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USE OF NEW DATA TO DETERMINE THE RELATIVISTIC RED SHIFT TO $3x10^{-17}$ AT NIST, BOULDER [1]

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ABSTRACT

We have estimated the relativistic red shift correction due to gravity at the National Institute of Standards and Technology (NIST) in Boulder, Colorado, USA, using a new survey and the new U.S. detailed geoid model, G99SSS. We have computed the frequency offset that a standard at NIST would suffer in several ways. We referenced the frequency offsets computed from the different methods to the same geoid surface, one defined with respect to the current best estimate of an ideal mean-Earth ellipsoid. The new results are: (1) -1797.61x10^{-16}, based on the global gravitational model EGM96, (2a) -1798.72x10^{-16}, based on the regional, high-resolution geoid model G96SSS, (2b) -1798.49x10^{-16}, based on the regional, high-resolution geoid model G99SSS, and (3) -1798.91x10^{-16}, based on the value provided in the National Geodetic Survey's data sheet for the NIST reference marker. The minus sign implies that clocks run faster in the laboratory in Boulder than a standard clock located on the geoid. The values from (2b) and (3) are expected to be the most accurate and are also *independent*. Based on these results, we estimate the frequency shift at the reference point at NIST to be -1798.7x10^{-16}, with an estimated standard uncertainty of ± 0.3x10^{-16}.

Keywords: geoid, geodetic leveling, geopotential, mean-Earth ellipsoid, relativistic red shift, vertical datum

1. THEORETICAL BACKGROUND

This is a continuation of previous work [1]. Whereas previously we used coordinates accurate to about 1 m, using a recent GPS survey we now have coordinates that should be accurate to 20 cm or better. In addition, there is now available a new regional model of the geoid for the continental U.S., which we also used in our computations.

With the advent of new primary frequency standards with uncertainties approaching 1 part in 10^{15}, there is a need for improved estimates of the relativistic red shift. This is an effect predicted by relativity theory as the sum of a special and a general relativistic effect. In general relativity, a clock at a higher gravitational potential runs faster relative to a clock at a lower potential. In relativity, *"higher"* potential means less negative, since the convention used is such that potential has (in general) negative value, approaching zero as a particle moves towards infinity away from an attracting body. Thus the effect of the geopotential on a clock would cause it to run faster as it moves away from the Earth, or in our case, higher above the geoid. Note that geodesy uses the *opposite* sign convention for geopotentials than that used in relativity theory. In geodesy, all potentials are positive, so that a higher potential would generally be closer to the Earth. In this paper we will use the geodetic convention, in which all geopotentials are positive.

A second relativistic effect enters, the so-called second-order Doppler shift of special relativity, in which a standard clock runs slower as it moves faster, relative to a clock at rest. The rotation of the Earth, therefore, gives rise to a centripetal potential that also changes the clock's frequency. We differentiate between the potential due to *gravitation* and that due to *gravity*: the former arises from the presence of attracting masses *only*, the latter contains in addition the centripetal potential due to the Earth's rotation [2, section 2-1]. It is the gravity potential that we need to consider here, therefore the term "gravitational red shift" is somewhat misleading and has been avoided herein.

A primary frequency standard that contributes to International Atomic Time (TAI) must be corrected to run at the rate clocks would run on the Earth's geoid. It is therefore necessary to determine the difference in gravity potential $(W_0 - W_P)$, between the geoid (0) and the location of a primary frequency standard (P), in order to correct for this frequency offset, according to [3]

$$(f_0 - f_P)/f = \Delta f/f = (W_P - W_0)/c^2, \qquad (1)$$

where $f = (f_0 + f_P)/2$, and c denotes the speed of light. Note that if the point P is above the geoid, we generally have $W_P < W_0$, using the convention in which potentials are positive. Hence, Δf is negative in this case, since this clock correction would make the clock in Boulder run slower to match the rate of a standard clock on the geoid.

The *geopotential number* $C = W_0 - W_P$ [2, page 56] is given by:

$$C = W_0 - W_P = \int_{H=0}^{H=H_P} g dH \ , \qquad (2)$$

where g is the magnitude of the gravity acceleration vector, and dH is the length increment along the positive upward plumb line. The path-independent line integral in equation (2) starts from a reference equipotential surface whose gravity potential is W_0 (on which every point has *orthometric* height equal to zero) and ends at the station location where $W = W_P$ and $H = H_P$.

The Earth's geoid is a unique equipotential surface that closely approximates in some prescribed fashion the Mean Sea

Surface (MSS). The geoid has to be defined and realized through the operational development of models [4,5]. The presence of a quasi-stationary (i.e., non-vanishing through averaging over long time periods) component within the Dynamic Ocean Topography (DOT) results in departures of MSS from an equipotential surface ranging, geographically, between -2.1 m and +1.3 m approximately. Due to these departures (and in some cases due to additional considerations related to mapping applications), different vertical datums refer to different equipotential surfaces. Therefore, given a datum-dependent C value, the determination of $\Delta f/f$ with respect to a unique equipotential surface requires the estimation of that datum's offset from that unique equipotential surface.

There exist global geoid models, developed through the combination of satellite tracking data, surface gravimetry, and satellite altimetry. EGM96 is a state-of-the-art such model, complete to degree and order 360, corresponding to a half-wavelength resolution of ~55 km at the equator [6]. The resolution of such global models is limited primarily by the available surface gravimetric data used in their development. Detailed (i.e., higher-resolution) local or regional geoid models are developed by incorporating the information contained within dense gravity and topography data into a global geoid model. This adds high (spatial) frequency details to the broader geoid features represented within a global model. G99SSS [7] is such a regional geoid model for the United States. Global and regional geoid models can also be used to estimate the geopotential number C, given the geocentric coordinates of the point P.

We distinguish therefore two general approaches for the computation of C (and hence $\Delta f/f$): one based on spirit leveling and gravity observations, and another based on the use of geoid models (either global or regional/local). It is important to recognize that each computational method or model used may yield a result that refers to a different equipotential surface. Since the various reference surfaces may be offset by several decimeters, estimation of their relative offsets becomes important if one desires to compare the various results at the level of a decimeter or less.

It is useful to recall the correspondence between the approximate magnitude changes of H, C, and $\Delta f/f$. Near the Earth's surface $g \approx 9.8 ms^{-2}$ and since $c = 2997924.58\ ms^{-2}$, a change in H by one meter implies roughly a 9.8 m^2s^{-2} change in C, and therefore a change in $\Delta f/f$ of -1.1×10^{-16}. Our present requirement is that $\Delta f/f$ be computed with an error not exceeding $\pm1\times10^{-16}$. Therefore, the total error in an absolute determination of the geopotential number C, consisting of the error in W_0 (absolute) and the error in $W_0 - W_P$ (relative), should not exceed ~ 9.8 m^2s^{-2} (equivalently, the absolute orthometric height H_P of our station should be determined to better than 1.0 m).

2. COMPUTATIONAL ASPECTS

In the following paragraphs we discuss the specific computations involved in the estimation of $\Delta f/f$, according to three methods. The first two methods are based on geoid model information (global and regional respectively), while the third method is based on spirit leveling and gravity observations. The first two methods share some (long-wavelength) errors, but the third method is *independent* of the other two.

2.1 Mean-Earth Ellipsoid

The concept of a *mean-Earth ellipsoid* [2, section 2-21] is of central importance in gravimetric geodesy (and in our specific application). This purely mathematical construct is a rotating ellipsoid of revolution (i.e., bi-axial), whose surface is also an equipotential surface of its gravity field. The gravity potential on its surface is pre-supposed to equal the gravity potential on the geoid. Four parameters are necessary and sufficient to define uniquely its size, shape, rotation, and gravity field. One may assume that these parameters are numerically equal to the corresponding parameters of the real Earth. Then, the departures of the geoid from such an "ideal" ellipsoid, called *geoid undulations* and denoted N, have vanishing zero-degree term (i.e., their average over the whole Earth equals zero). Therefore, by suppressing the zero-degree term in the spherical harmonic expansion of N, we "automatically" obtain geoid undulations that refer to this "ideal" mean-Earth ellipsoid, *without* the need to know the specific scale (semi-major axis) of this ellipsoid. Specification of the scale and the gravity field of this "ideal" ellipsoid require numerical specification of its defining parameters. These values can be determined only from analyses of various geodetic observations and therefore contain random (and possibly systematic) errors. Here we will define this "ideal" mean-Earth ellipsoid, in a *tide-free* system [8], by adopting the current best estimates for the values of the following parameters [9]:

Equatorial radius: $a = 6378136.46\ m$
Flattening: = 1/298.25765
Geocentric gravitational constant: (3)
$GM = 3.986004418\text{x}10^{14}\ m^3s^{-2}$
Mean rotational speed:
$\omega = 7292115\text{x}10^{-11}\ rad\ s^{-1}$

We should emphasize here that the mean-Earth ellipsoid defined by the above four values is only as "ideal" as the current accuracy of these values allows. The adopted defining values of equation (3) imply a value of the gravity potential on the geoid equal to:

$$W_0 = 62636856.88\ m^2s^{-2}\ , \qquad (4)$$

with an estimated error of $\pm1.0\ m^2s^{-2}$ [9].

2.2 Reference Point

For this work we compare gravity potential based on models with potential based on spirit leveling and gravity measurements. To accomplish this we used two different markers at NIST. The U.S. National Geodetic Survey surveyed three points on the NIST campus in September, 2000 [10]. One of these, identified as DMA (it was first surveyed by the Defense Mapping Agency in 1977 using the Transit system), was used to obtain geocentric Cartesian coordinates for use in evaluating models. This point is located on the flat roof above the fourth floor at NIST, Boulder. There is also a point on the side of the second floor of the building designated Q407. This point is part of the North American Vertical Datum 1988 (NAVD88), network of spirit leveling and gravity measurements. Since most of the frequency standards at NIST are on the second or third floors, it is more convenient to evaluate the relativistic red shift at an elevation equal to that of point Q407 rather than that of the DMA point.

Q407 is approximately 18.6 m distant horizontally from a point directly below the DMA point. We measured the Q407 point as 9.903 m below the DMA point. For evaluating the global and regional models, we used the coordinates of a point 9.903 m below the DMA point. This rather fictitious point is the point *P* we refer to in what follows. The change in gravity potential from point *P* to Q407 should be small in that it is due to a horizontal shift of only about 18.6 m. The relativistic red shift at *P* should agree with the value at the Q407 point to better than 10^{-18}, or 1 cm in terms of orthometric height.

2.3 Method 1

For method 1 we evaluated the EGM96 global model for the *gravitational* potential, i.e. the potential due to the Earth's attracting mass. To this we added the centripetal potential. For details on our evaluation of EGM96 see [1]. The EGM96 model is realized in the form of coefficients, $\overline{C}_{nm}$, for an expansion in terms of fully-normalized associated Legendre functions of the first kind [2, sections 1-11, 1-14], of degree *n* and order *m*. EGM96 [6] currently provides the most accurate estimate of a set of $\overline{C}_{nm}$, complete to degree and order 360.

The geocentric Cartesian coordinates of our reference point (P) 9.903 m below the DMA point, at the level of the Q407 marker for NIST, Boulder in ITRF97 are:

$$X_P = -1288394.075 \quad Y_P = -4721673.869 \quad Z_P = 4078630.782 \tag{5}$$

These coordinates are expected to be accurate to 20 cm or better.

We converted these coordinates to spherical coordinates $(r_P, \theta_P, \lambda_P)$ and evaluated the EGM96 gravitational model plus the centripetal potential. We obtained:

$$W_P = 62620700.75\ m^2 s^{-2}, \tag{6}$$

which, implies due to equations (1) and (4):

$$\Delta f / f = -1797.61 \times 10^{-16} \cdot \tag{7}$$

There are two types of error associated with the use of EGM96: (a) error of commission due to the fact that the coefficients $\overline{C}_{nm}$ are imperfectly known, and, (b) error of omission due to the truncation at degree 360. The commission error of EGM96 has two components. The first one (long wavelength) can be computed rigorously from the error covariance matrix that accompanies the part of the model up to degree and order 70. The second component, corresponding to degrees 71 to 360, can be computed only in terms of a global root mean square (RMS) estimate that does not account for the specific geographic location of our station. This estimate can be computed from the standard deviations of the EGM96 coefficients above degree 70. The omission error of EGM96 can also be estimated based on some theoretical model describing the decay of the gravitational spectrum of the Earth *globally*. Based on the EGM96 geoid error assessment in [6, sections 7.3.3.1 and 10.3.2], we estimate the total (commission plus omission) geoid undulation error of EGM96 at our reference point (P) to be approximately $\pm 0.6m$. Details of this error estimate are available in [1]. Combining this with an estimated error in ellipsoidal height of 0.2 *m* yields an error of the $\Delta f/f$ value given in (7) of 0.7×10^{-16}.

2.4 Method 2

A significant reduction of the omission error encountered with EGM96 can be effected through the use of a detailed regional geoid model. We have used the coordinates obtained from the new survey both with the 2'x2' gravimetric geoid model G96SSS [11] (which we also used in [1] along with the old survey coordinates), and the updated G99SSS [7] model. We estimated the geoid undulations at *P*, N_{GxxSSS}, (xx=96 or 99) using a bicubic spline to interpolate the grid on which each GxxSSS values is given (2'x2' for 96 and 1'x1' for 99). We computed the offset required to reference the N_{GxxSS} to an ideal mean-Earth ellipsoid. For both xx=96 and 99, this gave us (see [1] for details)

$$N_{GxxSSS}(ideal) = N_{GxxSSS} + 0.40\,m \cdot \tag{8}$$

We used the geodetic coordinates of P,

$$\varphi_P = 39°59'42.86105'' \quad \lambda_P = 254°44'14.54111'' \quad h_P = 1634.421m \tag{9}$$

in particular the ellipsoidal height h_P, to determine the orthometric height, H_P from

$$H_P(ideal) = h_P - N_{GxxSSS}(ideal) \tag{10}$$

We then determined the geopotential number, *C*, using Helmert's equation as in [2, equation 4-26, and 12].

2.4.1 Method 2a

Using G96SSS, the model we used previously in [1], we now obtained with the new coordinates:

$$C = 16166.08\, m^2 s^{-2}, \tag{11a}$$

which yields, from equation (1):

$$\Delta f / f = -1798.72 \times 10^{-16} \cdot \tag{12a}$$

Unlike EGM96, the GxxSSS regional geoid models are not accompanied by propagated error estimates. Their accuracy has been assessed only through comparisons with *independent* geoid undulation estimates obtained from GPS positioning and leveling observations [11,7]. Based on this uncertainty assessment, we estimate the error in N_{G96SSS} to be approximately 0.20 *m*. Considering also a 0.20 *m* error in the ellipsoidal height h_P, this implies an error for the $\Delta f/f$ value given in (12a) of 0.31×10^{-16}.

2.4.2 Method 2b

Using the new geoid model G99SSS, we now obtained with the new coordinates:

$$C = 16164.01\ m^2 s^{-2}, \quad (11b)$$

which yields, from equation (1):

$$\Delta f/f = -1798.49 \times 10^{-16} \quad (12b)$$

From comparisons with *independent* geoid undulation estimates obtained from GPS positioning and leveling observations [11] we estimate the error in N_{G99SSS} to be approximately 0.18 *m*. Considering the 0.20 *m* error in the ellipsoidal height h_P, this implies an error for the $\Delta f/f$ value given in (12b) of 0.30×10^{-16}.

2.5 Method 3

The $\Delta f/f$ values given in equations (7) and (12a, 12b) were computed based on a global and two regional geoid models, respectively. We turn now to the $\Delta f/f$ computation from spirit leveling and gravity measurements, as shown in equation (2). We performed this computation as documented in [1] and found

$$C_{NAVD88} = 16170.76 m^2 s^{-2}, \quad (13)$$

where the subscript "*NAVD88*" emphasizes the fact that this value refers to the equipotential surface that passes through the origin point (Father Point/Rimouski located in Quebec) of the North American Vertical Datum 1988 [12].

We adopted the value $\bar{d} = -0.30\ m$ as our current "best" estimate of the offset between the NAVD88 reference equipotential surface and the geoid surface. The minus sign implies that the equipotential surface passing through the origin of NAVD88 is *below* the geoid surface that is realized through the EGM96 model, when the latter is referenced to our current best estimate of a mean-Earth ellipsoid. We should also mention that in the above analysis the permanent tide effect was consistently accounted for. The offset value is expressed in the tide-free system.

Applying the offset $\bar{d}$ to estimate the correction dC necessary to convert C_{NAVD88} to C_{ideal} we find:

$$dC = -2.94\ m^2 s^{-2} \Rightarrow C(ideal) = 16167.82\ m^2 s^{-2} \quad (14)$$

which implies:

$$\Delta f/f = -1798.91 \times 10^{-16}. \quad (15)$$

Errors in the estimate of $\Delta f/f$ given in (15) arise from errors in the NAVD88 dynamic height value provided in the NGS data sheet for our reference marker, and errors in our estimation of the NAVD88 datum offset. The NGS data sheet for our reference marker contained no error estimates, other than the designation that "First order, Class II" leveling was performed to determine our station's height. Zilkoski et al. [12] discuss a comparison of NAVD88 heights with corresponding *independent* estimates from Canadian leveling observations over the USA-Canada border. Over 14 points the maximum difference found was 0.11 m. This value does not necessarily apply to our station; nevertheless (and in lieu of more precise information) a reasonable estimate of our station's dynamic height error may be about $\pm 0.15 m$. Considering an error of 0.20 *m* in our estimate of the NAVD88 datum's offset, we conclude that the $\Delta f/f$ value given in (15) is probably accurate to 0.28×10^{-16}.

3. COMBINED RED SHIFT ESTIMATE

The results from the three methods are summarized below in Table 1.

Table 1

Method	Red Shift parts in 10^{-16}	Uncertainty parts in 10^{-16}
1. EGM96	-1797.61	0.70
2a. G96SSS	-1798.72	0.31
2b. G99SSS	-1798.49	0.30
3. Leveling/Gravity	-1798.91	0.28

Method 3 is *independent* from the others. We accept G99SSS as an update to G96SSS and use only the results from methods 2b. and 3. to determine our final result. These differ by 0.42×10^{-16}, while our estimated errors from Table 1 imply a 0.41×10^{-16} uncertainty (1 sigma) for this difference.

Averaging methods 2b. and 3. to evaluate $\Delta f/f$, we estimate its value and uncertainty for our reference marker to be

$$(-1798.70 \pm 0.3) \times 10^{-16}. \quad (16)$$

We note that the uncertainty here is larger than our previous estimate of 0.2×10^{-16}. We consider this uncertainty estimate to be perhaps more realistic.

We should mention that we have not accounted here for luni-solar tidal effects. At this level of accuracy the effects of (at least) the semi-diurnal lunar tide M_2 (and possibly of other constituents) must be considered. One should therefore interpret our $\Delta f/f$ result as an average value over multiples of the main tidal constituents' periods.

4. SUMMARY AND FUTURE PROSPECTS

Based on our work, it appears that the existing measurements and models of the Earth's gravity field may not support estimates of the relativistic red-shift correction to better than the 10^{-17} level for frequency standards on the Earth. Since this number contributes to the error budget of a primary frequency standard in an RMS sense, this implies that a primary frequency standard in an Earth-bound laboratory will have difficulty contributing to TAI at better than the 10^{-16} level. In the next decade it seems reasonable to expect frequency standards to reach accuracies challenging our current accuracy in the determination of the red-shift correction.

Currently two geopotential mapping missions are in preparation, which are expected to support a significant advance in the present application: NASA's Gravity Recovery And Climate Experiment (GRACE), and ESA's Gravity Field and Steady-State Ocean Circulation (GOCE) missions. GRACE is scheduled for launch in 2001, and promises to deliver centimeter-level geoid undulation accuracy with half-wavelength resolution of 200 to 300 km. GOCE (scheduled for launch in 2005) is expected to improve even further the resolution, allowing cm-level geoid undulation accuracy down to ~80 km resolution [13]. The global geopotential models expected from these missions, in combination with locally available detailed surface gravity and topography data, may permit point geoid undulation determination approaching centimeter-level accuracy. In addition, radar altimeter data from satellites such as TOPEX/Poseidon and its follow-on Jason-1, in combination with the global geopotential models from GRACE and GOCE, should permit improvements in the determination of the equatorial radius of the mean-Earth ellipsoid, which directly affects the accuracy of W_0. These advances may permit determination of $\Delta f / f$ accurate to a few parts in 10^{18}.

On the opposite side, development of frequency standards accurate to 10^{-17} or better may provide one possibility for the verification and error calibration of geopotential differences estimated from data acquired in part from the GRACE and GOCE missions. This could be attempted following ideas such as those proposed originally by Bjerhammar [3]. In addition, frequency standards of such high accuracy, located on different continents, provide an alternative technique well recognized among geodesists for connecting different vertical datums. While there is promise for standards of such accuracies, methods for transferring such time and frequency measurements appear to be lacking. The current best time transfer methods appear to be at the level of 200 ps stability, or about 2×10^{-15} frequency transfer over 1 day [14]. In conclusion, it appears that technological advances in the development of frequency standards and advances in determination of the gravity field over the upcoming years are expected to benefit both disciplines in complementary ways.

5. REFERENCES

[1] Pavlis, N.K., M.A. Weiss, The Relativistic Red Shift with $2\text{x}10^{-17}$ Uncertainty at NIST, Boulder, Colorado, U.S.A., *Proceeds International Frequency Control Symposium*, June 2000, pp. 642-650.

[2] Heiskanen, W.A. and H. Moritz, *Physical Geodesy*, W.H. Freeman and Co., San Francisco, 1967.

[3] Bjerhammar, A., On a relativistic geodesy, *Bull. Géodésique, 59*, 207-220, 1985.

[4] Heck, B. and R. Rummel, Strategies for Solving the Vertical Datum Problem Using Terrestrial and Satellite Geodetic Data, IAG Symposia, Vol. 104, *Sea Surface Topography and the Geoid*, H. Sünkel and T. Baker (Eds.), Springer-Verlag, New York, 1990.

[5] Jekeli, C., The world of gravity according to Rapp, IAG Symposia, Vol. 119, *Geodesy on the Move*, R. Forsberg, M. Feissel, and R. Dietrich (Eds.), Springer-Verlag, Berlin, 1998.

[6] Lemoine, F.G., S.C. Kenyon, J.K. Factor, R.G. Trimmer, N.K. Pavlis, D.S. Chinn, C.M. Cox, S.M. Klosko, S.B. Luthcke, M.H. Torrence, Y.M. Wang, R.G. Williamson, E.C. Pavlis, R.H. Rapp, and T.R. Olson, The Development of the Joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) Geopotential Model EGM96, *NASA/TP-1998-206861*, Goddard Space Flight Center, Greenbelt, MD, July, 1998.

[7] NGS page on the internet for G99SSS is:
http://www.ngs.noaa.gov/GEOID/G99SSS/g99sss.html

[8] Rapp, R.H., R.S. Nerem, C.K. Shum, S.M. Klosko, and R.G. Williamson, Consideration of permanent tidal deformation in the orbit determination and data analysis for the TOPEX/Poseidon mission, *NASA Tech. Memo. 100775*, Goddard Space Flight Center, Greenbelt, MD, January, 1991.

[9] Bursa, M., Report of Special Commission SC3, Fundamental Constants (SCFC), Travaux de l' Association Internationale de Géodésie, Tome 30, Rapports Généraux et Rapports Techniques, IAG, 140 rue de Grenelle, 75700 Paris, France, 1995.

[10] NGS Data Sheets available on the internet by Permanent Identifier (PID) at:
http://www.ngs.noaa.gov/datasheet.html
PID's for points DMA and Q407 are: AI7564 and KK1350, respectively.

[11] Smith, D.A. and D.G. Milbert, The GEOID96 high-resolution geoid height model for the United States, *J. Geod.*, *73*, 219-236, 1999.

[12] Zilkoski, D.B., J.H. Richards, and G.M. Young, Results of the General Adjustment of the North American Vertical Datum of 1988, *Surv. and Land Info. Systems, 52(3)*, 133-149, 1992.

[13] ESA SP-1233(1), The Four Candidate Earth Explorer Core Missions, Gravity Field and Steady-State Ocean Circulation Mission, European Space Agency, 1999.

[14] Nelson, L.M., J. Levine, K.M. Larson, P. Hetzel, Comparing Primary Frequency Standards at NIST and PTB, *Proceeds International Frequency Control Symposium*, June 2000, pp 622-636.