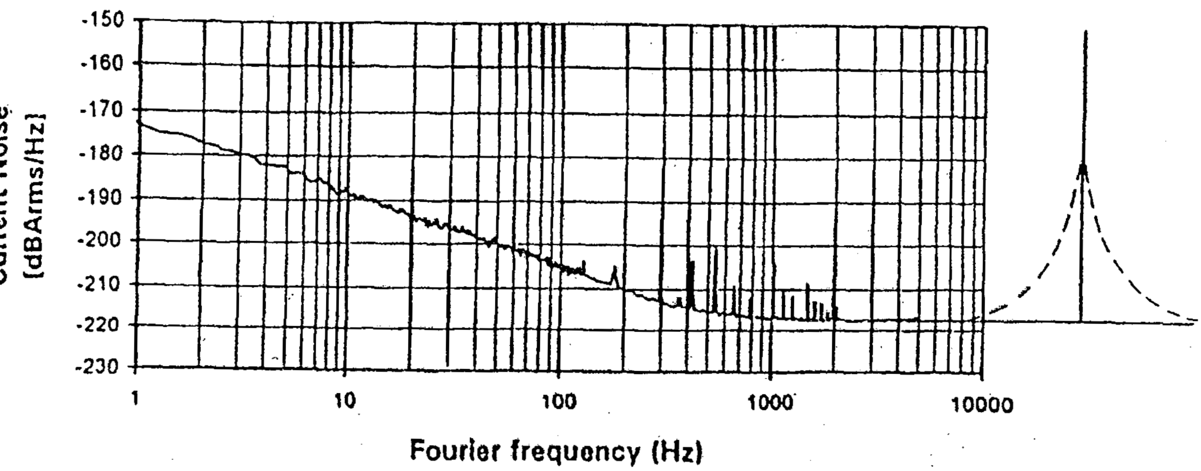


Fig. 1. Current noise of an amplifier as a function of frequency.

3. Basic Upconversion Problem

The solid line in Fig. 1 shows the output noise power per Hz bandwidth of an amplifier as a function of frequency. Near zero frequency the noise power typically varies as approximately 1/f where f is frequency. At higher frequencies (approximately 300 Hz for silicon BJTs), the noise power/Hz that can be transferred to a matched load is independent of frequency and given by

$$P_N = kTFG, \tag{4}$$

where k is Boltzmann's constant, T is temperature in kelvin, F is the noise figure, and G is the power gain. At 300 K this thermal noise is equal to -174 dBm/Hz for F = G = 1. When a coherent signal is introduced this thermal noise is divided equally between AM and PM noise and scales as $1/P_O$, where P_O is the output signal power:

$$1/2\, S_a(f) = \mathcal{L}(f) = kTFG/(2P_O). \tag{5}$$

For P_o = 0 dBm, F = G = 1, $1/2\, S_a(f) = \mathcal{L}(f)$ = -177 dBc/Hz. If the noise on the input signal is low enough, it is possible to observe that the amplifier adds, in addition to the thermal noise given in Eq. 4, excess noise power to the output signal. This excess noise, which is proportional to the input signal power, is illustrated by the dashed line in Fig. 1. When referenced to the center of the signal, this excess noise has the same 1/f characteristics as that observed near dc and is therefore commonly referred to up-converted dc or baseband noise. *In contrast to the thermal noise, circuit parameters determine whether the 1/f portion of the AM noise is higher, equal to, or lower than the 1/f portion of the PM noise.*

4. Early Work on 1/f Noise in Amplifiers And Frequency Multipliers

Halford et al. [4] and Andressen [5] were the first to show that the 1/f PM noise of CE amplifiers (Fig. 2) and frequency multipliers can be reduced by adding an unbypassed resistance R_E in the emitter leg of the transistor. Excess AM noise was studied by Healy [6]. Because no theory existed to explain

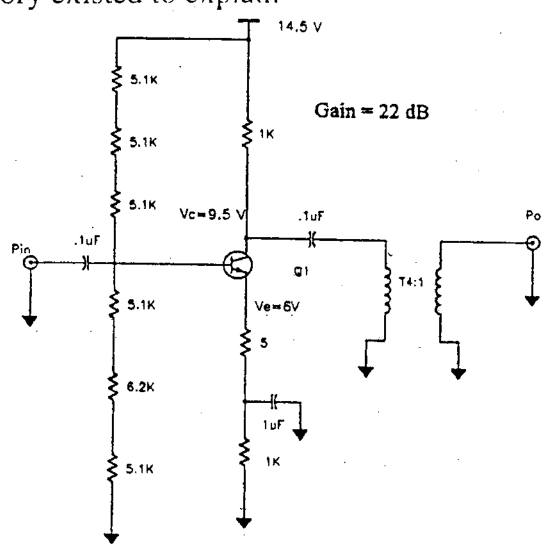


Fig. 2. CE amplifier with unbypassed emitter resistor.

the reduction of the PM noise with unbypassed emitter resistor or the effect on the AM noise it was not possible to design amplifiers with low 1/f noise except by trial and error.

Our work in this area started with [7,8] which showed that the 1/f AM and PM noise was often of similar amplitude. This occurred in many types of amplifiers and raised the possibility that the noise might be due to a common mechanism.

5. 1/f Noise in Linear Amplifiers

We first derive the complete general expression for added AM and PM noise in a two-port device such as an amplifier. The first order approximations to these expressions are then used to derive the added AM and PM noise for a CE amplifier with a single dominant pole. We then show in a simplified example how fluctuations in the emitter current cause fluctuations in the magnitude of the gain thereby inducing AM noise. We then include the current and voltage dependence of various transistor and circuit elements to develop a more complete theory of both AM and PM noise in CE amplifiers.

6. A General Expression of Added AM and PM Noise in Amplifiers

The general transfer function for an amplifier, G(s), can be written as

$$G(s) = G_o F_L(s) F_H(s), \tag{6}$$

where $s = j\omega$ ($\omega = 2\pi\nu_o$ = angular frequency), G_o is the mid-band gain, F_L is the low frequency dependence of the transfer function, and F_H describes the high frequency dependence of gain on frequency. For the purposes of the present discussion, we can concentrate on the behavior of F_H, as we are assuming that the amplifier is operating at a frequency well above its low frequency cut-off. F_H can be written as

$$F_H(s) = \frac{(1+s/\omega_{z1})(1+s/\omega_{z2})\ldots(1+s/\omega_{znh})}{(1+s/\omega_{p1})(1+s/\omega_{p2})\ldots(1+s/\omega_{pnh})}, \tag{7}$$

where ω_{p1}, ω_{p2}, ..., ω_{pnh} are positive numbers representing the angular frequencies of the n real poles and ω_{z1}, ω_{z2}, ..., ω_{znh} are real numbers representing the frequencies of the n zeros. (Note that these can be positive, negative, or infinite.) In many real-world cases the zeros are at such high frequencies that they are of little significance in determining the upper cutoff frequency of G(s). In this case the function $F_H(s)$ can be approximated by

$$F_H(s) = \frac{1}{(1+s/\omega_{p1})(1+s/\omega_{p2})\ldots(1+s/\omega_{pnh})}. \tag{8}$$

An alternative representation which will prove useful in the following discussion is to analyze G(s) in terms of the gain and phase. That representation leads to G(s) being described by

$$G(s) = G_O |F_H(s)| e^{j\vartheta}, \quad \vartheta = \tan^{-1}\frac{\mathrm{Im}(F_H(s))}{\mathrm{Re}(F_H(s))} \tag{9}$$

It should be noted that the two representations are equivalent.

Finally in the case which we will analyze in detail, the CE amplifier, one pole is dominant, at which point we can further approximate $F_H(s)$ as

$$F_H(s) = \frac{1}{1 + s/\omega_{pl}}, \tag{10}$$

Equation 9 can now be written as

$$G(s) = G_o\left(\frac{1}{1+\delta^2}\right)^{1/2} e^{j\vartheta}, \vartheta = \mathrm{Tan}^{-1}\delta, \delta = \left(\frac{\omega}{\omega_{pl}}\right). \tag{11}$$

If we consider the input signal to be perfect, that is

$$V_{in} = V_o \cos\omega t, \tag{12}$$

then the output signal is given by

$$\begin{aligned} V_{out} &= V_{in}G = V_o G_o\left(\frac{1}{1+\delta^2}\right)^{1/2} e^{j\vartheta} \cos(\omega t) \\ &= V_o G_o\left(\frac{1}{1+\delta^2}\right)^{1/2} \cos(\omega t + \vartheta). \end{aligned} \tag{13}$$

The AM noise added by an amplifier can be written as

$$\frac{1}{2} S_a(f) \cong \frac{1}{2}\left(\frac{\Delta G_V}{G_V}\right)^2 \frac{1}{BW} + \frac{kTFG}{2P_o}, \tag{14}$$

where G_V is the magnitude of the voltage gain. The PM noise added by an amplifier can be written as

$$\frac{1}{2} S_\phi(f) \cong \frac{1}{2} \Delta\vartheta^2 \frac{1}{BW} + \frac{kTFG}{2P_o}. \tag{15}$$

Equations 14 and 15 also include the contribution of the thermal noise. These expressions are completely general and can be used on any two port element such as an amplifier, attenuator, or level shifter, as long as the output frequency is the same as the input frequency, by expressing G_V as a function of the circuit parameters at the operating carrier frequency. Equations 14 and 15 can also be extended to cover two-port elements where the frequency is changed by multiplying by the square of the ratio of output to input frequency [9].

7. Single Pole Approximation for Gain of Common-Emitter Amplifier

In this section we use the above results to develop a physical model which explains the up-conversion of baseband noise to create excess AM and PM noise in CE amplifiers. The equation describing gain in a practical CE amplifier is very complicated; therefore, several approximations will be used to simplify the equations so that the underlying principles are more easily seen.

In the dominant pole case at frequencies ω much smaller than the 3 dB bandwidth (ω_{pl}), that is $\delta = (\frac{\omega}{\omega_{pl}}) < 1$, Eq. 11 can be further approximated and simplified to

$$G = G_o\left(\frac{1}{1+\delta^2}\right)^{1/2} e^{j\vartheta} = G_o\left(1-\frac{\delta^2}{2}+\frac{3\delta^4}{8}\ldots\right)e^{j(\delta+\delta^3..)} = G_o\left(1-\frac{\delta^2}{2}\right)e^{j\delta}. \tag{16}$$

This result can be used to simplify Eq. 13 to

$$V_{out} = V_{in}G = V_o G_o\left(1-\frac{\delta^2}{2}\right)\cos(\omega t+\delta)\cdot \tag{17}$$

Equation 14 can be expanded to yield

$$\frac{1}{2}S_a(f) \cong \frac{1}{2}\left(\frac{\Delta G_o}{G_o}\right)^2 \frac{1}{BW} + \frac{1}{2}\delta^2\Delta\delta^2\frac{1}{BW} + \frac{kTFG}{2P_o}. \tag{18}$$

Equation 18 can also be expressed as

$$\frac{1}{2}S_a(f) \cong \frac{1}{2}\left(\frac{\Delta G_o}{G_o}\right)^2 \frac{1}{BW} + \frac{1}{2}\delta^2\left(S_\phi(f)-\frac{kTFG}{2P_o}\right)+\frac{kTFG}{2P_o}, \tag{19}$$

where

$$\frac{1}{2}S_\phi(f) \cong \frac{1}{2}\Delta\delta^2\frac{1}{BW}+\frac{kTFG}{2P_o}. \tag{20}$$

Fig. 2 shows the schematic of a simple CE amplifier. In the approximation that the load impedance can be represented by R_L, and ω **is** much smaller than ω_{pl} (that is, the phase shift δ is small) we can write the gain as

$$G = G_o\left(\frac{1}{1+\delta^2}\right)^{1/2} e^{-j\vartheta} = G_o(1-\frac{\delta^2}{2})e^{-j\delta}, \tag{21}$$

$$G_o = -\frac{R_L}{r_e+R_E+r_g/\beta},$$

$$r_g = \left(r_{bb}+\frac{R_S R_{BIAS}}{R_S+R_{BIAS}}\right)K, \tag{22}$$

$$\delta \cong \omega C_{bc}G_o\left(r_e+R_E+r_g\right)+ \omega C_{be}G_o r_g r_e\left(1-\frac{R_E}{R_E+r_e}\right)M/R_L+.. \tag{23}$$

$$M = \frac{1}{1 + (\omega C_{be}\beta)^2 \left(\frac{r_e R_E}{r_e + R_E} \right)^2}, \qquad (24)$$

$$K = \frac{1 + (\omega C_{be}\beta)^2 r_e \left(\frac{r_e R_E}{r_e + R_E} \right),}{1 + (\omega C_{be}\beta)^2 \left(\frac{r_e R_E}{r_e + R_E} \right)^2} \qquad (25)$$

where the R's are defined in Fig. 2, r_g is the total effective input impedance, and δ is the phase shift due to C_{bc}, and C_{be}. A further simplification can be made for CE amplifiers with large gain or $C_{be} \frac{r_e}{R_L} \left(1 - \frac{R_E}{R_E + r_e} \right) << C_{bc}$. In this case and for $K \approx 1$, and $M \approx 1$, the gain becomes

$$G_o \cong - \frac{R_L}{r_e + R_E + r_g / \beta},$$

$$r_g = \left(r_{bb} + \frac{R_S R_{BIAS}}{R_S + R_{BIAS}} \right), \qquad (26)$$

$$\delta \cong \omega C_{bc} G_o \left(r_e + R_E + r_g \right).$$

When the emitter current is noise modulated as

$$I_E = I_{dc} + \Delta I_E \cos(\Omega t + \theta), \qquad (27)$$

the intrinsic emitter resistance r_e is also modulated. For $\Omega << \nu_o$ and $\gamma = \frac{\Delta I_E}{I_E} << 1$, the average value of r_e is given by

$$r_e \cong \frac{K}{I_{dc}[1 + \gamma \cos(\Omega t + \theta)]} \cong r_e[1 - \gamma \cos(\Omega t + \theta)], \qquad (28)$$

with K = 26 ohms/mA emitter current for a Si BJT. We can now express the gain as

$$G_o \cong \frac{-R_L}{r_e + R_E + r_g / \beta} \left[1 + \frac{\gamma r_e \cos(\Omega t + \theta))}{r_e + R_E + r_g / \beta} \right]. \qquad (29)$$

Averaging over the phase of the modulation yields

$$\overline{G_o^2} \cong \left(\frac{R_L}{r_e + R_E + r_g / \beta} \right)^2 \left[1 + \frac{\gamma^2}{2} \left(\frac{r_e}{r_e + R_E + r_g / \beta} \right)^2 \right]. \qquad (30)$$

The single sideband AM noise spectrum added by the amplifier due to current modulation follows the spectrum of the current modulation and is given by

$$1/2\ S_a(f) \cong 1/4 \left(\frac{r_e}{r_e + R_E + r_g/\beta} \right)^2 \gamma^2(f). \tag{31}$$

Considering the possible variation of the other parameters affecting the gain and phase of this simplified CE amplifier, we obtain a more complete expression for the AM noise given by

$$\begin{aligned} \frac{1}{2} S_a(f) \cong\ & \frac{1}{4}\left(\frac{r_e}{r_e + R_E + r_g/\beta}\right)^2 \gamma^2 + \frac{1}{4}\left(\frac{r_g/\beta}{r_e + R_E + r_g/\beta}\right)^2 \left(\frac{\Delta r_g}{r_g}\right)^2 + \frac{\delta^2}{4}\left(\frac{r_e}{r_e + R_E + r_g/\beta}\right)^2 \left(\omega C_{bc} G_o (r_e + R_E + r_g)\right)^2 \gamma^2 + \frac{kTFG}{2P_o} \\ & + \frac{1}{4}\left(\frac{R_E}{r_e + R_E + r_g/\beta}\right)^2 \left(\frac{\Delta R_E}{R_E}\right)^2 + \frac{1}{4}\left(\frac{\Delta R_L}{R_L}\right)^2 \\ & + \frac{\delta^2}{4}\left[\left(\omega G_o (r_e + R_E + r_g)\right)^2 \Delta C_{bc}^2 + \left(\omega C_{bc} G_o\right)^2 (\Delta r_e^2 + \Delta R_E^2 + \Delta r_g^2)\right]. \end{aligned} \tag{32}$$

Equation 32 can be considerably simplified if we make use of the following approximations,

$$r_g > (r_e + R_E), \quad \Delta r_e = \gamma\, r_e. \tag{33}$$

$$\begin{aligned} \frac{1}{2} S_a(f) \cong\ & + \frac{G_o^2}{4}\left[\left(\frac{\gamma r_e}{R_L}\right)^2 + \left(\frac{\Delta r_g}{\beta R_L}\right)^2 + \left(\frac{\Delta R_E}{R_L}\right)^2\right] + \frac{(\omega G_o)^4 (C_{bc} r_g)^2}{4}\left((r_g \Delta C_{bc})^2 + C_{bc}^2\left(\Delta R_E^2 + \Delta r_g^2\right)\right). \\ & + \frac{1}{4}\left(\frac{\Delta R_L}{R_L}\right)^2 + \frac{kTFG}{2P_o} \end{aligned} \tag{34}$$

The first term in the above equation scales as $\left(\frac{r_e}{r_e + R_E + r_g/\beta}\right)^2$, while the contribution of the terms associated with ΔC_{bc} scale as $\left(\frac{r_g}{r_e + R_E + r_g/\beta}\right)^4$ Both sets of terms are reduced by increasing R_E, although the AM noise originating from capacitance modulation is reduced much faster than that originating from current modulation. The first term is also decreased by increasing the dc emitter current I_E. The limit to which I_E can be increased depends on the maximum ratings of the transistor and its environment.

The PM noise added by the CE amplifier in the same approximation used above for the AM noise yields

$$1/2\ S_\phi(f) = L(f) = \frac{1}{4}\left(\omega G_o (r_e + R_E + r_g)\right)^2 \Delta C_{bc}^2 + \frac{1}{4}\left(\omega C_{bc} G_o\right)^2\left(\Delta r_e^2 + \Delta R_E^2 + \Delta r_g^2\right) + \frac{KTFG}{2P_o}$$

$$\frac{1}{4}(\omega C_{bc}G_o)^2\left(\Delta r_e^2 + \Delta R_E^2 + \Delta r_g^2\right) + \frac{KTFG}{2P_o}$$

$$+\frac{1}{4}\left(\frac{r_e}{r_e + R_E + r_g / \beta}\right)^2 \left(\omega C_{bc}G_o(r_e + R_E + r_g)\right)^2 \gamma^2 \qquad (35)$$

For $\Delta r_e = \gamma r_e$, $r_g > (r_e + R_E)$, Eq. 35 can be reduced to

$$L(f) = \frac{1}{4}(\omega G_o)^2\left[(r_g \Delta C_{bc})^2 + (C_{bc})^2\left(\frac{\gamma r_g r_e G_o}{R_L}\right)^2\right] \qquad (36)$$

$$+\frac{1}{4}(\omega G_o)^2 (C_{bc})^2\left(\Delta R_E^2 + \Delta r_g^2\right) + \frac{kTFG}{2P_o}.$$

The PM noise due to capacitance modulation is reduced as $\left(\frac{R_L r_g}{r_e + R_E + r_g / \beta}\right)^2$, while the contribution of the terms associated with current modulation scale as $\left(r_e r_g R_L\right)^2\left(\frac{1}{r_e + R_E + r_g / \beta}\right)^4$. Both sets of terms are reduced by increasing R_E and reducing r_g, although the PM noise originating from current modulation is reduced much faster with R_E than those originating from capacitance modulation. Increasing the dc emitter current I_E primarily affects the term originating from current modulation.

Current modulation originates from the intrinsic baseband modulation generated within the transistor and by noise in the power supply. The modulation of C_{bc} is induced by baseband modulation of the base collector voltage. This baseband voltage modulation can originate from the current modulation and finite dc gain or from the power supply.

8. Measurement of AM and PM Noise Induced by Current Modulation in a CE Amplifier

Current noise was injected in the emitter of the transistor to test the functional form of Eq. 31. Current noise of order 22 X 10^{-12} A_{rms}^2/Hz was used, which was large enough that the induced AM and PM noise was usually above the noise floor of our measurement system. Table 1 shows the measured AM noise sensitivity to γ ($\Delta I_E/I_E$) and the predictions calculated using Eq. 31 for a 2N2222A and a microwave transistor (with much lower C_{bc}) for R_E = 0 Ω and 10 Ω. The characteristics of the transistors are given in Table 2. Column A in Table 1 shows the measured AM noise sensitivity to γ as a function of Fourier frequency offset from a 5 MHz carrier for the two transistors with R_E= 0, while column B shows the calculated value for $r_g/\beta \cong 1$ Ω and $r_e \cong 1.5$ Ω. Column C shows the measured AM noise sensitivity with R_E = 10 Ω and Column D shows the calculated values. The calculated and measured improvements in the AM noise sensitivity to γ due to increasing R_E from 0 to 10 Ω agree very well for both transistors. Column E shows the measured PM noise sensitivity to γ for R_E = 0 Ω while column G shows the PM noise sensitivity to γ for R_E = 10 Ω. The calculated values using the last term in Eq. 35 are shown in columns F and H. For the 2N2222A, the calculated values are in agreement with the measured values. Eq. 35 predicts that the PM noise (due to current noise) at $R_E = 10\ \Omega$ is reduced relative to the PM noise at $R_E = 0\ \Omega$ by the square of the reductions observed for the AM noise. The measured reduction of PM noise follows this relatively closely for the 2N2222A. The measured PM noise sensitivity to γ for the microwave transistor is shown in column E. This PM sensitivity should be reduced relative to that of the 2N2222A by the ratio of $(C_{bc1}/C_{bc2})^2$. For these two transistors that ratio is approximately 22 dB. The measured reduction is about 5 dB higher. Measurements at $R_E = 10\ \Omega$ for the microwave transistor were limited by the noise floor of the measurement system.

	A	B	C	D
	2N2222A			
Fourier frequency	AM sensitivity $R_E = 0\ \Omega$ [dBc/Hz rel to γ=1]	AM theory $R_E = 0\ \Omega$ [dBc/Hz rel to γ=1]	AM sensitivity $R_E = 10\ \Omega$ [dBc/Hz rel to γ=1]	AM Theory $R_E = 10\ \Omega$ [dBc/Hz rel to γ=1]
100 Hz	-8.5	-10.5	-23.4	-25.1
50 Hz	-8.6	-10.5	-23.3	-25.1
20 Hz	-8.6	-10.5	-22.9	-25.1
10 Hz	-8.6	-10.5	-22.7	-25.1
5 Hz	-8.9	-10.5	-23.3	-25.1
	microwave transistor			
Fourier frequency	AM sensitivity $R_E = 0\ \Omega$	AM theory $R_E = 0\ \Omega$	AM sensitivity $R_E = 10\ \Omega$	AM theory $R_E = 10\ \Omega$
100 Hz	-8.5	-10.5	-23.1	-25.1
50 Hz	-8.5	-10.5	-23.4	-25.1
20 Hz	-8.5	-10.5	-23.3	-25.1
10 Hz	-8.5	-10.5	-22.9	-25.1
5 Hz	-8.6	-10.5	-22.5	-25.1

Table 1a. AM noise sensitivities to current noise for a CE amplifier when $\gamma = \Delta I_E/I_E = 1.9 \times 10^{-5}$, $I_E = 25$ mA, and $V_{CB} \approx 9$ V.

Table 1 along with Eqs. 34 and 35 can be used to predict the AM and PM levels due to current fluctuations in a CE amplifier. For a given current noise (ΔI_E), dc emitter current (I_E) and unbypassed emitter resistor (R_E), one can estimate the resulting level of the AM noise. One can also calculate the maximum allowed value of γ ($\Delta I_E/I_E$) for a desired AM noise level. In a similar way, one can predict the resulting PM noise due to current fluctuations, but the values of C_{cb} and r_g are required.

The collector-base capacitance varies with the dc voltage between the collector and base terminals

	E	F	G	H
	2N2222A			
Fourier frequency	PM noise $R_E = 0\ \Omega$ [dBc/Hz rel to γ=1]	PM theory $R_E = 0\ \Omega$ [dBc/Hz rel to γ=1]	PM noise $R_E = 10\ \Omega$ [dBc/Hz rel to γ=1]	PM theory $R_E = 10\ \Omega$ [dBc/Hz rel to γ=1]
100 Hz	-16.3	-15.6	-45.6	-43.7
50 Hz	-16.2	-15.6	-45.2	-43.7
20 Hz	-15.6	-15.6	-45.8	-43.7
10 Hz	-16.3	-15.6	-45.4	-43.7
5 Hz	-16.3	-15.6	-45.6	-43.7
	microwave transistor			
Fourier frequency	PM noise $R_E = 0\ \Omega$ [dBc/Hz rel to γ=1]	PM theory $R_E = 0\ \Omega$ [dBc/Hz rel to γ=1]	PM noise $R_E = 10\ \Omega$ [dBc/Hz rel to γ=1]	PM theory $R_E = 10\ \Omega$ [dBc/Hz rel to γ=1]
100 Hz	-43.2	-37.4	limited by	-65.5
50 Hz	-42.6	-37.4	system	-65.5
20 Hz	-42.1	-37.4	floor	-65.5
10 Hz	-42.4	-37.4	<-48.6	-65.5
5 Hz	-41.8	-37.4		-65.5

Table 1b. PM noise sensitivities to current noise for a CE amplifier when $\gamma = \Delta I_E/I_E = 1.9 \times 10^{-5}$, $I_E = 25$ mA, and $V_{CB} \approx 9$ V

	2N2222A	Microwave transistor
f_T	300 MHz	8 GHz
C_{be}	25 pF	1.6 pF
C_{bc} (V_{CB} = 9V)	8 pF	.65 pF
β	$75 < \beta < 375$	$50 < \beta < 300$
F	≤ 4 dB	1.5 dB

Table 2. Transistor parameters for Table 1

9. Measurements of AM and PM Noise Induced by Collector-Base Voltage Modulation in a CE Amplifier

(V_{CB}) according to Eq. 37. (In this equation, K_O is a constant that includes the permittivity of the material and the doping concentration, V_{bi} is the built-in potential of the p-n junction, and n is a parameter that depends on the doping profile of the junction.) Collector-base voltage modulation will therefore modulate C_{bc}.

$$C_{bc} = \frac{K_0}{(V_{bi} + V_{CB})^n}. \tag{37}$$

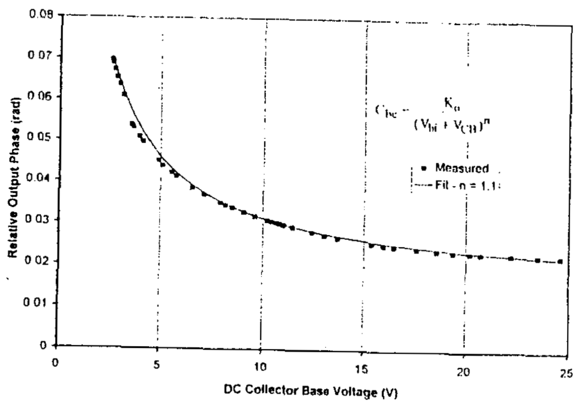


Fig. 3. Relative phase shift of a 20 MHz CE amplifier as a function of V_{CB}.

Fig. 3 shows the relative phase shift across a 20 MHz CE amplifier (shown in Fig. 2) as a function of collector-base voltage (V_{CB}) when the active element was a 2N2222A transistor. This graph shows the dependence of C_{bc} on V_{CB}, and can be used to obtain the exponent n in the phase equation of the amplifier (Eq. 38). The optimum fit, shown as a dotted line in Fig. 3, occurred when $n \cong 1$. As shown in Fig. 3, the measured phase shift follows Eq. 38 fairly well for V_{BC} from 3 to 24 V.

$$\text{phase} = \vartheta = K_1\omega + K_2\omega C_{bc} = K_1\omega + \frac{K_3\omega}{(V_{bi} + V_{CB})^n}. \tag{38}$$

The PM sensitivity to voltage fluctuations can be obtained by squaring the partial derivative of the phase with respect to V_{CB}:

$$\left(\frac{\partial\vartheta}{\partial V_{CB}}\right)^2 = \left(\frac{nK_3\omega}{(V_{bi} + V_{CB})^{n+1}}\right)^2 \tag{39}$$

To measure the effect of ΔV_{CB} on the AM and PM noise, voltage noise was added to the collector terminal of the CE amplifier. Filtering on the base reduced any direct current modulation to an insignificant level. Fig. 4 shows the PM noise sensitivity to V_{CB} fluctuations measured with a 2N2222A transistor in the CE circuit (for $R_E = 30\ \Omega$) as a function of V_{CB} at 5, 10, and 20 MHz carrier frequencies. The dotted lines are predictions on how the PM should vary with V_{CB} based on Eq. 39 for n = 1. The form of the PM noise follows closely the predictions from 6 to 17 V and scales as ω^2 as expected.

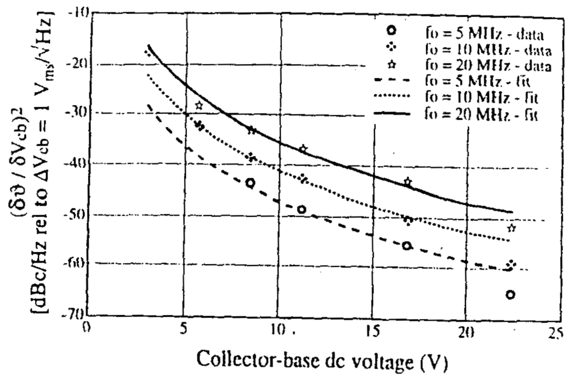


Fig. 4. PM noise sensitivity to ΔV_{CB} in a CE amplifier.

The change of PM sensitivity to V_{CB} fluctuations was also measured for different R_Es (for V_{CB} = 8V). Table 3 shows the PM sensitivity measurements and predicted values (from Eq. 35) as a function of Fourier frequency offset from a 5 MHz carrier for the CE amplifier in Fig. 3 when voltage noise was injected at the collector terminal. Columns A and D show the measured PM sensitivity to V_{CB} fluctuations for $R_E = 0\ \Omega$ and $R_E = 10\ \Omega$ respectively. Columns B and E show the PM sensitivity values predicted by the first term in Eq. 35. For the calculations it was assumed that half of the output capacitance in the transistor was due to parasitics and the other half due to the junction capacitance (Eq. 37).

The measured and predicted values for the PM sensitivity when a 2N2222A was used differ by approximately 4 dB. This is probably due to the approximations made in Eq. 35 and the uncertainty in some of the values used (r_g, C_{bc}). From theory, the reduction in PM when R_E is added should be proportional to the reduction in gain. Both the measured and calculated sensitivities show approximately a 12 dB reduction for R_E changed from 0 Ω to10 Ω. The measured gain reduction was 13.5 dB. When a microwave transistor was used, the PM sensitivity to V_{CB} fluctuations for $R_E = 0\ \Omega$ was reduced by 33 dB, compared to the predicted reduction of 28 dB. The predicted value was obtained from Eqs. 35 and 39 when using n = 0.55 and C_{bc} = 0.3 pF. These values where obtained from the C_{bc} vs. V_{CB} plot in the specification sheet of the transistor. For R_E = 10 Ω the noise was limited by the noise floor of the measurement system. The AM sensitivity to V_{CB} fluctuations is shown in columns C ($R_E = 0\ \Omega$) and F ($R_E = 10\ \Omega$) of Table 3. The AM sensitivity when using a 2N2222A was reduced by approximately twice the amount of reduction in PM sensitivity when an unbypassed emitter resistor was added ($R_E = 10\ \Omega$), as predicted by theory. The PM noise is thus a first order effect in the up-conversion of V_{CB} fluctuations, while the AM noise is a second order effect.

10. Amplifiers with Improved 1/f Noise

Figure 5 shows an amplifier design that makes use of the information of Tables 1 and 3 [3,10]. Figure 6 shows a comparison between the 1/f PM noise of an amplifier similar to Fig. 2 and that of Figure 5. The reduction in the 1/f noise is approximately 20 dB. See [3,10] for a more detailed discussion on the amplifiers and on guidelines for designing low-noise amplifiers.

Figure 7 shows the exceptionally low 1/f PM noise in a 100 MHz Si BJt based amplifier following the same design rules as those of Figure 5[design rules]

	A	B	C	D	E	F
	2N2222A					
Fourier frequency	PM sensitivity $R_E = 0\ \Omega$ [dBc/Hz rel to $\Delta V_{CB} = 1V_{rms}/\sqrt{Hz}$]	PM theory $R_E = 0\ \Omega$ [dBc/Hz rel to $\Delta V_{CB} = 1V_{rms}/\sqrt{Hz}$]	AM sensitivity $R_E = 0\ \Omega$ [dBc/Hz rel to $\Delta V_{CB} = 1V_{rms}/\sqrt{Hz}$]	PM sensitivity $R_E = 10\ \Omega$ [dBc/Hz rel to $\Delta V_{CB} = 1V_{rms}/\sqrt{Hz}$]	PM theory $R_E= 10\ \Omega$ [dBc/Hz rel to $\Delta V_{CB} = 1V_{rms}/\sqrt{Hz}$]	AM sensitivity $R_E = 10\ \Omega$ [dBc/Hz rel to $\Delta V_{CB} = 1V_{rms}/\sqrt{Hz}$]
100 Hz	-40.7	-36.2	-42.4	-52.3	-48.6	-63
50 Hz	-40.7	-36.2	-42.6	-52.4	-48.6	-62.8
20 Hz	-40.6	-36.2	-42.8	-52.4	-48.6	limited by
10 Hz	-40.6	-36.2	-43.1	-52.4	-48.6	system
5 Hz	-40.6	-36.2	-43.1	-52.4	-48.6	noise floor
	microwave transistor					
100 Hz	-73.6	-63.9	limited by	limited by	-76.3	limited by
50 Hz	-74.3	-63.9	system	system	-76.3	system
20 Hz	-73.2	-63.9	noise floor	noise floor	-76.3	noise floor
10 Hz	limited by	-63.9	<-63	<-77	-76.3	<-69
5 Hz	sys. floor	-63.9			-76.3	

Table 3. AM and PM noise sensitivities to collector-base voltage fluctuations for a CE amplifier with V_{CB} = 8 V and I_E = 25 mA. For these measurements the output power level was adjusted to be approximately 6 dBm and $\Delta V_{CB} \cong 2.8 \times 10^{-4}\ V_{rms}/\sqrt{Hz}$.

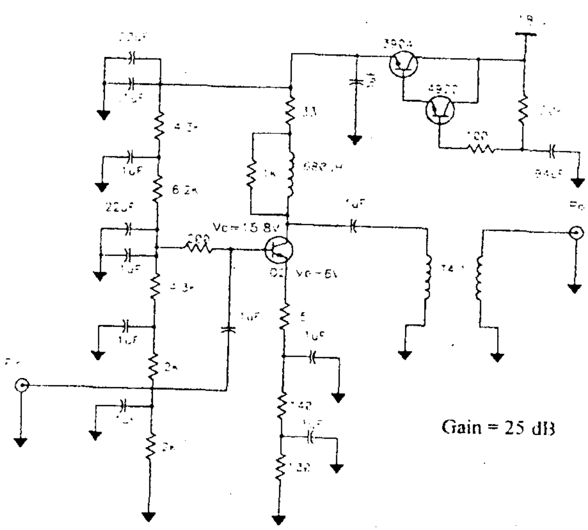


Figure 5. CE amplifier #2 with design changes to reduce 1/f noise

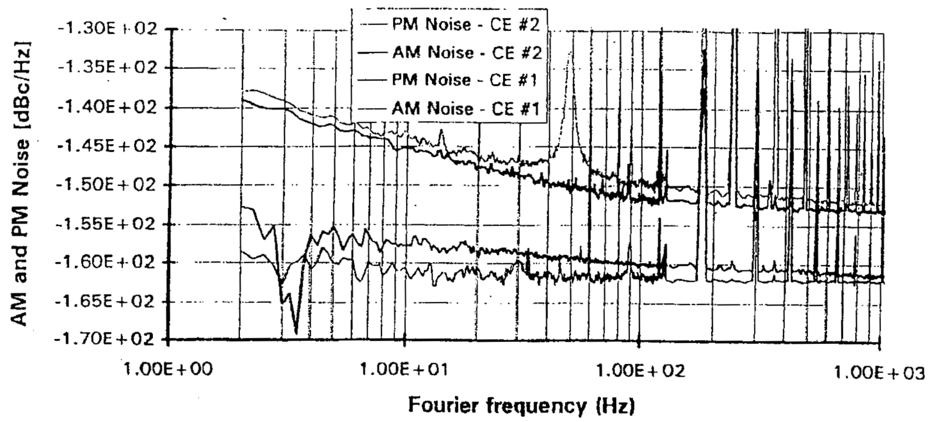


Figure 6. Comparison of the AM and PM noise of the CE amplifier #1 from Fig. 2 and CE amplifier #2 from Fig. 5 [3].

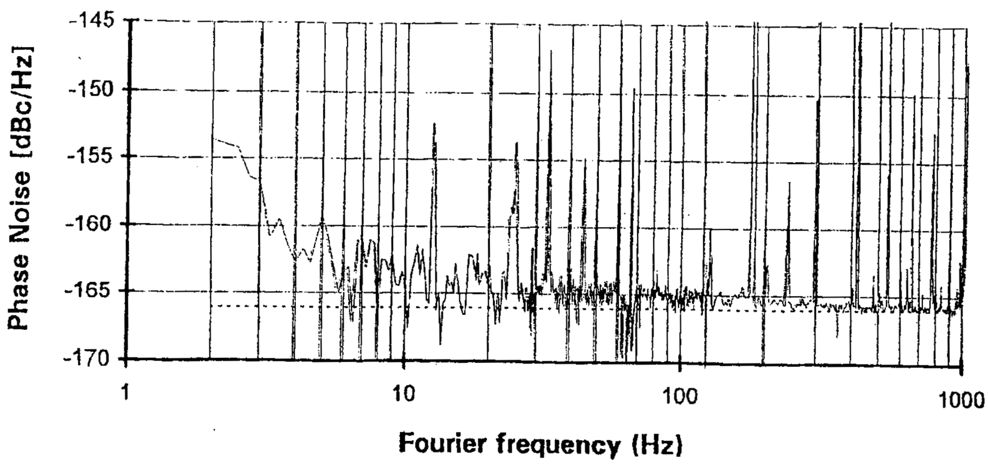
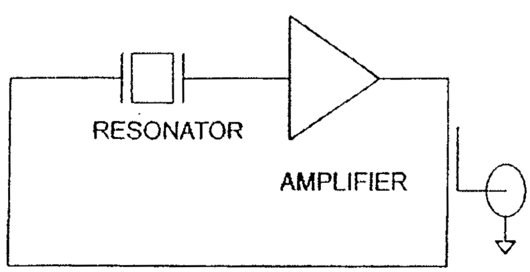


Figure7. PM noise of a low 1/f 100 MHz amplifier. The gain is 18.5 dB [3].

11. PM noise in Oscillators

The PM noise in a loop oscillator, schematically shown in Fig. 8, can be approximated using Leeson's model [11]. The basic point is that the gain around the loop must be 1 and the phase must be $2n\pi$, where n is an integer.



Gain = 1.0000... $\phi = n(2\pi)$ n = 0, 1, 2, 3 ...

Figure 8. Schematic of loop oscillator