

## Practical implications of relativity for a global coordinate time scale

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Frontiers of technology now need synchronization between remote clocks to an accuracy of about a nanosecond. Rate changes arising from the velocity and gravitational potential of a transported clock used for synchronization of a network must be accounted for. In addition, one cannot assume that the earth is an inertial frame, i.e., not spinning. If classical Einstein synchronization is used, where from the midpoint between clocks at A and at B, one simultaneously sends light pulses to A and B to synchronize them, two problems arise. First, the synchronization process will not be transitive; i.e., if A is synchronized with B and B with C, then A will not necessarily be synchronized with C. Second, starting at a point on the equator and transporting a portable clock eastward (westward), while establishing a synchronized time grid on the way, will result in a discontinuity upon returning to the original point of about  $-200$  ns ( $+200$  ns); minus ( $-$ ) means that the portable clock will be late. This paper will discuss the construction of a coordinate clock network on the earth's surface which does not have these problems; i.e., synchronization is transitive, and there is no discontinuity. This may be done by adjusting clocks to read coordinate time on an underlying nonrotating local inertial frame. The theoretical and practical implications of setting up such a coordinate clock network using either electromagnetic signals (e.g., laser, Loran-C) or portable clocks will be discussed. It will be shown how this network may be applied in making UTC or any other global scale more useful for state-of-the-art navigation and communication systems.

### INTRODUCTION

The clock accuracy currently needed at different locations on the earth's surface or on board satellites poses some interesting problems because of relativistic effects. Unfortunately, not all that has been published on this topic has been correct, and a significant amount of confusion exists in the field. There is need for a treatment which addresses this topic with operational applications in view and with sufficient completeness to provide a useful working document. We shall in this paper consider relativistic corrections to time synchronization procedures of the order of or greater than 1 ns ( $10^{-9}$  s = 1 ns) or to a normalized frequency accuracy of  $10^{-14}$ , which should satisfy most user needs for several years to come.

A spatially distributed set of clocks, synchronized by portable clocks or electromagnetic signals, with rates adjusted to maintain synchronization, comprises a coordinate time system. In establishing such

a system the accuracy of today's clocks and of time comparison techniques requires that relativistic effects due to the differences in gravitational potential and velocity of the clocks, even if they are stationary on the surface of the earth, be taken into account. The time dispersion of a good clock ensemble over a few days, the time errors in a state-of-the-art clock transportation, and the time errors in communicating time by satellite techniques are all of the order of a few nanoseconds. In contrast the relativistic time corrections associated with the spinning earth may be larger than 200 ns, and other relativistic effects may be of this order or even much larger in the case of satellite synchronization. Table 1 summarizes the accuracy capabilities of some of the state-of-the-art techniques. From this table it can be seen that it may soon be necessary in some special situations to include relativistic effects to subnanosecond accuracy.

It has been shown [Ashby, 1975; Hafele and Keating, 1972a, b] that for nanosecond accuracies, one cannot assume that the earth provides an inertial

TABLE 1. Time transfer accuracy

Method	Accuracy, ns	Typical Distance, km	Reference
Portable clock	≈5	3,000	<i>Reiss</i> [1976]; <i>Rueger</i> [1978]; <i>Williams</i> [1976]
VLBI	0.1*	15,000	<i>Fanselow</i> [1977]; <i>Rogers et al.</i> [1978]; <i>Schilizzi and Campbell</i> [1978]
Laser shuttle	0.6*	600	<i>Reinhardt et al.</i> [1977]
GPS	100 (1*)	worldwide	<i>Anderle</i> [1978]; <i>Buisson et al.</i> [1977]; <i>MacDoran</i> [1978]; <i>Rutman</i> [1978]; <i>Schuchman and Spiker</i> [1977]
TV network	20	150	<i>Lavanceau and Shepard</i> [1977]
Clock flyover	10 (1*)	8,000	<i>Besson</i> [1970]; <i>Reisse</i> [1976]; <i>Williams</i> [1976]
Laser geodimeter	0.01*	50	J. Levine (personal communication, 1978)

\*Theoretical potential accuracy.

frame, i.e., that it is not spinning. Consider a rotating reference frame equipped with a network of standard clocks at rest in that frame. Synchronization of the network by means of the usual Einstein procedure is not self-consistent; for example, starting at a point on the earth's equator and carrying a portable clock eastward (westward) while establishing a synchronized set of clocks on the way will give rise to a discrepancy upon returning to the original point of about  $-200$  ns ( $+200$  ns); minus ( $-$ ) means the portable clock will be late. This effect is due to rotation.

In principle, to obtain a coordinate time system without such discrepancies, one may introduce a new 'coordinate time' grid in the following way. Imagine an underlying nonrotating frame, or local inertial frame unattached to the spin of the earth, but with its origin at the center of the earth (geocentric). In this nonrotating frame, introduce a fictitious set of standard clocks available anywhere, all of them being synchronized via the Einstein procedure, and let them run at agreed upon rates such that synchronization is maintained. Now, introduce the rotating earth with a set of standard clocks distributed around the earth. To each of these standard clocks a set of corrections may be applied to generate 'coordinate clock time' such that at each instant the coordinate clock is synchronous with the fictitious standard clock at rest in the local inertial frame, whose location coincides with the earth-based standard clock at that instant. This set of clocks will therefore all be keeping coordinate time. In other words, coordinate time is equivalent to time measured by standard clocks in the local inertial frame. This is illustrated in Figure 1, in which the coordinate time is indicated as resulting from relativistic corrections applied to the elapsed time measured using standard clocks on the spinning

earth. We will discuss the practical implementation of the coordinate time concept in what follows.

The process of implementing such a coordinate time system is complicated by the presence of the gravitational fields of the earth, sun, and moon and by the motion of the earth's center, as well as by a variety of motions that clocks of interest can have. The center of mass of the earth-moon system is falling in an elliptical orbit around the sun; the earth not only orbits about this center of mass but spins on its axis, and clocks in airplanes or satellites have additional motions about the earth. All these effects must be considered in calculating relativistic corrections to be applied to standard clocks near the earth's surface to arrive at a coordinate time system. Because of these effects, the local coordinate time discussed in this paper differs from the classical coordinate time of general relativity (as it occurs, for example, in the Schwarzschild metric), which provides an idealized theoretical model for ephemeris time.

If the solar system were isolated from the gravitational effects of other masses, we could consider readings of standard clocks which are distant from the solar system and at rest with respect to its barycenter as corresponding to the ideal ephemeris time mentioned above. *Thomas* [1975] and *Moyer* [1976] have studied the relationship between this time and the time of an atomic clock fixed on the earth. Thus expressions for effects such as annual terms arising from the earth's orbital motion are available. However, these are not in a form convenient for the discussion of time transfer between different clocks near the earth's surface, where seasonal effects common to all clocks are of less importance than effects arising from earth rotation or clock motion.

A standard clock falling along with the center

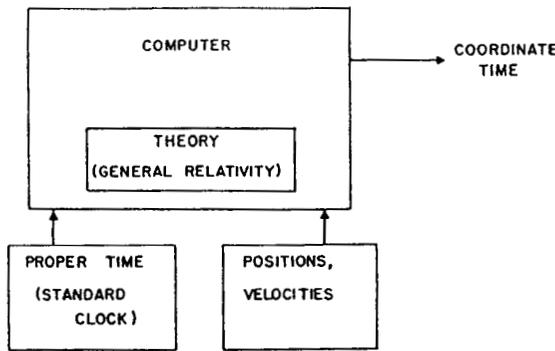


Fig. 1. Obtaining local coordinate time by correcting proper time readings on standard clocks for relativistic effects.

of mass of the earth would beat at a rate which disagrees with the rate of ideal ephemeris time because of Doppler shifts and gravitational red shifts, which both vary with time. The readings of such a clock may nevertheless be used as a reference for the introduction of coordinate time based on a local inertial frame, with standard clocks in the neighborhood of the earth synchronized with this reference clock. Then in the local frame the gravitational potential differences due to the sun's mass can, for purposes of time synchronization, be neglected over a region large enough to include the earth-moon system (see Appendix A).

The gravitational effect of the earth's mass and the rotation of the earth introduce additional small shifts in the rates at which standard clocks on the earth's surface beat relative to the above mentioned reference clock. Such effects are common to clocks at rest at mean sea level and can be suppressed by appropriate redefinition of the rate at which coordinate clocks beat.

Adoption of a local coordinate time system such as that proposed here would have significant advantages in the elimination of ambiguities in clock comparisons. The proposed system has the property of transitivity, path-dependent effects on time transfer by portable clocks and electromagnetic signals are properly accounted for, and the different types of time transfer processes are in agreement.

METHODS OF ESTABLISHING A COORDINATE TIME GRID

*Synchronization by portable clocks*

Suppose that having selected a reference point on the geoid (the geoid is the equipotential surface

at mean sea level), standard clocks are transported from point to point on the earth's surface. Let the infinitesimal increment of proper time measured on the standard clock be denoted by  $ds$  (in seconds) and the corresponding increment of coordinate time by  $dt$  (in seconds). Then the elapsed coordinate time during the process of transport may be calculated from the equation (derivations of results given in this section may be found in Appendix A):

$$\Delta t = \int_{\text{path}} ds \left[ 1 - \frac{g(\phi)h}{c^2} + \frac{1}{2}(v/c)^2 + \frac{\omega a_1 v_E \cos \phi}{c^2} \right] \quad (1)$$

where  $g(\phi)$  is the acceleration of gravity,  $v$  is the ground velocity of the clock having an eastward component  $v_E$ ,  $h$  is the altitude above the geoid,  $\omega$  is the angular velocity of rotation of the earth,  $a_1$  is the earth's equatorial radius, and  $\phi$  is the geographical latitude. The standard clock readings are thus corrected according to (1) for red shift, ground speed, and earth rotation in order to arrive at coordinate time. Let us consider each term individually for the moment, though in practice it is necessary to account for all three terms. In (1) the constant rate differences due to the earth's gravitational potential on the geoid have been suppressed.

*Clock at rest with respect to the earth (red shift).* Note that for a clock at rest on the surface of the earth,  $v = v_E = 0$ , and (1) reduces to

$$\Delta t = \Delta s \left( 1 - \frac{g(\phi)h}{c^2} \right) \quad (2)$$

so that a standard clock at rest on the earth's surface needs only to have its rate corrected for the red shift before being used to measure elapsed coordinate time at that point. The fractional red shift correction is  $1.09 \times 10^{-13}/\text{km}$ .

*Clock with significant ground speed (time dilation or second-order Doppler shift).* For a typical jet plane speed ( $270 \text{ m/s} = 604 \text{ mph}$ ) the time dilation effect represented by the term  $\frac{1}{2}(v/c)^2$  in (1) is a normalized correction of  $4 \times 10^{-13}$ . In an 8-hour flight the coordinate time would advance only 12 ns. In a satellite or rocket, however, this term could be an order of magnitude larger.

*Infinitesimally slow transport on the geoid (earth rotation).* In the limit as  $v \rightarrow 0$  and for  $h = 0$ , (1) reduces to

$$\Delta t = \Delta s + (\omega a_1 / c^2) \int ds v_E \cos \phi \quad (3)$$

*Example 1:* Transport along a meridian; then  $v_E = 0$ , and so we could use this result to synchronize coordinate clocks over the entire surface of the earth by selecting one location as a reference and transporting a standard clock slowly along a meridian to one of the poles and then back along another meridian to an arbitrary position on the earth's surface. The proper time elapsed on the transported clock would read coordinate time.

*Example 2:* Transport along a parallel of latitude; then  $v = v_E$ , and  $\int ds v_E \cos \phi = L \cos \phi$ , where  $L$  is the distance traversed eastwards. The coordinate time elapsed will be

$$\Delta t = \Delta s + \omega a_1 L \cos \phi / c^2 \quad (4)$$

If the portable clock is brought back to the starting point so that its path describes one complete circuit on the earth's surface, then an alternative form of (4) is

$$\Delta t = \Delta s + 2\omega A_E / c^2 \quad (5)$$

where  $A_E$  is the area enclosed by the path as projected on the equatorial plane, considered positive if it is traversed in the sense of rotation of the earth. Note that the correction term in (4) and (5) is independent of velocity, provided that the velocity is small. This result implies that the proper time elapsed on a standard clock carried eastward around the globe will be

$$2\pi\omega a_1^2 \cos^2 \phi / c^2 = 207.4 \cos^2 \phi \text{ ns} \quad (6)$$

less than that on a standard clock which remains at rest, while a standard clock carried slowly westward in the geoid would lead a standard clock which remained at rest by  $207.4 \cos^2 \phi$  ns.

*Synchronization by direct transmission of electromagnetic signals*

Consider synchronization by means of transmission of electromagnetic signals more or less parallel to the earth's surface, as in a television transmission. The coordinate time elapsed between transmission and reception may be expressed as follows:

$$\Delta t = \int d\sigma (1 + \omega a_1 c_E \cos \phi / c^2) / c \quad (7)$$

where  $\int d\sigma$  is the proper length, or standard distance, measured between the two clocks. The only significant effect is due to rotation of the earth. The eastward component of the light velocity is  $c_E$ .

*Example 1:* Light signals sent along a meridian; for this case the eastward velocity of the light,  $c_E$ , is zero, and the elapsed coordinate time is

$$\Delta t = \int_{\text{path}} d\sigma / c \quad (8)$$

*Example 2:* Signals sent along a parallel of latitude eastward; then

$$\Delta t = (1 + \omega a_1 \cos \phi / c) \int_{\text{path}} d\sigma / c \quad (9)$$

It should be noted that for 1-ns accuracy the proper distance along the path in the first term of the above equation must be known to an accuracy of 30 cm or better.

For the case of transmission over a distance of 160 km the second term in the above equation for a latitude of  $40^\circ$  is 0.7 ns, which in terms of fractional error is  $10^{-6}$ . The laser geodimeter, which uses two-way microwave transmissions for synchronization, has design goals of  $5 \times 10^{-8}$  (J. Levine, personal communication, 1978).

*Synchronization by two-way satellite transponder*

The situation is as diagrammed in Figure 2. Ground stations A and B have clocks which are to be synchronized by transmission from A to a transponder aboard an orbiting satellite, thence to station B. This is immediately followed by transmission back from B to the satellite and to A. As viewed from a local inertial geocentric reference frame, the ground stations are in motion due to rotation of the earth with angular velocity  $\omega$  about the axis

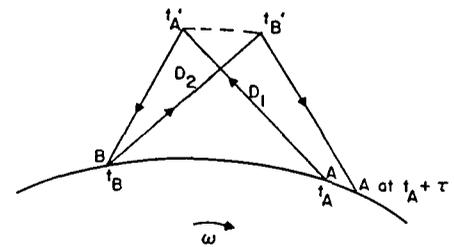


Fig. 2. Synchronization via two-way transponder. A signal leaves ground station A at time  $t_A$ , arrives at the satellite at time  $t'_A$ , and is retransmitted to ground station B, where it arrives at time  $t_B$ . A signal is then transmitted back to the satellite, where it arrives at time  $t'_B$  and is retransmitted to the ground station A, where it arrives at time  $t_A + \tau$ . The diagram is drawn with the polar axis pointing into the paper, so that longitudes are measured positive in the counterclockwise sense (westward).

represented by the unit vector  $\hat{k}$ . The normal to the plane of the satellite orbit is denoted by the unit vector  $\hat{n}$ . The initial position of ground station A is denoted by the vector  $\vec{r}_A$  at the instant  $t_A$  of transmission of the first signal. The position of ground station B at the instant of arrival (and retransmission) of the signal is denoted by  $\vec{r}_B$ . Finally, the position of the satellite at the instant the first signal arrives,  $t'_A$ , is denoted by  $\vec{r}'_A$ .  $D_1$  is the proper distance between the initial position of station A and the satellite at the instant the signal first arrives at the satellite; similarly,  $D_2$  is defined as the distance between the position of station B when the signal arrives and is retransmitted and the position of the satellite when the retransmitted signal arrives at the satellite. Let  $\tau$  be the coordinate time required for the signal to go out from station A and return, and let  $t_B$  be the time of arrival of the signal at station B. Then it can be shown that (results in this and subsequent sections are derived in Appendix B)

$$t_B = t_A + \frac{1}{2}\tau + \vec{v} \cdot [\vec{r}_B - \vec{r}'_A + D_2(\vec{r}_A - \vec{r}'_A)/D_1] / c^2 + \omega(1 + D_2/D_1)\vec{r}'_A \cdot \hat{k} \times \vec{r}_A / c^2 \quad (10)$$

In (10),  $\vec{v}$  denotes the velocity of the satellite at time  $t'_A$ . In the special case of a satellite in a circular orbit, with angular velocity  $\omega_s$  in its orbit, the above expression simplifies to

$$t_B = t_A + \frac{1}{2}\tau + \omega_s(\hat{n} \times \vec{r}'_A) \cdot (\vec{r}_B + D_2\vec{r}_A/D_1) / c^2 + \omega(1 + D_2/D_1)\vec{r}'_A \cdot \hat{k} \times \vec{r}_A / c^2 \quad (11)$$

Thus the motions of the earth and of the satellite give rise to corrections which can be of the order of hundreds of nanoseconds. In the case of a geosynchronous satellite,  $\omega_s = \omega$ , and (11) simplifies further to

$$t_B = t_A + \frac{1}{2}\tau + \omega(\vec{r}'_A \times \hat{k}) \cdot (\vec{r}_A - \vec{r}_B) / c^2 \quad (12)$$

For example, consider a geosynchronous satellite midway between two ground stations whose longitude difference is  $90^\circ$  and which are both situated on the equator. Under such circumstances the term involving  $\omega_s$  in (11) is zero ( $D_2 = D_1$ ). Thus only the motion of ground station A during the time of the experiment,  $\tau$ , gives rise to a correction, which is  $-308$  ns in this example. That is, it takes 308 ns less than half the normal trip time for a signal to go from station A to station B if B is west of A. Plots of the relativistic correction of (12) are given in Figure 3. If the satellite orbit has appreciable eccentricity, or for an extraterrestrial

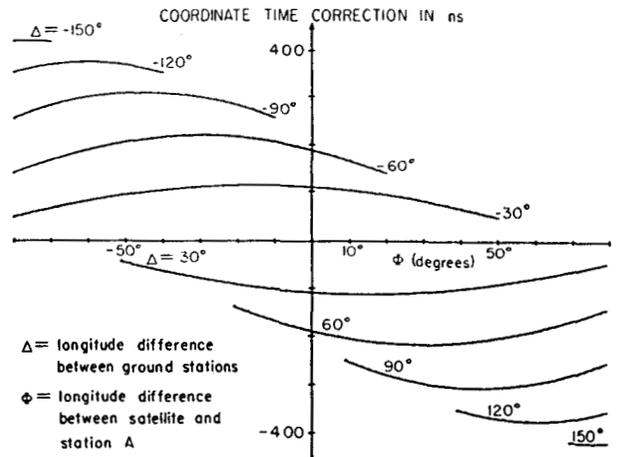


Fig. 3. Corrections to time transfer via two-way transponder on board a geosynchronous satellite for two equatorial ground stations. Each curve corresponds to a fixed longitude difference between ground stations A and B, measured positive westward. The horizontal coordinate is the difference in longitude between the satellite and ground station A, measured positive westward.

transponder in a noncircular orbit, (10) should be used.

*Time transfer via satellite clocks*

*Transfer to the satellite.* Consider, as in Figure 4, the synchronization of a clock on board a satellite with a master ground station at A. A signal is sent from A at  $t_A$  to the satellite, where it arrives at time  $t'_A$ . It is then immediately sent back to the ground station, where it arrives at time  $t_A + \tau$ , while the ground station has rotated into a new position.

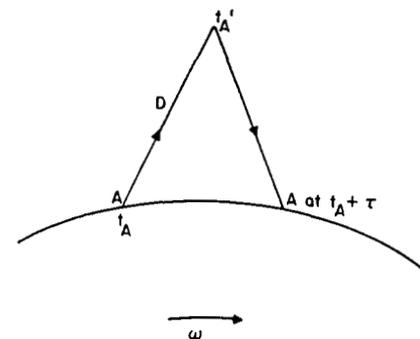


Fig. 4. Synchronization of satellite clock with master ground station. The signal leaves the ground station position A at time  $t_A$ , arrives at the satellite at time  $t'_A$ , and is retransmitted to the ground station, where it arrives after a trip time  $\tau$ .

TABLE 2. Values of constants useful in the calculation of relativistic effects [Rapp, 1974]

Constant	Definition	Values
$\omega$	rotational angular velocity of the earth	$7.2921 \times 10^{-5} \text{ s}^{-1}$
$J_2$	quadrupole moment coefficient for earth	$1.083 \times 10^{-3}$
$GM_e$	earth's mass times gravitational constant	$3.9860 \times 10^{14} \text{ m}^3/\text{s}^2$
$a_1$	earth's equatorial radius	$6.37814 \times 10^6 \text{ m}$
$g(\phi)$	acceleration of gravity on the geoid as a function of latitude $\phi$	$9.7803 + 0.0519 \sin^2 \phi \text{ m/s}^2$
$2\omega/c^2$	coefficient of area in equation (5)	$1.6227 \times 10^{-6} \text{ ns/km}^2$

First we should note that owing to red shift and Doppler effects, the satellite clock rate must be corrected using the following expression:

$$\Delta t = \int_{\text{path}} ds [1 + \epsilon/c^2 + 2GM_e/c^2r - 2GM_e a_1^2 J_2 P_2(\cos \theta)/c^2 r^3] \quad (13)$$

where the last term is due to the quadrupole moment of the earth;  $M_e$  is the mass of the earth,  $r$  is the radial distance of the satellite from the earth's center of mass,  $a_1$  is the earth's equatorial radius,  $\theta$  is the colatitude, and  $P_2(x) = \frac{1}{2}(3x^2 - 1)$ .  $J_2$  is the quadrupole moment coefficient; values of the constants are given in Table 2. The quantity  $\epsilon$  is the classical energy per unit mass of the orbiting spacecraft due to gravitational potential and kinetic energies and is given by

$$\epsilon = \frac{1}{2}v^2 + V \quad (14)$$

where  $V$  is the gravitational potential (see (B11)).

A ground-based clock is also subject to corrections which are given by

$$\Delta t = \int ds(1 - V_0/c^2 - g(\phi)h/c^2) \quad (15)$$

where  $V_0$  is the effective potential on the geoid, including effects due to rotation. An expression for  $V_0$  is given in (B18). The  $V_0$  term was suppressed in (1); in order to make (13) and (15) consistent with (1) the factor  $1 - V_0/c^2$  must be absorbed into  $ds$ , and then (13) becomes

$$\Delta t = \int_{\text{path}} ds [1 + \epsilon/c^2 + V_0/c^2 + 2GM_e/c^2r - 2GM_e a_1^2 J_2 P_2(\cos \theta)/c^2 r^3] \quad (16)$$

Effects arising from the quadrupole potential are a few parts in  $10^{12}$ .

For synchronization of a satellite clock (see Figure 4) the coordinate time  $t'_A$  of arrival of the signal at the satellite is given by

$$t'_A = t_A + \frac{1}{2}\tau + \omega \bar{r}'_A \cdot \hat{k} \times \bar{r}_A / c^2 \quad (17)$$

If the ground station were not in motion due to the earth's rotation, the relationship between  $t'_A$  and  $t_A$  would be just  $t'_A = t_A + \frac{1}{2}\tau$ , the usual result of the standard Einstein synchronization procedure. To estimate the magnitude of this correction in a typical case, consider a ground station located at spherical polar coordinate position  $(a, \theta_A, \phi_A)$  and a satellite at  $(r'_A, \theta'_A, \phi'_A)$ . Equation (17) then reduces to

$$t'_A = t_A + \frac{1}{2}\tau + \omega ar'_A \sin \theta_A \sin \theta'_A \sin(\phi'_A - \phi_A) / c^2 \quad (18)$$

and for a near earth satellite,  $\omega ar'_A / c^2 \approx 33 \text{ ns}$ . In Figure 5 are plotted the corrections of (17) for a satellite in a 12-hour equatorial orbit.

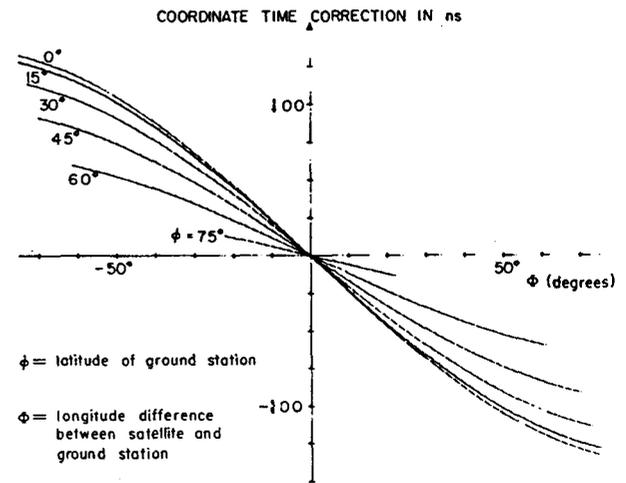


Fig. 5. Correction for coordinate time transfer to satellite clock due to earth rotation, corresponding to Figure 4. Each curve corresponds to a ground station at a specific latitude. The horizontal coordinate is the difference of longitude between the satellite and the ground station, measured positive westward. The satellite is assumed to be in a 12-hour equatorial orbit. (Note: the correction varies as the cosine of the satellite latitude.)

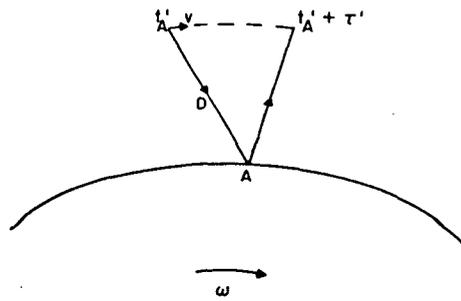


Fig. 6. Time transfer from satellite clock to ground station. A signal originates at coordinate time  $t'_A$  at the satellite, arrives at the ground at time  $t_A$ , and is retransmitted to the satellite, where it arrives at time  $t'_A + \tau$ .

**Transfer from satellite to ground station.** In this process (see Figure 6) we imagine a signal transmitted at time  $t'_A$  from a satellite at initial position  $\vec{r}'_A$  to a ground station which is at  $\vec{r}_A$  at the time of arrival  $t_A$  and thence back to the satellite which has moved a distance  $\vec{v}\tau'$  during the trip time  $\tau'$ . Then

$$t_A = t'_A + \frac{1}{2}\tau' - \vec{v} \cdot (\vec{r}'_A - \vec{r}_A) / c^2 \quad (19)$$

Plots of the relativistic corrections of (19) are given in Figure 7 for a geostationary satellite. For a satellite in a circular orbit,  $\vec{v} \cdot \vec{r}'_A = 0$ . Then if the normal to the orbit is  $\hat{n}$  and the satellite has angular velocity  $\omega_s$  in its orbit, (19) reduces to

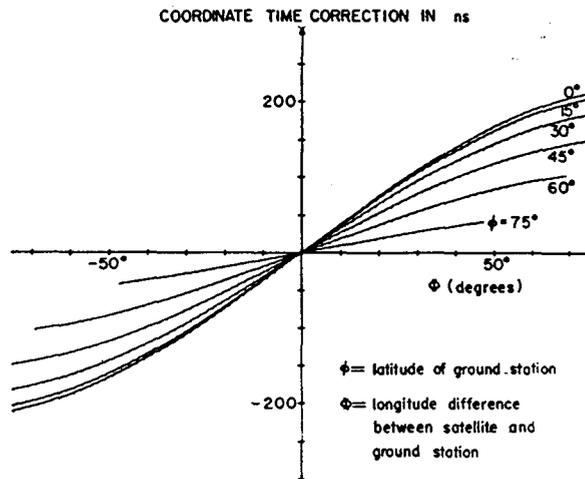


Fig. 7. Correction for coordinate time transfer from geosynchronous clock to ground station. Each curve corresponds to a ground station at a specific latitude. The horizontal coordinate is the difference of longitude between the satellite and the ground station, measured positive westward.

$$t_A = t'_A + \frac{1}{2}\tau' + \omega_s \vec{r}'_A \cdot (\hat{n} \times \vec{r}_A) / c^2 \quad (20)$$

To estimate the magnitude of this effect, consider a ground station on the equator and a satellite in equatorial orbit. The correction can be written

$$t_A = t'_A + \frac{1}{2}\tau' + [GM_s / c^2 r'_A]^{1/2} a_1 \sin(\varphi_s - \varphi_A) / c \quad (21)$$

where  $\varphi_s - \varphi_A$  is the longitude difference. For a near earth satellite,

$$[GM_s / c^2 r'_A]^{1/2} a_1 / c \approx 560 \text{ ns}$$

The results obtained in (17) and (19) can be combined in several ways to describe different procedures for time transfer between ground stations or between satellites. We shall discuss these from the point of view of transfer between ground stations.

**Lasso-type experiment [Rutman, 1978].** In this case, (17) is applied twice, as the experiment involves signals sent from two ground stations to a retroreflector on board a satellite or jet plane.

In Figure 8, let  $t'_A$  and  $t'_B$  be the times of arrival of pulses originating at ground stations at  $t_A$  and  $t_B$ , respectively, and let  $\tau' = t'_B - t'_A$ , assuming that  $\tau'$  is sufficiently small that relativistic corrections of (16) can be neglected. Let  $\tau_A$  and  $\tau_B$  be the trip times for the round trips of signals from A and B, respectively. Then the coordinate time of the clock at B is related to the coordinate time of the clock at A by

$$t_B = t_A + \frac{1}{2}(\tau_A - \tau_B) + \tau' + \omega(\vec{r}'_A \cdot \hat{k} \times \vec{r}_A - \vec{r}'_B \cdot \hat{k} \times \vec{r}_B) / c^2 \quad (22)$$

**Flyover [Besson, 1970].** Next we consider as in Figure 9 time transfer by means of a transported clock in a satellite or jet plane in which the moving

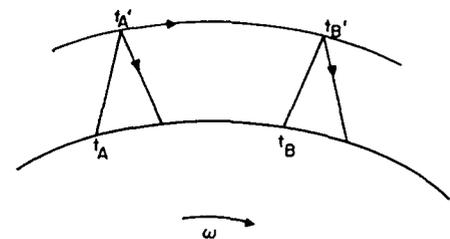


Fig. 8. Lasso-type experiment involving retroreflectors on board a satellite or airplane [Rutman, 1978]. Signals are sent from two ground stations A and B, at times  $t_A$  and  $t_B$ , respectively, to the satellite where they arrive at  $t'_A$  and  $t'_B$  and are then sent back.

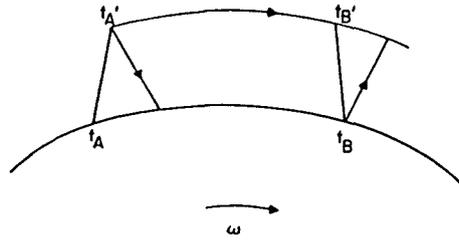


Fig. 9. Time transfer between remote ground stations using electromagnetic signals between ground stations and a clock flying overhead [Besson, 1970].

clock at A' is set by means of signals from ground station A, then the clock moves to B', and thence sends a signal to B and back. Let  $t_A$  be the departure time of the signal from A,  $t'_A$  its arrival time at the clock at A',  $t'_B$  the departure time of the signal from B', and  $t_B$  the arrival time at B. Let  $\tau = t'_B - t'_A$ , and let  $\tau_A$  and  $\tau'_B$  be the round trip times of the signals. Then

$$t_B = t_A + \tau + \frac{1}{2}(\tau_A + \tau'_B) + \omega \bar{r}'_A \cdot \hat{k} \times \bar{r}_A / c^2 + \bar{v} \cdot (\bar{r}_B - \bar{r}'_B) / c^2 \quad (23)$$

where  $\bar{v}$  is the velocity of the satellite at  $\bar{r}'_B$ .

The time  $\tau$  is subject to relativistic corrections according to (16) or (1), depending on whether the clock is transported by jet or by satellite. In order for relativistic corrections to the time  $\tau$  to be negligible (corrections of  $<1$  ns) so that all that is required of the moving clock is that it be stable, then for typical jet plane speeds (270 m/s) the most important term is due to earth rotation. The relativistic corrections in this case are negligible if  $\tau < 12$  min. Besson [1970] and coworkers were evidently the earliest to explicitly consider relativistic corrections in this type of experiment.

*Modified clock flyover.* A very similar experiment to the flyover is one in which, as in Figure

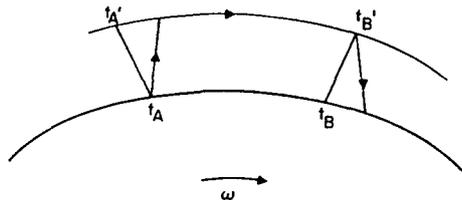


Fig. 10. Time transfer between remote ground stations where the directions in which the electromagnetic signals are sent are the reverse of those in Figure 9.

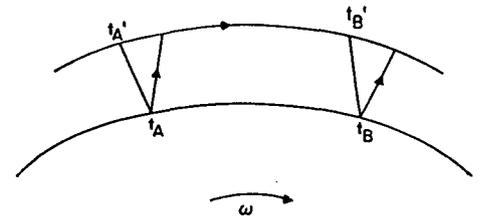


Fig. 11. Transfer between remote ground stations with satellite clock used as reference.

10, the signals are sent in reverse order compared with the ordinary flyover. Let a synchronizing signal be sent from the satellite clock at  $t'_A$  to ground station A, arriving at  $t_A$ , and let ground station B send a signal to the satellite clock at  $t_B$ , arriving at  $t'_B$ ;  $\tau'_A$  and  $\tau_B$  are the signal trip times. Let  $\tau' = t'_B - t'_A$ . Then for coordinate time at B,

$$t_B = t_A + \tau' - \frac{1}{2}(\tau'_A + \tau_B) - \bar{v} \cdot (\bar{r}_A - \bar{r}'_A) / c^2 - \omega \bar{r}'_B \cdot \hat{k} \times \bar{r}_B / c^2 \quad (24)$$

*Synchronization via master satellite clock.* Finally, we consider synchronization of ground stations by transmission of signals from an orbiting clock. The notation is indicated in Figure 11, with  $\tau'_A$  and  $\tau'_B$  the round trip times of the signals which leave the satellites at  $t'_A$  and  $t'_B$  and arrive at the ground stations at  $t_A$  and  $t_B$ . Then from (19);

$$t_B = t_A + \tau' + \frac{1}{2}(\tau'_B - \tau'_A) + \bar{v}'_B \cdot (\bar{r}'_B - \bar{r}_B) / c^2 - \bar{v}'_A \cdot (\bar{r}'_A - \bar{r}_A) / c^2 \quad (25)$$

where  $\tau' = t'_B - t'_A$ .

*One-way satellite transmissions.* The procedures previously discussed are not applicable to synchronization by means of a GPS satellite, which involves transmissions directly to a ground station

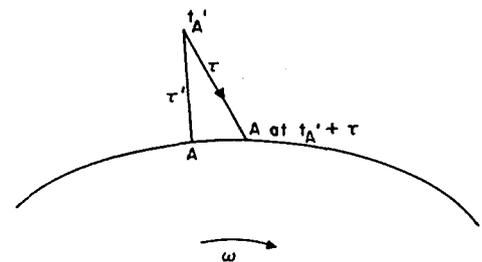


Fig. 12. Corrections due to earth rotation for one-way satellite transmissions, such as in the GPS system [Schuchman and Spiker, 1977; Buisson et al., 1977].

without a return signal. In this case, position information relating to the satellite is transmitted with the synchronizing signal. If the position of the ground station is accurately known, then the coordinate clock at the ground can be set by taking account of the motion of the ground station during the propagation time  $\tau$  of the signal from the satellite to the ground station. In Figure 12, let  $\bar{r}'_A$  be the position of the satellite at the instant of transmission and  $\bar{r}_A$  be the position of the earth station at the same instant. The difference between  $\tau$  and the time  $\tau'$  it would have taken for the signal to propagate if the ground station had not moved is given by

$$\tau = \tau' - \omega \bar{r}'_A \cdot \hat{k} \times \bar{r}_A / c^2 \quad (26)$$

This is similar in form to the result obtained in (17) except for a sign; the behavior of this correction can thus be inferred from the plot of Figure 5.

#### EXPERIMENTS IN CLOCK SYNCHRONIZATION

In this section we shall briefly review some recent experimental work which is particularly relevant for the implementation of a coordinate time scale as discussed in this paper.

##### *Past experiments*

*The Hafele and Keating [1972a, b] experiment.* This particular experiment is a classical one in terms of verifying the relativistic effects resulting from the spinning earth because the results clearly show the significant ( $\approx 200$  ns) discontinuities resulting when one develops a coordinate time scale using a reference frame tied to the spinning earth, treating it as though it were an inertial frame. From this experiment it becomes clearly evident that one should look for alternative reference frames. The geocentric local inertial reference frame proposed [Ashby, 1975] and developed in this paper in an operational direction avoids the ambiguous discontinuity problems cited above and provides a coordinate grid for the earth and for jets and satellites in the vicinity of the earth at the 1 ns or better accuracy level.

In principle, one could interpret the Hafele-Keating experiment, using (1), which is based on a geocentric inertial coordinate system, to show that as the clocks circumnavigated the earth, their geocentric coordinate time would agree before and after the trip, within the uncertainties due to time disper-

sion in the clocks and due to error in the knowledge of the flight path. It is probably worth restating at this point that the size of the correction due to earth rotation which is required to generate coordinate time for a clock returning to its point of departure is proportional to the projection on the equatorial plane of the circumnavigated area, using the usual geocentric coordinate. The proportionality coefficient is  $1.6227 \times 10^{-6}$  ns/km<sup>2</sup>, and the sign of the correction is positive (negative) when the area is circumscribed clockwise (counterclockwise) as viewed from the south pole. The Hafele-Keating data nicely support these conclusions.

In an earlier experiment, Besson [1970] and coworkers investigated relativistic correction in a flight which retraced its path in such a way that the projected area was zero. The resulting offset was due to red shift and Doppler effect; the correction of (1) due to earth rotation vanished.

*Laser pulse synchronization of flying clocks.* Alley and coworkers [Reisse, 1976; Williams, 1976] performed an experiment corresponding to the situation diagrammed in Figure 4 with an ensemble of clocks in a jet aircraft and another on the ground. A laser pulse was sent from the ground to a retroreflector and detector aboard the aircraft, and time transfers from the ground station to the aircraft were repeatedly made. In this case the largest effect is from the gravitational term in (1). Since the aircraft was only approximately 15 km east of the ground station, or less, the term arising from earth rotation in (18) is too small to be detected. Expected offsets of the flying clock obtained by applying the path integral in (1) were of the order of 45 ns; actual time transfers using the laser agreed with the expected offsets to about 1.5%.

*The Vessot gravitational red shift rocket probe.* This experiment [Vessot et al., 1976] featured a state-of-the-art hydrogen maser frequency standard, which exhibited about a part in  $10^{14}$  frequency stability during its rocket trip. The trip lasted about 8,000 s, going from Wallops Island (37.8°N, 75.5°W) on the east coast of the United States to a point in the Atlantic Ocean (29.3°N, 47.5°W) and reaching at apogee an elevation of about 10,000 km. The primary tracking station was at Merritt Island (28.5°N, 80.7°W). A rough estimate of the size of the correction implied by (18) is  $\approx 21$  ns at apogee.

To develop the theory for the experiment, Vessot et al. [1976] chose a nonrotating geocentric coordinate system. The theory predicted up to about 4

TABLE 3. Data for portable clock trip from USNO to NBS and return

Measurement	Result, ns	Time, UTC	Date
PC - UTC(USNO, system 1)	2,448	1517	Aug. 25, 1977
PC - UTC(USNO, system 1)	2,458	1217	Aug. 26, 1977
PC - UTC(NBS)	2,729	2007	Aug. 26, 1977
PC - UTC(USNO, system 1)	2,488	0313	Aug. 27, 1977
PC - UTC(USNO, system 1)	2,506	2146	Aug. 27, 1977
PC - UTC(USNO, system 1)	2,535	1116	Aug. 29, 1977

$\times 10^{-10}$  fractional frequency change at apogee primarily due to the red shift, and experiment agreed with theory to about the  $1 \times 10^{-13}$  level.

Though the experiment was not a clock experiment (the maser was stabilized after launch), one can still integrate the frequency to infer time departures. The maser advanced about 0.15 ns during the first 30 min of observation and advanced again about 0.1 ns during the last 10 min with respect to the predicted values. The latter departure is believed to be due to error in the orbit determination during that time. Effects due to the moving refractive medium associated with the spinning earth are currently being investigated (R. Vessot, personal communication, 1978), which may further improve the precision of the measurement.

#### A recent portable clock trip

During August 25-29, 1977, a high-performance cesium portable clock (PC) was transported from the U.S. Naval Observatory (USNO) in Washington, D.C., to the National Bureau of Standards (NBS) in Boulder, Colorado, and back. Table 3 gives the resulting six measured time differences

between the PC and two different time scales. The comparisons with USNO are plotted in Figure 13. It should be kept in mind that the UTC scales can be considered to measure elapsed coordinate time, since, for example, the rate of UTC(NBS) is adjusted to correct for gravitational red shift according to (2) in order to maintain synchronization.

To obtain a comparison between the UTC(USNO) and UTC(NBS) time scales, the data can be processed in several ways. In keeping with the point of view adopted in this paper the PC is regarded as a standard clock from which elapsed proper time may be read, while (1) is used to calculate the elapsed coordinate time.

We will consider two possible options. The first is extrapolation: for the prediction times of interest the best model for the time dispersion of a cesium beam frequency standard is random walk of the time fluctuations. For example, white noise frequency fluctuations

$$\sigma_x(\tau) = \tau\sigma_y(\tau) = 5 \times 10^{-12} \tau^{1/2}$$

is the confidence estimate for this particular PC. The optimum prediction algorithm for this case is

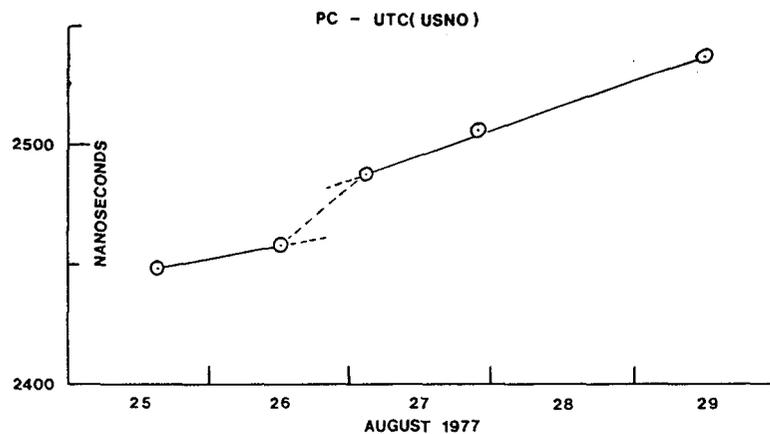


Fig. 13. Comparison of portable clock time minus UTC(USNO) coordinate time before and after trip from Washington, D.C., to Boulder, Colorado, and return.

simply an extrapolation of the mean frequency from the closest time measurement to the point of prediction. (Quite often in practice systematic deviations are larger than random deviations; however, we will ignore the systematic deviations for the moment.) Using (USNO system 1) as the reference coordinate time scale, one calculates from measurements 1 and 2 in Table 3 the mean normalized frequency offset of the PC as  $+1.3 \times 10^{-13}$ . Extrapolating from measurement 2 to 3 gives rise to a PC correction of  $-3.7$  ns; i.e., the PC gains 3.7 ns during the interval 2-3.

The elapsed coordinate time during this interval, from (1), contains three relativistic corrections that need to be added: first,  $-gh/c^2$ , which amounts to  $-12.4$  ns for a 3-hour flight from Dulles airport (Washington, D.C.) to Stapleton (Denver, Colorado) at a height of 10.5 km (34,400 feet). In other words the PC will gain owing to the red shift effect, and 12.4 ns should be subtracted from its time to bring it into agreement with coordinate time. The second-order Doppler term  $\frac{1}{2}(v/c)^2$  amounts to  $+4.4$  ns for this situation at a ground speed of 270 m/s (604 mph). For the above path the last term in (1) yields a correction of  $-9.6$  ns east to west and  $+9.6$  ns west to east. Combining all of the coordinate time corrections for the PC during the trip west gives

$$\begin{aligned} \Delta t &= \text{PC}(2007 \text{ UT}) - \text{PC}(1217 \text{ UT}) - 3.7 \text{ ns} \\ &\quad - 12.4 \text{ ns} + 4.4 \text{ ns} - 9.6 \text{ ns} \\ &= \text{PC}(2007 \text{ UT}) - \text{PC}(1217 \text{ UT}) - 21.3 \text{ ns} \end{aligned} \quad (27)$$

Therefore the extrapolated coordinate time transferred from USNO with the PC at the time (2007 UT) of comparison with NBS is

$$\text{UTC}(\text{USNO}, 2007 \text{ UT}) = \text{UTC}(\text{USNO}, 1217 \text{ UT}) + \Delta t \quad (28)$$

Subtracting measurement 2 from measurement 3 and adding (27) and (28) yield

$$\text{UTC}(\text{USNO}, \text{system 1}) - \text{UTC}(\text{NBS}) = 249.7 \text{ ns} \quad (29)$$

at 2007 UT on August 26, 1977, for the forward extrapolation.

Similarly, one may extrapolate backward to measurement time 3 using the measurement time interval 4-6 to determine the mean normalized frequency offset of the PC ( $+2.3 \times 10^{-13}$ ). Using the same

flight parameters but changing the sign of the relativistic correction due to earth rotation yields

$$\text{UTC}(\text{USNO}, \text{system 1}) - \text{UTC}(\text{NBS}) = 245.4 \text{ ns} \quad (30)$$

at 2007 UT on August 26, 1977, for the backward extrapolation.

Equations (29) and (30) are independent estimates and can be combined in proportion to the inverse square of the confidence of the estimates to give

$$\text{UTC}(\text{USNO}, \text{system 1}) - \text{UTC}(\text{NBS}) = 247.4 \text{ ns} \quad (31)$$

at 2007 UT on August 26, 1977, for the combined extrapolated values.

A second option for analyzing the data is by use of interpolation. As was mentioned earlier, it is often the case for portable clock trips that systematic effects in the PC are larger than the random fluctuations assumed above; in this case it is better to use interpolation. This, of course, requires that the PC return to its origin. One can then calculate the mean normalized frequency offset for the trip (measurement 4 - measurement 2 =  $5.6 \times 10^{-13}$ ). The assumption of symmetry for a trip like the above may often be a good assumption even in the case of systematic effects on the PC and is also valid for the red shift and second-order Doppler corrections. Accounting for the last two corrections plus the PC's average offset from before and after the trip predicts that the PC should have gained 25.6 ns during the round trip flight, when in fact it gained 30 ns. The 4.4-ns error is a factor of 4 larger than the random fluctuation and is probably mainly due to systematics and/or errors in the estimated flight altitude, velocity, and time.

The known asymmetric term is the last correction in (1). Interpolating and applying this correction yield

$$\text{UTC}(\text{USNO}, \text{system 1}) - \text{UTC}(\text{NBS}) = 245.6 \text{ ns} \quad (32)$$

at 2007 UT on August 26, 1977.

Equations (31) and (32) are based on independent frequency estimates for the PC and can be further combined. There is no simple objective way to do this, as each situation will be different. In the current case, because of evidence of significant systematics the last result, (32), is probably more valid. A reasonable final result would therefore be

$$\text{UTC(USNO, system 1)} - \text{UTC(NBS)} = 246 \pm 2 \text{ ns} \quad (33)$$

at 2007 UT on August 26, 1977, based on the consistency and nature of the above experiment.

#### *Possible future experiments*

*Geostationary satellite experiment.* Starting July 1978 a joint experiment is planned using two geostationary satellites, CTS and Symphonie. In North America, NBS, NRC, and USNO will be able to view CTS and will participate. The BIH will participate from Europe and will share common viewing of Symphonie with NRC. The approximate coordinates of CTS and Symphonie are (116°W, 0°N) and (11.5°W, 0°N), respectively. The approximate coordinates of the BIH, NBS, NRC, and USNO are: BIH, 3.5°W, 48.8°N (Pleumeur-Bodou); NBS, 40.0°N, 105.3°W; NRC, 45.4°N, 75.9°W; USNO, 38.9°N, 77.1°W. From Figure 3, one can then make estimates of the corrections required for generation of coordinate time. It is also necessary to modify the values obtained from Figure 3 using (12), since the ground stations are not on the equator. Taking the BIH as station A and NRC as station B gives an approximate correction of -158 ns. Then using NRC as station A and sequentially NBS and USNO as station B gives approximate corrections of -67 ns and +7.5 ns, respectively.

Portable clocks will be carried between stations, concurrent with the above experiments. After applying coordinate corrections for the specific paths taken, the portable clocks should then, to within the uncertainties of the measurements, bring agreement between geocentric coordinate time at the various timing centers obtained by satellite and obtained by portable clocks. The portable clock correction between the BIH and the NBS in Boulder, Colorado, is typically 50 ns. The accuracy of the experiment will probably be limited by the quality of the portable clocks used (~100 ns). Potentially, the experiment could be done with an accuracy of about 10 ns, as limited by the satellite and by the methods of transmission. If a high-quality cesium clock were used in the 'flyover' mode with a laser or microwave signal to provide the down-link to the timing centers, then the accuracy for coordinate time transfer could be better than 10 ns.

*The Lasso experiment [Rutman, 1978].* The coordinate time corrections for one specific Lasso-

type experiment are given by (22). The accuracy of this technique should be about 1 ns. For a BIH to PTB time comparison the geocentric coordinate time correction due to the spin of the earth would be 3.3 ns. The corrections for red shift and second-order Doppler shifts would have to be calculated after the velocity and altitude of the flight were known. Because of the excellent accuracy of this experiment, if a passive hydrogen clock in the flyover mode using a laser down-link were used for comparison, such an experiment could have an overall accuracy of 1 ns.

*Very long baseline interferometry.* Because of its great potential accuracy of 0.1 ns [Carter *et al.*, 1978; Fanselow, 1977; Robertson *et al.*, 1978; Rogers *et al.*, 1978; Schilizzi and Campbell, 1978] a comparison between remote clocks synchronized by VLBI and by portable clocks or by satellite transmissions should reveal some large effects. For remote VLBI stations the last term in (1) can approach 100 ns as a correction to the transported clock time. Since VLBI data are customarily analyzed using nonrotating coordinates, no corrections are required for synchronization using VLBI alone.

In the future, VLBI is expected to operate regularly between the United States and Europe and possibly also between the United States, Australia, and Japan for determination of polar motion, crustal movement, and UT1. The accuracy of comparisons between frequency standards laboratories may then be limited mainly by transfers over distances of  $\approx 1000$  km within continents.

*Line-of-sight microwave synchronization.* The geodimeter is believed capable of about 0.01-ns accuracy in time transfer over distances of about 50 km. If at latitude 40°N a circuit consisting of four geodimeter legs arranged in a square having sides of 50 km were constructed, the area of this circuit projected on the equatorial plane is about 1600 km<sup>2</sup>. Suppose that the geodimeter were used to transfer coordinate time around the circuit and back to the starting point. The coordinate time corrections required would be equal to the area times the coefficient of area from (5), which is  $1.6 \times 10^{-6}$  ns/km<sup>2</sup>. This gives a correction of +0.0026 ns, which unfortunately is less than the order of accuracy of the experiment.

#### CONCLUSION

Because of relativistic corrections associated with the spinning earth it is proposed that a geocentric

nonrotating coordinate frame be adopted for purposes of comparing clocks near the surface of the earth. There is ample evidence, some of which has been cited above, to support this concept. Significant simplification occurs in the elimination of ambiguities and path-dependent effects and in the implementation of a coordinate time scale which has the property of transitivity. Accuracies of 1 ns or better in coordinate clock comparisons may straightforwardly be achieved by applying corrections depending on altitude, velocity, and path of a portable clock. Similarly, coordinate time synchronization corrections depending upon locations of timing centers and satellites may be calculated where appropriate using results given in this paper.

It is recommended that the geoid be used as a reference position for determination of the rate of a geocentric coordinate clock and that the UTC scale as generated by BIH be used to define the time origin and reference from which geocentric coordinate time will be disseminated.

In the past the coordinate time corrections discussed here have not generally been employed. First, there has not been general agreement to do so, and second, these corrections have not been of great significance, because most portable clock trips have been such that the return trip was over the same path as the outward trip, rather than circumnavigations of the earth, and because most comparisons have been between Europe and the United States. In the future an accurate coordinate time scale will be distributed over the entire globe. As GPS and other systems are developed, confusion will be avoided if such a geocentric coordinate time scale is adopted.

APPENDIX A: SYNCHRONIZATION BY PORTABLE CLOCKS

This analysis is carried out within the framework of the general theory of relativity; modifications which might be necessary in some other gravitational theory such as that due to Brans and Dicke will be quite small. The basic assumptions and results of general relativity which we need are as follows.

1. There exists a metric tensor  $g_{\mu\nu}$  ( $\mu, \nu = 0, 1, 2, 3$ ) such that the space-time interval  $ds$  defined by

$$-c^2 ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{A1}$$

is invariant with respect to arbitrary coordinate transformations. (We use the notation and sign

conventions of *Weber* [1961], in which  $g_{00} < 0$ .)

2. Propagation of light rays from  $x^\mu$  to  $x^\mu + dx^\mu$  is described by the vanishing of  $ds$ :

$$0 = g_{\mu\nu} dx^\mu dx^\nu \tag{A2}$$

3. The proper time (in seconds) elapsed on a standard clock (e.g., an atomic clock) transported along the space-time path element  $dx^\mu$  is given by

$$ds = \frac{1}{c} \sqrt{-g_{\mu\nu} dx^\mu dx^\nu} \tag{A3}$$

4. Of particular importance in this paper is the interpretation of the coordinate  $x^0$ , which is a global coordinate time having units of length. We shall write for the coordinate time  $t$  in seconds,  $t = x^0/c$ . In the general theory it is assumed that at each point of space an observer can be equipped with a coordinate clock which can be used to measure the coordinate time of any event occurring at that point. The importance of coordinate time may be appreciated by considering the example of the gravitational red shift of a standard clock. Standard clocks will have different rates depending on the gravitational potential at the clock's position. In contrast, coordinate clock rates will be independent of position if the gravitational field is static.

5. The metric coefficients  $g_{\mu\nu}$  may be calculated from the field equations of general relativity. Furthermore, because in the vicinity of the earth's surface all gravitational fields are weak it will be sufficient to work in the linearized approximation in which

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{A4}$$

with  $\eta_{\mu\nu} = (-1, 1, 1, 1)$  the metric tensor of flat space, and

$$h_{\mu\nu} \ll 1 \tag{A5}$$

so that squares and products of  $h_{\mu\nu}$  are negligible.

In isotropic coordinates, to a sufficient approximation we have

$$-ds^2 = -(1 + 2\Phi/c^2)(dt)^2 + (1 - 2\Phi/c^2)(dx^2 + dy^2 + dz^2)/c^2 \tag{A6}$$

where  $\Phi$  is the gravitational potential. In the solar system the only bodies we need account for are the earth, sun, and moon; however, the earth cannot be treated as a point mass because of its flattening. Thus we may write

$$\Phi(\vec{r}) = V_s + V_M + W(\vec{r}) \quad (\text{A7})$$

where  $V_s$  and  $V_M$  are the potentials due to the sun and the moon, respectively, at the observation point  $\vec{r}$ .  $W(\vec{r})$  is the potential at observation point  $\vec{r}$  due to the flattened earth.

When the potential of (A7) is substituted into (A6), the coordinate time  $t$  can be interpreted as ephemeris time, since a standard clock placed at rest sufficiently far from all masses would read elapsed coordinate time  $t$ .

It is worthwhile at this point to explain in detail the point of view adopted in this paper. Since standard clocks beat at rates which are affected by gravitational red shifts and Doppler shifts due to time dilation, we shall propose that a consistent scheme of establishing a network of synchronized clocks be adopted by using standard clocks, together with (A6), to measure elapsed coordinate time,  $\Delta t$ .

This may be illustrated in Figure 1, in which measured elapsed proper time,  $\Delta s$ , on a standard clock is combined with information on positions and velocities of the moving standard clocks, to produce the elapsed coordinate time  $\Delta t$ . This procedure has the advantage that, because of the global nature of the coordinate time  $x^0$  in general relativity, if coordinate clock A is synchronized with coordinate clock B and coordinate clock B is synchronized with coordinate clock C, then A will be synchronized with C.

Consider, for example, the problem of establishing a network of synchronized clocks on the surface of the earth. Neglect for the moment the gravitational potential. Because laboratories maintaining atomic time are in motion due to the earth's spinning, time transfer between laboratories will be path dependent. It is therefore suggested that coordinate clocks fixed on the surface of the earth or in motion in planes or satellites be adjusted to read the coordinate time  $t$  which would be read on clocks at rest in an underlying nonrotating local inertial frame. By such a procedure, one can obtain a consistent coordinate clock network in which synchronization has the property of transitivity.

Next let us consider the effect of the sun's gravitational potential, in (A7). In the literature the suggestion frequently recurs that a clock on board a satellite will undergo a periodic shift in rate owing to the fact that the gravitational potential of the sun changes as the clock moves in its orbit around the earth. This effect is much more subtle than

is generally realized. Analysis of this situation using circular orbits shows that there are actually four such effects, all of the same magnitude, but with two of the effects having one sign and two having the opposite sign, hence the net effect to quite a high degree of accuracy is negligible because of the mutual cancellation of these four effects.

The first of these effects is the gravitational red shift. Imagine a local reference frame which maintains, say, the  $x$  axis pointing toward the sun. Of two clocks at different positions along the  $x$  axis the one closer to the sun should beat at a slower rate due to gravitational red shift. The fractional amount by which the closer clock is red shifted is  $g_s \Delta r / c^2$ , where  $g_s = GM_s / r_s^2$  is the gravitational field strength of the sun at the earth's orbit,  $M_s$  is the sun's mass, and  $r_s$  is the orbit radius, assuming that the orbit is circular.  $\Delta r$  is the difference in distance of the two clocks from the sun.

However, since one clock is at a greater radius than the other, its velocity in a heliocentric system will be greater, and it will be red shifted more due to time dilation or second-order Doppler shifts. Let  $v_0$  be the orbital velocity of the clock at radius  $r_s$  and  $v = v_0(1 + \Delta r / r_s)$  be the velocity of the clock at radius  $r_s + \Delta r$ . This clock would be almost at rest in the local inertial reference frame. The fractional Doppler shift compared with a clock at  $r_s$  will be

$$\sqrt{1 - v_0^2/c^2} - \sqrt{1 - v^2/c^2} \approx v_0^2 \Delta r / (c^2 r_s) \quad (\text{A8})$$

But since the orbit is determined by the condition  $GM_s / r_s^2 = v_0^2 / r_s$ , the Doppler shift cancels the gravitational red shift to terms linear in the distance from the origin. *Hoffman* [1961] has previously discussed the effect of this cancellation on clock rates.

There must yet be a third effect because a local inertial frame does not maintain one axis pointing toward the sun; with respect to such a direction the local inertial frame rotates once a year. When this rotation is accounted for, an additional contribution which is of the same sign and magnitude as the gravitational red shift is obtained.

There must thus exist a fourth effect because one of the principle tenets of relativity theory is that gravitational fields can be transformed away locally by introducing an appropriate freely falling inertial frame. To find this fourth effect it is necessary to carry out the actual construction of the local inertial frame—introduce a tetrad of mutu-

ally orthogonal four-vectors, integrate the equations of motion, find the transformation from coordinate time in the heliocentric system to coordinate time in the local frame, and calculate the metric in the local frame. This is too lengthy to include here, but the resulting coordinate time transformation is as follows:

$$X^0 = \left(1 - \frac{3\mu}{r_s}\right)^{-1/2} \left\{ ct + \sqrt{\frac{\mu/r_s}{1 - 2\mu/r_s}} (-x \sin [\omega t] + z \cos [\omega t]) \right\} \quad (A9)$$

where  $\mu = GM_s/c^2$ ,  $X^0$  is the heliocentric coordinate time,  $x$  and  $z$  are local rectangular spatial coordinates measured in the local inertial frame, and  $\omega^2 = GM_s/r_s^3$ , with  $\omega$  the orbital angular speed. Here the coordinate  $y$  does not appear because it is assumed to be measured normal to the plane of the orbit. The prescription for constructing such coordinates is discussed by *Manasse and Misner* [1963]. Note that the transformation between  $t$  and  $X^0$  involves terms linear in the distances  $x$  and  $z$ .

The result of these linear terms in the coordinate time transformation gives a correction which is of the same magnitude as, and of opposite sign to, the gravitational red shift. Hence locally, the presence of the sun does not affect relative clock rates.

Now let us consider the process of synchronization of clocks which are placed near the surface of the rotating earth, which we may now consider to be located with its center of mass at the origin of a local inertial frame. In local inertial coordinates with origin at the center of mass of the earth the metric tensor now takes the form

$$g_{00} = -1 - 2W(\bar{r})/c^2 \quad (A10)$$

$$g_{i0} = 0 \quad (A11)$$

$$g_{ij} = \delta_{ij}(1 - 2W(\bar{r})/c^2) \quad (A12)$$

where  $W(\bar{r})$  is the potential due solely to the earth's gravitational field. We now transform to rotating coordinates  $(x', y', z', t')$  by means of

$$x' = x \cos \omega t + y \sin \omega t \quad (A13)$$

$$y' = y \cos \omega t - x \sin \omega t \quad (A14)$$

$$z' = z \quad (A15)$$

$$t' = t \quad (x^{0'} = x^0) \quad (A16)$$

where  $\omega$  is the angular velocity of rotation of the earth, whose value is given in Table 2. The metric then becomes in linear approximation

$$g_{00} = -1 - \frac{2}{c^2} \left\{ W(\bar{r}) - \frac{1}{2}\omega^2(x'^2 + y'^2) \right\} \quad (A17)$$

$$g_{0i} = g_{i0} = -y' \omega / c \quad (A18)$$

$$g_{0z} = g_{z0} = x' \omega / c \quad (A19)$$

$$g_{ij} = \delta_{ij}(1 - 2W(\bar{r})/c^2) \quad (A20)$$

Thus keeping only linear terms,

$$-ds^2 = - \left\{ 1 + \frac{2}{c^2} (W(\bar{r}) - \frac{1}{2}\omega^2(x'^2 + y'^2)) \right\} (dt')^2 + 2(\bar{\omega} \times \bar{r}') \cdot d\bar{r}' dt' / c + [1 - 2W(\bar{r})/c^2] \cdot [(dx')^2 + (dy')^2 + (dz')^2] / c^2 \quad (A21)$$

The neglected terms are, for propagation of light, at most

$$\frac{2}{c^2} \frac{GM_e}{a_1} \frac{\omega a_1}{c} (\Delta x^0)^2$$

and over a path length of 10,000 km are completely negligible. For objects moving with speed  $v$  the neglected terms would be of order  $v/c$  smaller and hence negligible.

Henceforth we drop primes on the coordinates and take the metric to be

$$-ds^2 = - \left\{ 1 + \frac{2}{c^2} (W(\bar{r}) - \frac{1}{2}\omega^2(x^2 + y^2)) \right\} + 2\bar{\omega} \times \bar{r} \cdot d\bar{r} dt / c^2 + [1 - 2W(\bar{r})/c^2] \cdot [dx^2 + dy^2 + dz^2] / c^2 \quad (A22)$$

The potential due to the earth's flattened distribution of mass may, with accuracy sufficient for our purposes, be taken as [Caputo, 1967; Garland, 1965]

$$W(\bar{r}) = -\frac{M_e G}{r} \left\{ 1 - [a_1/r]^2 J_2 P_2(\cos \theta) \right\} \quad (A23)$$

where  $\theta$  is the polar angle measured from the rotation axis and the constants  $M_e G$ ,  $J_2$ , and  $a_1$  can be obtained from Table 2.  $P_2$  is the Legendre polynomial,

$$P_2(x) = \frac{1}{2}(3x^2 - 1) \quad (A24)$$

To express this potential in more convenient form, in terms of geographical latitude  $\phi$  and altitude  $h$

above the geoid, we use the expansion

$$r \approx a_1 - a_1 f \sin^2 \phi + h \quad (\text{A25})$$

where  $f$  is the flattening of the earth, defined as unity minus the ratio of polar to equatorial radii of the earth. Also,

$$\cos^2 \theta \approx \sin^2 \phi (1 - 4f \cos^2 \phi) \approx \sin^2 \phi \quad (\text{A26})$$

Then if we put

$$V(\vec{r}) = W(\vec{r}) - \frac{1}{2} \omega^2 (x^2 + y^2) \quad (\text{A27})$$

we have

$$V(\vec{r}) = -\frac{M_e G}{r} \left\{ 1 + \frac{J_2 a_1^2}{2r^2} (1 - 3 \cos^2 \theta) \right\} - \frac{1}{2} \omega^2 r^2 (1 - \cos^2 \theta) \quad (\text{A28})$$

The geoid is the equipotential at altitude  $h = 0$  or  $r = a_1 (1 - f \sin^2 \phi)$ . The value of the potential is, to sufficient accuracy,

$$V_0 = -\frac{M_e G}{r} \left\{ 1 + \frac{1}{2} J_2 (1 - 3 \cos^2 \theta) \right\} - \frac{1}{2} \omega^2 a_1^2 (1 - \cos^2 \theta) \quad (\text{A29})$$

and this may be solved for  $r$  to give

$$r = -\frac{M_e G}{V_0} \left[ 1 + \frac{1}{2} J_2 (1 - 3 \sin^2 \phi) \right] - \frac{1}{2} \omega^2 a_1^2 (1 - \sin^2 \phi) / V_0 \approx -\frac{M_e G}{V_0} \left[ 1 + \frac{1}{2} J_2 + \frac{\omega^2 a_1^3}{2M_e G} \right] \cdot \left[ 1 - \frac{1}{2} \left( \frac{\omega^2 a_1^3}{M_e G} + 3J_2 \right) \sin^2 \phi \right] \quad (\text{A30})$$

Identifying the above expression for the radius of the geoid as a function of angle with (A25) for  $h = 0$ , we obtain the following approximate expression for the flattening  $f$ :

$$f = \frac{\omega^2 a_1^3}{2M_e G} + \frac{3J_2}{2} \approx \frac{1}{298} \quad (\text{A31})$$

To obtain an expression for the acceleration of gravity, we calculate the expansion of  $V(\vec{r})$  to first order in  $h$ :

$$g = \frac{\partial V}{\partial r} \Big|_{r=a_1(1-f\sin^2\phi)}$$

$$= \frac{M_e G}{a_1^2} + \frac{3M_e G J_2}{2a_1^2} - \omega^2 a_1 + \sin^2 \phi \left\{ \frac{2M_e G f}{a_1^2} - \frac{9M_e G J_2}{2a_1^2} + \omega^2 a_1 \right\} = 9.7803 + 0.0519 \sin^2 \phi \quad \text{m/s}^2 \quad (\text{A32})$$

We thus obtain for  $g_{00}$ :

$$g_{00} = -\left( 1 + \frac{2}{c^2} (V_0 + g(\phi)h) \right) \quad (\text{A33})$$

The term  $2V_0/c^2$  is of order

$$2M_e G/a_1 c^2 \approx 1.4 \times 10^{-9}$$

For a standard clock at rest on the earth at mean sea level (where  $h = 0$ , the geoid in this model) the proper time increment  $ds$  would be roughly

$$ds = \left( 1 - \frac{M_e G}{a_1 c^2} \right) dt \quad (\text{A34})$$

and during a flight of duration 10 hours the  $V_0$  term would produce a correction of order 50,000 ns. This is a large correction, but it is the same for all clocks, since nearly all available standard clocks are placed in fact near the geoid. Therefore if we redefine the rates of all coordinate clocks so as to absorb this term, we may write

$$g_{00} = -(1 + 2g(\phi)h/c^2) \quad (\text{A35})$$

The metric is then to sufficient accuracy

$$-ds^2 = -(1 + 2g(\phi)h/c^2)(dt)^2 + (1 - 2W(\vec{r})/c^2)(dx^2 + dy^2 + dz^2)/c^2 + 2\vec{\omega} \times \vec{r} \cdot d\vec{r} dt/c^2 \quad (\text{A36})$$

It should be noted that the expression  $g(\phi)$  appearing in (A32) and (A35) includes rotation.

With the aid of (A36) we are now in a position to analyze various processes of clock synchronization by means of portable clock transport. Consider first a series of standard clocks at rest at altitudes  $h = 0$  (on the geoid). The proper time elapsed will be given by

$$ds = dt \quad (\text{A37})$$

This shows that by distributing a system of standard clocks about on the geoid at rest all such clocks will beat at the same rate, equal to the rate at which the global coordinate clocks will beat. This result was discussed by *Cocke* [1966a, b, c]. There is,

of course, still the problem of initializing the clocks.

If in (A36) the increment  $d\vec{r}$  is replaced by  $d\vec{r} = \vec{v}dt$ , where  $\vec{v}$  is the velocity of the clock relative to the ground, then on solving for  $dt^2$  and taking a square root we obtain

$$dt = ds \left[ 1 - \frac{g(\phi)h}{c^2} + \frac{1}{2} \frac{v^2}{c^2} + \frac{\vec{\omega} \times \vec{r} \cdot \vec{v}}{c^2} \right] \quad (\text{A38})$$

Therefore on measuring the elapsed proper time  $ds$  on a standard clock and applying the corrections indicated in (A38) the elapsed coordinate time can be calculated. It is this sense in which Figure 1 is intended to illustrate the approach used in this paper.

An alternative form for the third term of (A38) may be written in vector form as

$$\frac{1}{c^2} \int_{\text{path}} \vec{\omega} \cdot \vec{r} \times \vec{v} ds$$

But  $\vec{v}ds = d\vec{r}$ , the change of position, and  $\frac{1}{2}\vec{r} \times d\vec{r}$  is the vector element of area swept out by the radius vector to the clock as it moves on the surface of the earth. Therefore this term can be expressed as

$$\frac{2}{c^2} \vec{\omega} \cdot \vec{A} = 2\omega A_E/c^2 \quad (\text{A39})$$

where  $A_E$  is the projection of the area swept out by the radius vector from the center of the earth to the clock onto the equatorial plane. The area is considered positive if it is traversed in the sense of rotation of the earth [Saburi, 1976].

For propagation of an electromagnetic signal,  $ds^2 = 0$ , and (A36) can be expressed as

$$0 = (dt)^2 \left[ 1 + 2g(\phi)h/c^2 - 2\omega a_1 c_E \cos\phi/c^2 \right] - (1 - 2W(\vec{r})/c^2)(dx^2 + dy^2 + dz^2)/c^2 \quad (\text{A40})$$

where  $c_E$  is the eastward component of velocity of the signal. The term  $W(\vec{r})/c^2$  may, for propagation paths within about 12,000 m of the earth's surface, be incorporated into the measurement of proper distance. The term  $gh/c^2$  is negligible, so solving for  $dt$ ,

$$dt = d\sigma(1 + \omega a_1 c_E \cos\phi/c^2) \quad (\text{A41})$$

where

$$d\sigma^2 = \left( 1 + \frac{2M_e G}{a_1 c^2} \right) (dx^2 + dy^2 + dz^2) \quad (\text{A42})$$

Use of (A41) to calculate coordinate time elapsed along the path of an electromagnetic signal will give results consistent with those obtained by a portable clock using (A38).

#### APPENDIX B: SYNCHRONIZATION BY ELECTROMAGNETIC SIGNALS

In this appendix we derive results discussed in the text in those cases involving the synchronization of clocks by transmission of electromagnetic signals, with possibly very large altitude differences between the locations of the clocks. The earth is regarded as spinning with respect to a local inertial frame with angular velocity  $\omega$ .

##### Synchronization via two-way transponder

Referring to Figure 2, the locations of ground station A when the signals are transmitted and received are  $\vec{r}_A$  (at time  $t_A$ ) and  $\vec{r}_A + \omega\vec{k} \times \vec{r}_A \tau$  (at time  $t_A + \tau$ ), respectively. The location of station B on reception is  $\vec{r}_B$ . The satellite locations on reception of the first and second signals are denoted by  $\vec{r}'_A$  (at time  $t'_A$ ) and  $\vec{r}'_A + \vec{v}(t'_B - t'_A)$  (at time  $t'_B$ ). Here the time interval  $t'_B - t'_A$  is assumed sufficiently small that the velocity may be treated as constant. Since  $t'_B - t'_A$  is typically of order 0.2 s or less and satellite velocities are of order 5 km/s, this should be a very good approximation.

We first define distances  $D_1$  and  $D_2$  as the true distances of the first leg of the transmission from each ground station, viewed from the local inertial frame:

$$D_1 = [(\vec{r}'_A - \vec{r}_A)^2]^{1/2} \quad (\text{B1})$$

$$D_2 = [(\vec{r}'_A + \vec{v}(t'_B - t'_A) - \vec{r}_B)^2]^{1/2} \quad (\text{B2})$$

Then for the time of propagation from A to A' we have

$$c^2(t'_A - t_A)^2 = (\vec{r}'_A - \vec{r}_A)^2$$

thus

$$t'_A = t_A + D_1/c \quad (\text{B3})$$

For the propagation time from A' to B,

$$\begin{aligned} c^2(t_B - t'_A)^2 &= (\vec{r}'_A - \vec{r}_B)^2 \\ &= (\vec{r}'_A + \vec{v}(t'_B - t'_A) - \vec{r}_B - \vec{v}(t'_B - t'_A))^2 \\ &= D_2^2 + 2(\vec{r}_B - \vec{r}'_A) \cdot \vec{v}(t'_B - t'_A) \end{aligned} \quad (\text{B4})$$

where on the right we have expanded in terms of

small corrections to the distance  $D_2$ . On taking the square root of (B4) and carrying out the expansion to only first-order terms in  $\bar{v}$ , the result is

$$t_B = t'_A + D_2/c + \frac{(\bar{r}_B - \bar{r}'_A) \cdot \bar{v}(t'_B - t'_A)}{cD_2} \quad (\text{B5})$$

and combining with (B3),

$$t_B = t_A + (D_1 + D_2)/c + (\bar{r}_B - \bar{r}'_A) \cdot \bar{v}(t'_B - t_A - D_1/c)/cD_2$$

To make use of the above results we need corresponding expressions for the return transmission from B. For the time of propagation from B to B',

$$c^2(t'_B - t_B)^2 = D_2^2$$

Thus

$$t'_B = t_B + D_2/c \quad (\text{B6})$$

The above two equations involving  $t'_B$  and  $t_B$  may be solved to yield to first order

$$t_B = t_A + (D_1 + D_2)/c + 2\bar{v} \cdot (\bar{r}_B - \bar{r}'_A)/c^2 \quad (\text{B7})$$

$$t'_B = t_A + (D_1 + 2D_2)/c + 2\bar{v} \cdot (\bar{r}_B - \bar{r}'_A)/c^2 \quad (\text{B8})$$

For the final leg the propagation time from B' to A is given by

$$c^2(t_A + \tau - t'_B)^2 = (\bar{r}_A + \omega\tau\hat{k} \times \bar{r}_A - \bar{r}'_A - \bar{v}(t'_B - t'_A))^2$$

The right side of the above equation may be expanded in terms of small corrections to the distance  $D_1$ . Then

$$c^2(t_A + \tau - t'_B)^2 = D_1^2 - 2\omega\tau\hat{k} \times \bar{r}_A \cdot \bar{r}'_A - 2\bar{v} \cdot (\bar{r}_A - \bar{r}'_A)(t'_B - t_A)$$

On taking the square root and carrying out the further expansion in small corrections to  $D_1^2$ ,

$$t_A + \tau - t'_B = D_1/c - (\omega\tau\hat{k} \times \bar{r}_A \cdot \bar{r}'_A + \bar{v} \cdot (\bar{r}_A - \bar{r}'_A)(t'_B - t'_A))/cD_1$$

Using the results obtained in (B8) and (B3) for  $t'_B$  and  $t'_A$ , the following expression for  $\tau$ , the total trip time, is obtained:

$$\tau = 2(D_1 + D_2)/c + \frac{2}{c^2} \bar{v} \cdot \left( \bar{r}_B - \bar{r}'_A - \frac{D_2}{D_1} (\bar{r}_A - \bar{r}'_A) \right) - 2\omega(D_1 + D_2)\hat{k} \times \bar{r}_A \cdot \bar{r}'_A/c^2 D_1 \quad (\text{B9})$$

Now to express the coordinate time  $t_B$  of the station at B in terms of the measured time  $\tau$  and the corrections, use (B9) and (B7) and eliminate the unknown distances  $D_1$  and  $D_2$  from the zeroth-order terms, leaving them only in the first-order corrections where approximate values can be used. The result is

$$t_B = t_A + \frac{1}{2}\tau + \bar{v} \cdot \left( \bar{r}_B - \bar{r}'_A + \frac{D_2}{D_1} (\bar{r}_A - \bar{r}'_A) \right) + c^2 + \frac{\omega(D_1 + D_2)}{c^2 D_1} \hat{k} \times \bar{r}_A \cdot \bar{r}'_A \quad (\text{B10})$$

In this form, one can clearly see the effect of motion of the satellite and of the ground stations on the synchronization procedure. If it were not for these corrections, the equation would read  $t_B = t_A + \frac{1}{2}\tau$ , which is what one would expect from the usual Einstein synchronization procedure in an inertial frame.

#### Rate correction for satellite clock

A clock on board an orbiting satellite is subjected to gravitational blue shifts and Doppler shifts due to velocity, with respect to a clock on the ground. To derive the appropriate correction which must be applied to the standard clock in the satellite in order that coordinate time be obtained, we may to sufficient accuracy write the potential of the earth as

$$V = -\frac{GM_e}{r} \left[ 1 - \left( \frac{a_1}{r} \right)^2 J_2 P_2(\cos \theta) \right] \quad (\text{B11})$$

where  $M_e$  is the earth's mass,  $a_1$  is the earth's equatorial radius,  $\theta$  is the colatitude, and the term in  $J_2$  arises from the nonsphericity of the earth. Values of the constants  $GM_e$ ,  $a_1$ , and  $J_2$  are given in Table 2. Also,  $P_2(x) = \frac{1}{2}(3x^2 - 1)$ .

From (A6) the coordinate time  $dt$  elapsed on a standard clock which measures elapsed proper time  $ds$  is then given by

$$ds^2 = dt^2 \left[ 1 + \frac{2V}{c^2} - \frac{v^2}{c^2} \right] \quad (\text{B12})$$

or

$$dt = ds \left[ 1 + \frac{1}{c^2} \left( \frac{1}{2}v^2 - V \right) \right] \quad (\text{B13})$$

By conservation of energy we have

$$\frac{1}{2}v^2 + V = \text{const} = E/m \equiv \epsilon \quad (\text{B14})$$

where  $E/m$  is the classical energy per unit mass of the satellite (which can be expressed in terms of orbital elements). Therefore the expression for the coordinate time rate correction is

$$dt = ds \left[ 1 + \frac{E}{mc^2} - \frac{2V}{c^2} \right] \quad (\text{B15})$$

or

$$\Delta t_{\text{sat}} = \int_{\text{path}} ds \left[ 1 + \frac{E}{mc^2} + \frac{2GM_e}{rc^2} \left( 1 - \left( \frac{a_1}{r} \right)^2 J_2 P_2(\cos \theta) \right) \right] \quad (\text{B16})$$

In comparing the above rate correction with that which is appropriate for a ground-based clock, from (1) we have

$$\Delta t_{\text{gnd}} = \int ds \left[ 1 - \frac{V_0}{c^2} - \frac{g(\phi)h}{c^2} \right] \quad (\text{B17})$$

where  $h$  is the altitude above the geoid of the ground station and

$$V_0 = -\frac{GM_e}{a_1} \left( 1 + \frac{1}{2}J_2 \right) - \frac{1}{2}\omega^2 a_1^2 \quad (\text{B18})$$

is the effective potential on the geoid, including effects due to rotation.

#### Synchronization of satellite clock

Now referring to Figure 4, for the time  $t'_A - t_A$  required for the signal to propagate from the ground station to the satellite we have

$$c^2(t'_A - t_A)^2 = (\bar{r}'_A - \bar{r}_A)^2 \equiv D^2$$

where  $D$  is the distance traversed from the viewpoint of the local inertial frame. Thus

$$t'_A = t_A + D/c \quad (\text{B19})$$

For the return signal the position of the ground station is  $\bar{r}_A + \omega \hat{k} \times \bar{r}_A \tau$ . Hence

$$c^2(t_A + \tau - t'_A)^2 = (\bar{r}'_A - \bar{r}_A - \omega \hat{k} \times \bar{r}_A \tau)^2$$

Since the correction involving  $\tau$  on the right-hand side of the above equation is small, we may expand as follows:

$$\begin{aligned} c^2(t_A + \tau - t'_A)^2 &= D^2 - 2(\bar{r}'_A - \bar{r}_A) \cdot \hat{k} \times \bar{r}_A \omega \tau \\ &= D^2 \left[ 1 - \frac{2\bar{r}'_A \cdot \hat{k} \times \bar{r}_A \omega \tau}{D^2} \right] \end{aligned}$$

Thus taking the square root and keeping only first-order corrections,

$$c(t_A + \tau - t'_A) = D \left[ 1 - \frac{\bar{r}'_A \cdot \hat{k} \times \bar{r}_A \omega \tau}{D^2} \right]$$

Thus

$$t'_A = t_A + \tau - \frac{D}{c} + \frac{\bar{r}'_A \cdot \hat{k} \times \bar{r}_A}{cD} \omega \tau \quad (\text{B20})$$

To a first approximation,  $\tau = 2D/c$ , and this may be substituted into the correction term to obtain

$$t'_A = t_A + \tau - \frac{D}{c} + \frac{2\omega}{c^2} \bar{r}'_A \cdot \hat{k} \times \bar{r}_A$$

Using (B19) to eliminate  $D$ , the result is

$$t'_A = t_A + \frac{1}{2}\tau + (\omega/c^2) \bar{r}'_A \cdot \hat{k} \times \bar{r}_A \quad (\text{B21})$$

In this form the coordinate time  $t'_A$  of arrival of the signal at the satellite clock is expressed in terms of the measured trip time  $\tau$  and a calculated correction which can be several hundred nanoseconds.

#### Synchronization of ground station

The notation is indicated in Figure 6. Letting  $D$  be the proper distance between the initial position  $\bar{r}'_A$  of the satellite and the ground station  $\bar{r}_A$  for the propagation time of the signal from  $A'$  to  $A$  we have

$$c^2(t_A - t'_A)^2 = D^2$$

or

$$t_A = t'_A + D/c \quad (\text{B22})$$

Then during the propagation of the signal to the ground station and back to the satellite the satellite will have moved a distance  $\bar{v}\tau$ , where  $\bar{v}$  is the velocity. Thus

$$\begin{aligned} c^2(t'_A + \tau - t_A)^2 &= (\bar{r}'_A + \bar{v}\tau - \bar{r}_A)^2 \\ &\equiv D^2 + 2\tau \bar{v} \cdot (\bar{r}'_A - \bar{r}_A) \end{aligned}$$

On taking square roots and carrying out the expansion in small quantities, we obtain

$$t'_A + \tau - t_A = \frac{D}{c} + \frac{\tau}{cD} \bar{v} \cdot (\bar{r}'_A - \bar{r}_A) \quad (\text{B23})$$

Using (B22) and (B23) to eliminate  $D$ , the result may be expressed as

$$t_A = t'_A + \frac{1}{2}\tau - \bar{v} \cdot (\bar{r}'_A - \bar{r}_A)/c^2 \quad (\text{B24})$$

*Lasso-type experiment (derivation of (22))*

The notation is explained in Figure 8. The time elapsed between receptions of the signals at the satellite is here denoted by

$$\tau' = t'_B - t'_A \quad (\text{B25})$$

For the first signals from A to the satellite and back, (17) gives

$$t'_A = t_A + \frac{1}{2}\tau_A + \omega \bar{r}'_A \cdot \hat{k} \times \bar{r}_A / c^2 \quad (\text{B26})$$

and for the second set of signals, correspondingly,

$$t'_B = t_B + \frac{1}{2}\tau_B + \omega \bar{r}'_B \cdot \hat{k} \times \bar{r}_B / c^2 \quad (\text{B27})$$

Using the above equations to eliminate  $t'_B - t'_A$  and express  $t_B$  in terms of  $t_A$ , we have

$$\begin{aligned} t_B &= t'_B - \frac{1}{2}\tau_B - \omega \bar{r}'_B \cdot \hat{k} \times \bar{r}_B / c^2 \\ &= t'_A + \tau' - \frac{1}{2}\tau_B - \omega \bar{r}'_B \cdot \hat{k} \times \bar{r}_B / c^2 \\ &= t_A + \frac{1}{2}(\tau_A - \tau_B) + \tau' \\ &\quad + \omega(\bar{r}'_A \cdot \hat{k} \times \bar{r}_A - \bar{r}'_B \cdot \hat{k} \times \bar{r}_B) / c^2 \end{aligned} \quad (\text{B28})$$

This is (22) of the main text.

*Flyover (derivation of (23))*

For the initial synchronization of a clock in a satellite or jet in which the satellite clock is set we have from (17)

$$t'_A = t_A + \frac{1}{2}\tau_A + \omega \bar{r}'_A \cdot \hat{k} \times \bar{r}_A / c^2 \quad (\text{B29})$$

For the subsequent synchronization of the ground station B starting with a signal from the satellite at time  $t'_B$ , (19) gives

$$t_B = t'_B + \frac{1}{2}\tau'_B - \bar{v} \cdot (\bar{r}'_B - \bar{r}_B) / c^2$$

Letting

$$\tau' = t'_B - t'_A \quad (\text{B30})$$

we can then calculate  $t_B$  in terms of  $t_A$ . First eliminating  $t'_B$  in favor of  $\tau'$ ,

$$t_B = \tau' + t'_A + \frac{1}{2}\tau'_B - \bar{v} \cdot (\bar{r}'_B - \bar{r}_B) / c^2 \quad (\text{B31})$$

then using (B25) for  $t'_A$  gives

$$\begin{aligned} t_B &= t_A + \frac{1}{2}(\tau_A + \tau'_B) + \tau' \\ &\quad + \omega \bar{r}'_A \cdot \hat{k} \times \bar{r}_A - \bar{v} \cdot (\bar{r}'_B - \bar{r}_B) / c^2 \end{aligned} \quad (\text{B32})$$

where  $\bar{v}$  is the velocity of the satellite at  $\bar{r}'_B$ .

*Derivation of (24)*

The process is diagramed in Figure 10. For the initial synchronization, using the signal from A'

to A, (19) gives

$$t_A = t'_A + \frac{1}{2}\tau'_A - \bar{v} \cdot (\bar{r}'_A - \bar{r}_A) / c^2 \quad (\text{B33})$$

where  $\bar{v}$  is the initial satellite velocity. Then for the second part of the transfer, (17) gives

$$t'_B = t_B + \frac{1}{2}\tau_B + \omega \bar{r}'_B \cdot \hat{k} \times \bar{r}_B / c^2 \quad (\text{B34})$$

Also, defining  $\tau' = t'_B - t'_A$ , we can solve for  $t_B$  in terms of  $t_A$  as follows:

$$\begin{aligned} t_B &= t'_B - \frac{1}{2}\tau_B - \omega \bar{r}'_B \cdot \hat{k} \times \bar{r}_B / c^2 \\ &= t'_A + \tau' - \frac{1}{2}\tau_B - \omega \bar{r}'_B \cdot \hat{k} \times \bar{r}_B / c^2 \\ &= t_A + \tau' - \frac{1}{2}(\tau'_A + \tau_B) + \bar{v} \cdot (\bar{r}'_A - \bar{r}_A) / c^2 \\ &\quad - \omega \bar{r}'_B \cdot \hat{k} \times \bar{r}_B / c^2 \end{aligned} \quad (\text{B35})$$

which is (24) of the main text.

*Derivation of (25)*

Referring to Figure 11 and the accompanying text where the notation is explained, (19) may be applied twice to obtain

$$t_A = t'_A + \frac{1}{2}\tau'_A - \bar{v}'_A \cdot (\bar{r}'_A - \bar{r}_A) / c^2 \quad (\text{B36})$$

$$t_B = t'_B + \frac{1}{2}\tau'_B - \bar{v}'_B \cdot (\bar{r}'_B - \bar{r}_B) / c^2 \quad (\text{B37})$$

where  $\bar{v}'_A$  and  $\bar{v}'_B$  are the satellite velocities at the instants of transmission  $t'_A$  and  $t'_B$ , respectively. Then putting

$$\tau' = t'_B - t'_A \quad (\text{B38})$$

it follows immediately that

$$\begin{aligned} t_B &= t_A + \tau' + \frac{1}{2}(\tau'_B - \tau'_A) \\ &\quad + (\bar{v}'_B \cdot (\bar{r}'_B - \bar{r}_B) - \bar{v}'_A \cdot (\bar{r}'_A - \bar{r}_A)) / c^2 \end{aligned} \quad (\text{B39})$$

*One-way transmissions (derivation of (26))*

From Figure 12 the propagation delay  $\tau'$  which would occur if the earth were not rotating is given by

$$c^2 \tau'^2 = (\bar{r}_A - \bar{r}'_A)^2$$

The true propagation delay, however, will be different owing to the motion  $\omega \tau \hat{k} \times \bar{r}_A$  of the ground station during the time  $\tau$ . Thus

$$\begin{aligned} c^2 \tau^2 &= (\bar{r}_A + \omega \tau \hat{k} \times \bar{r}_A - \bar{r}'_A)^2 \\ &= (\bar{r}_A - \bar{r}'_A)^2 - 2\omega \tau \bar{r}'_A \cdot \hat{k} \times \bar{r}_A \end{aligned}$$

Thus taking square roots and expanding,

$$c\tau = c\tau' (1 - \omega \tau \bar{r}'_A \cdot \hat{k} \times \bar{r}_A / (c\tau')^2)$$

and thus the first-order correction is

$$\tau = \tau' - \omega \vec{r}'_{\Lambda} \cdot \hat{k} \times \vec{r}_{\Lambda} / c^2 \quad (\text{B40})$$

## REFERENCES

- Anderle, R. J. (1978), Geodetic applications of the Navstar global positioning system, paper presented at Second International Symposium on Problems Related to Redefinition of North American Geodetic Networks, April 22-24, Washington, D.C.
- Ashby, N. (1975), An earth-based coordinate clock network, *Nat. Bur. Stand. Tech. Note*, 659, 34 pp.
- Besson, J. (1970), Comparison of national time standards by simple overflight, *IEEE Trans. Instrum. Meas.*, IM-19(4), 227-232.
- Buisson, J. A., R. L. Easton, and T. B. McCaskill (1977), Initial results of the Navstar GPS NTS-2 satellite, *Proceedings of the 9th Annual Precise Time and Time Interval Applications and Planning Meeting*, pp. 177-200, Technical Information and Administrative Support Division, Goddard Space Flight Center, Greenbelt, Md.
- Caputo, M. (1967), *The Gravity Field of the Earth*, 202 pp., Academic, New York.
- Carter, W. E., D. S. Robertson, and Michael D. Abell (1978), An improved polar motion and earth rotation monitoring service using radio interferometry, in *Proceedings of International Astronomical Union Symposium No. 82*, in press, International Astronomical Union, London.
- Cocke, W. J. (1966a), Relativistic effects for terrestrial clock synchronization, *Phys. Rev. Lett.*, 16(15), 662-664.
- Cocke, W. J. (1966b), Erratum, *Phys. Rev. Lett.*, 16(17), 779.
- Cocke, W. J. (1966c), Erratum, *Phys. Rev. Lett.*, 16(26), 1233.
- Fanselow, J. L. (1977), VLBI and its current applications within the solar system, *Proceedings of the 9th Annual Precise Time and Time Interval Applications and Planning Meeting*, pp. 85-96, Technical Information and Administrative Support Division, Goddard Space Flight Center, Greenbelt, Md.
- Garland, G. D. (1965), *The Earth's Shape and Gravity*, 420 pp., Pergamon, New York.
- Hafele, J. C., and R. E. Keating (1972a), Around-the-world atomic clocks: Predicted relativistic time gains, *Science*, 177(4044), 166-168.
- Hafele, J. C., and R. E. Keating (1972b), Around-the-world atomic clocks: Observed relativistic time gains, *Science* 177(4044), 168-170.
- Hoffman, B. (1961), Noon-midnight red shift, *Phys. Rev.*, 121(1), 337-342.
- Lavanceau, J. D., and L. F. Shepard (1977), Real time distribution via passive TV networks, *Proceedings of the 9th Annual Precise Time and Time Interval Applications and Planning Meeting*, pp. 249-276, Technical Information and Administrative Support Division, Goddard Space Flight Center, Greenbelt, Md.
- MacDoran, P. F. (1978), Series GPS geodetic system, *Eos Trans. AGU*, 59, 1052.
- Manasse, F. K., and C. W. Misner (1963), Fermi normal coordinates and some basic concepts of differential geometry, *J. Math. Phys.*, 4(6), 735-745.
- Moyer, T. D. (1976), Transformation from proper time on earth to coordinate time in solar system barycentric space-time frame of reference, *Tech. Memo. 33-786*, 66 pp., Jet Propul. Lab., Calif. Inst. of Technol., Pasadena.
- Rapp, R. H. (1974), Current estimates of mean earth ellipsoid parameters, *Geophys. Res. Lett.*, 1(1), 35-38.
- Reinhardt, V. S., D. A. Premo, M. W. Fitzmaurice, S. C. Waldrip, and P. O. Arvenka (1977), Nanosecond time transfer via shuttle laser ranging experiment, *Proceedings of the 9th Annual Precise Time and Time Interval Applications and Planning Meeting*, pp. 319-342, Technical Information and Administrative Support Division, Goddard Space Flight Center, Greenbelt, Md.
- Reisse, R. A. (1976), The effect of gravitational potential on atomic clocks as observed with a laser pulse time transfer system, Ph.D. thesis, 215 pp., Univ. of Md., College Park.
- Robertson, D. S., W. E. Carter, B. E. Corey, C. C. Counselman III, I. I. Shapiro, J. J. Wittels, H. F. Hinteregger, C. A. Knight, A. E. E. Rogers, J. W. Ryan, T. A. Clark, R. J. Coates, C. Ma, and J. M. Moran (1978), Recent results of radio interferometric determinations of polar motion and earth rotation, in *Proceedings of International Astronomical Union Symposium No. 82*, in press, International Astronomical Union, London.
- Rogers, A. E. E., C. A. Knight, H. F. Hinteregger, A. R. Whitney, C. C. Counselman III, I. I. Shapiro, S. A. Gourevitch, and T. A. Clark (1978), Geodesy by radio interferometry: Determination of a 1.24-km baseline vector with ~5 mm repeatability, *J. Geophys. Res.*, 83(B1), 325-334.
- Rueger, L., and A. Bates (1978), Nova satellite experiment, in *Proceedings of the Nineteenth General Assembly of URSI, Union Radio Scientifique International*, Helsinki, Finland.
- Rutman, J. (1978), Application of space techniques to time and frequency dissemination and synchronization, *Proceedings of the European Workshop on Space Oceanography, Navigation, and Geodynamics, ESASP-137*, pp. 311-316, Eur. Space Agency, Neuilly, France.
- Saburi, Y. (1976), Observed time discontinuity of clock synchronization in rotating frame of the earth, *J. Radio Res. Labs.*, 23(12), 255-265.
- Schilizzi, R. T., and J. Campbell (1978), Very long baseline interferometry in geophysics, Geodesy and Astronomy, *Proceedings of the European Workshop on Space Oceanography, Navigation, and Geodynamics, ESASP-137*, pp. 329-337, Eur. Space Agency, Neuilly, France.
- Schuchman, L., and J. Spiker (1977), Time transfer via GPS, *Proceedings of the 9th Annual Precise Time and Time Interval Applications and Planning Meeting*, pp. 167-168, Technical Information and Administrative Support Division, Goddard Space Flight Center, Greenbelt, Md.
- Thomas, J. B. (1975), Reformulation of the relativistic conversion between coordinate time and atomic time, *Astron. J.*, 80(5), 405-411.
- Vessot, R. F. C., M. W. Levine, E. M. Mattison, T. E. Hoffman, E. A. Imbier, M. Tetu, G. Nystrom, J. J. Kelt Jr., H. F. Trucks, and J. L. Vaniman (1976), Space-borne hydrogen maser design, *Proceedings of the 8th Annual Precise Time and Time Interval Applications and Planning Meeting*, pp. 227-334, Technical Information and Administrative Support Division, Goddard Space Flight Center, Greenbelt, Md.
- Weber, J. (1961), *General Relativity and Gravitational Waves*, 200 pp., Interscience, New York.
- Williams, E. R. (1976), A direct measurement of the relativistic effect of the gravitational potential on the rates of atomic clocks flown in aircraft, 174 pp., Ph.D. thesis, Univ. of Md., College Park.