Laser-to-microwave frequency division using synchrotron radiation

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Calculations are made to demonstrate the feasibility of obtaining one-step frequency division from optical or infrared laser frequencies to a subharmonic in the microwave spectral region. The cyclotron orbit of a single relativistic electron in a Penning trap is driven with a Gaussian laser beam focused to a spot diameter $\sim\lambda$; the laser subharmonic frequency is measured from the electron synchrotron radiation. The uncertainty in orbit dimensions is limited to $\lambda/2$ by radiative cooling and the technique of motional sideband excitation.

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I. INTRODUCTION

The need for precise frequency measurement in the infrared and optical regions is well known.[1] Present successful techniques[1] use the method of harmonic mixing to intercompare optical and infrared laser frequencies and to compare these laser frequencies with the more conventional microwave frequency standards. It would of course be desireable to simplify this process as much as possible; for example, if the most stable and accurate frequency sources are eventually realized in the infrared or optical regions, one needs to translate the frequency to very low values to obtain precise and accurate timing. With this in mind, it is suggested that frequency translation from the infrared or optical region to the microwave region might be accomplished by a single device which is a type of frequency divider as described below.

II. PRINCIPLE OF FREQUENCY DIVISION

The more conventional techniques of frequency multiplication or harmonic mixing use the harmonic generation and mixing properties resulting from a nonlinear device (e.g., crystal with nonlinear response for harmonic generation) which is driven by one or more linear excitations.[1] It is noted, however, that harmonic response can also be obtained by driving a purely harmonic oscillator by a nonlinear (spatially inhomogeneous) field. This technique has been used in high-energy particle accelerators for many years.[2] For example, particles in synchrotrons are sometimes driven by radiation from microwave cavities (localized to a small portion of the cyclotron orbit) at frequencies which are higher harmonics of the cyclotron orbit frequency. It is suggested then, to carry this technique to its practical limit; that is, one can drive the cyclotron orbit of a charged particle (electron) in a magnetic field at a very high harmonic of the cyclotron frequency. The power supplied by the harmonic excitation is balanced by the synchrotron radiation. If the orbit is stable, then the cyclotron frequency is an exact submultiple of the driving frequency and can be measured from the emitted synchrotron or cyclotron radiation.

To illustrate this technique, consider the example of Fig. 1. Radiation from a collimated Gaussian laser beam (polarized in the x direction) travels in the $-z$ direction and is focused to a spot diameter S_0 (beam-waist diameter at focal point). The center of the beam waist is made to coincide with the orbit path of a single electron confined to the x,y plane (see Sec. III). As the electron passes through this region it experiences an electric field whose amplitude in the x direction can approximately[3] be given by

$$E_x = \phi\left(\frac{2}{\pi}\right)^{1/2}\frac{2}{S_0}\exp\left(-\frac{4(x^2+y^2)}{S_0^2}\right)\cos(\omega_l t+\delta), \qquad (1)$$

where $\omega_l = 2\pi c/\lambda = 2\pi\nu_l$ is the laser angular frequency, t is the time (where $t=0$ coincides with the electron crossing the y axis), δ is the phase factor, and ϕ is the amplitude factor (in volts). Assuming the change in energy (ΔW) for one pass through the laser field is small compared to the total electron energy and that the spot size is small compared to the orbit diameter, we have

$$\Delta W \cong \int_{-\infty}^{\infty} eE_x\,dx = \sqrt{2}e\phi\cos\delta\exp\left[-\left(\frac{\pi}{2}\frac{c}{v}\frac{S_0}{\lambda}\right)^2\right], \qquad (2)$$

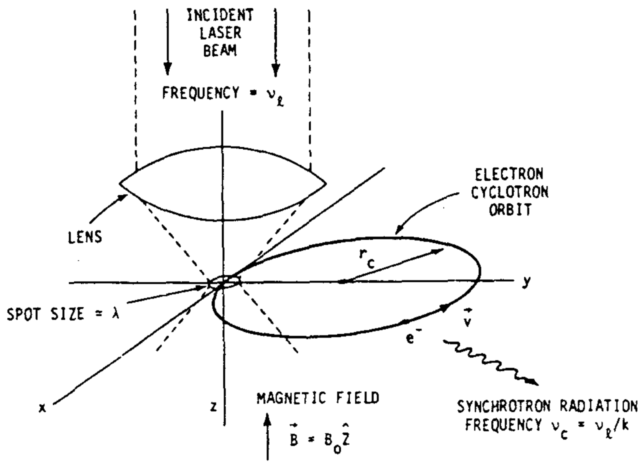

FIG. 1. Schematic depicting the principle of the synchrotron frequency divider. The electron-cyclotron orbit is stabilized by balancing the synchrotron radiation (frequency ν_c) with power supplied by the focused laser beam (frequency ν_l). For the phase-lock condition, the measured cyclotron frequency is an exact submultiple (k) of the laser frequency.

where the substitution $t = x/v$ (v is the electron velocity) has been used. We can write the incident laser power in the form

$$P_l = \frac{cv_l}{4\pi}\int_A\int_{t=0}^{1/v_l} E_x^2\, dA\, dt = \frac{c}{8\pi}\phi^2.$$

If $P_l = 1$ mW, then $\phi = 0.87$ V, and for $v = 0.8c$ and $S_0 = \lambda$, $\Delta W = 4.1\times 10^{-21}$ ($\cos\delta$) J/pass. If the cyclotron frequency $\nu_c = 50$ GHz ($r_c = 0.076$ cm), then the power (P_e) absorbed by the electron is $P_e = 2\times 10^{-10}(\cos\delta)$ W. In steady state, this same power is radiated by the electron, and it is assumed that a substantial fraction can be coupled out at frequency ν_c. Note in Eq. (2) that ΔW depends critically on the ratio $cS_0/(v\lambda)$; this expresses the fact that ΔW is small unless the electron experiences a nearly constant phase of the laser field as it passes through the interaction region. If the extent of this interaction region could be made substantially less than λ (for example with some kind of cavity), then the speed of the electron could be made correspondingly less.

III. ELECTRON CONFINEMENT

The electron-cyclotron orbit can be provided by a static magnetic field $\mathbf{B} = B_0\hat{z}$ ($2\pi\nu_c = eB_0/mc$, m is the electron mass). However, deviations from the circular electron orbit path must be $\lesssim \frac{1}{4}\lambda$, or ΔW averages to zero after several passes. This can be accomplished by confining the electron in a Penning trap with hyperbolic electrodes.[4] The electric potential inside such a trap is given by $\psi = V_0[r^2 - 2(z^2 - Z_0^2)]/(R_0^2 + 2Z_0^2)$, where r is the radial coordinate, R_0 and Z_0 are the characteristic dimensions of the trap,[4] and V_0 is the voltage applied between the trap electrodes. The axially symmetric trap is formed with two "endcap" electrodes which conform to $\psi = 0$ equipotentials and a "ring" electrode which conforms to the equipotential $\psi = V_0$. In the nonrelativistic limit, the motion is comprised of purely harmonic motion along the axial (z) direction at frequency ν_z and the motion in the xy plane is described by the sum of the two vectors $\mathbf{r}_c$ and $\mathbf{r}_m$, which rotate at frequencies ν_c' and ν_m. These frequencies are given by $\nu_z = [eV_0/m(R_0^2 + 2Z_0^2)]^{1/2}/\pi$, and ν_c' and ν_m are solutions of the equation $\nu^2 - \nu_c\nu + \frac{1}{2}\nu_z^2 = 0$. When $\nu_c \gg \nu_z$, then $\nu_m \simeq \nu_z^2/2\nu_c$ and $\nu_c' \simeq \nu_c$.

To include relativistic effects, consider only the case where $|\mathbf{r}_m|$ and $z \lesssim \frac{1}{4}\lambda$, which can be accomplished by radiatively cooling[4] the axial (z) motion and suppressing the magnetron motion by the techinque of motional sideband excitation.[5,6] The electron orbit is then nearly circular with frequency ν_c'; however, the above expressions must take into account that the electron mass is the relativistic mass $m = \gamma m_0$, where m_0 is the electron rest mass and $\gamma = [1 - (v/c)^2]^{-1/2}$. Note that this special solution is possible when $|\mathbf{r}_m|$ and $z \lesssim \frac{1}{4}\lambda$ only because the electron velocity (and therefore γ) are nearly constant.

Treating the problem quantum mechanically, one can show that the amplitudes of the axial and magnetron motion are given by [7]

$$z_n = [\hbar(n + \tfrac{1}{2})/\pi\nu_z\gamma m_0]^{1/2}$$

and

$$|\mathbf{r}_m| = r_q = [\hbar(q + \tfrac{1}{2})/\pi(\nu_c - 2\nu_m)\gamma m_0]^{1/2},$$

where n and q are the axial and magnetron-oscillation quantum numbers, respectively. Expressed in terms of temperature (T_z) we have (when $h\nu_z \ll kT_z$)

$$h\nu_z(n + \tfrac{1}{2}) = kT_z = m\omega_z^2\langle z^2\rangle.$$

The axial motion can be thermalized to the ambient temperature, which could be that of liquid He. The magnetron motion could be "cooled" by the technique of motional sideband excitation.[5,6] Treating the problem classically (see the Appendix) the magnetron radius can be shrunk to a value

$$r_m^2 \cong 2(\omega_m/\omega_z)\langle z^2\rangle,$$

where $\langle z^2\rangle$ is the mean-square thermally excited axial amplitude. Assuming $R_0 = 1.6Z_0 = 0.1$ cm, $V_0 = 10$ kV, $v/c = 0.8$, $S_0 = \lambda$, $\nu_c = 50$ GHz, and $T_z = 4$ K, we have $\nu_z = 7.8$ GHz, $n \cong 10.3$, $z_{10.3} = 175$ nm, $r_m \cong 50$ nm, and $B_0 \cong 3.0$ T. Hence the path of the orbit can be confined to $< \frac{1}{4}\lambda$ provided $\lambda \gtrsim 700$ nm in this example.

IV. CYCLOTRON ORBIT STABILITY

Under the proper conditions, the cyclotron orbit will phase lock to a subharmonic of the laser frequency. This is generally true if the condition $\frac{1}{2}\pi > \delta > 0$ is satisfied and if the laser has rather modest frequency and amplitude stability. Consider that $\delta = \frac{1}{4}\pi$ initially. Then $P_e \cong 1.4\times 10^{-10}$ W, and this same power must be dissipated by the synchrotron radiation. The power radiated by an electron orbiting in a magnetic field in free space is[2]

$$P_f = \frac{4\pi\nu_c}{3}\frac{e^2}{r_c}\left(\frac{v}{c}\right)^3\gamma^4.$$

For the example conditions discussed above, $P_f = 2.5\times 10^{-13}$ W. To achieve the necessary increase in damping rate, it would be necessary to split the ring electrode along a plane containing the z axis and couple the two halves to a microstrip which is then made to resonate at frequency ν_c' ($Q \simeq 500$). This circuit would then be coupled into frequency-measurement electronics to determine ν_c'. One could alternatively reduce the laser power but it is desirable to couple the maximum power out in order to maximize the signal-to-noise ratio.

Stability is achieved because if the energy of the electron increases slightly, due for example to increased laser power or reduction in Q of the microwave resonator, then the cyclotron frequency of the electron decreases slightly (due to the increase in relativistic mass) and it arrives at the laser spot experiencing a slightly advanced phase on the next pass, i.e., δ in Eq. (1) increases slightly. From Eq. (2) this implies ΔW decreases slightly and therefore the electron energy decreases. In this sense the electron-cyclotron frequency phase locks to the laser frequency. To be more specific, suppose that the division factor $\nu_l/\nu_c' = k$. (In the above example if $\lambda = 600$ nm, $k = 10\,000$.) We now consider the response of the electron cyclotron frequency to various instabilities in the system.

A. Laser frequency stability requirements

An obvious upper limit for the phase fluctuations in the laser is given by the condition that $|\delta| \lesssim \frac{1}{4}\pi$ in a time less than the cyclotron orbit time. A stronger limit is, however, given by the condition that the cyclotron frequency be able to track the laser frequency as it jitters or drifts. An approximate limit on how fast the cyclotron frequency can slew is given by the rate at which the cyclotron energy decays. (This is also approximately equal to the maximum rate at which the energy can increase, which is the condition when $\delta \to 0$.) In a more thorough treatment of the problem one must consider the effect of stimulated emission.[8] This would actually create a stronger tendency to lock as the cyclotron frequency became less; for simplicity we neglect stimulated emission here. We have

$$\frac{d\nu_c'}{dt} = -\frac{\nu_c'}{E}\frac{dE}{dt} \simeq -\frac{\nu_c'}{E}P_e,$$

where E is the total relativistic electron energy. In the phase-lock condition, $\nu_l = k\nu_c'$; therefore to maintain phase lock, the maximum rate of frequency change that can be tolerated by the laser is

$$\left|\frac{d\nu_l}{dt}\right|_{\max} = k\frac{\nu_c'}{E}P_e,$$

which equals 5×10^{17} Hz/sec for the example parameters chosen above. Stabilized lasers[9] have shown frequency slew rates as small as 50–500 kHz/sec, and therefore this condition is satisfied.

B. Laser amplitude stability requirements

Similarly, rather modest amplitude stability requirements are imposed on the laser. When the laser input power is "matched" to the extracted power, ($\delta \simeq \frac{1}{4}\pi$) then phase lock is maintained except for extremely large amplitude fluctuations in a time $< 1/\nu_c'$. Over longer times, if the laser power decreases by more than a factor of 2 from the matched condition [$\phi < \sqrt{2}\phi(\text{matched})$], then ΔW becomes too small to maintain phase lock.

In the phase-locked condition, we note that amplitude changes cause phase fluctuations given by the condition that ΔW must remain constant. From Eq. (2), we note that changes in ϕ must therefore be accompanied by changes in δ. This implies that the change in phase at the output frequency (ν_c') is equal to $\Delta\delta/k$.

C. Magnetic field stability requirements

If the magnetic field strength changes, then phase lock will still be maintained since the cyclotron radius (r_c) will change to keep ν_c' constant. This is true to the extent that the cyclotron orbit path continues to pass through the laser spot. From the expression $\omega_c = eB/(\gamma m_0 c)$ we find that for ν_c' to remain constant we have

$$\frac{dr_c}{r_c} = \left(\frac{v}{c}\gamma\right)^{-2}\frac{dB_0}{B_0}.$$

If we require $dr_c < 0.1\lambda$, then for the example discussed above we must have $dB_0/B_0 < 7.4\times10^{-5}$, which is easily satisfied. The limit on how fast B_0 can change is given by how fast the energy can change for constant ω_c'. We have

$$\frac{1}{B_0}\frac{dB_0}{dt} = \frac{1}{E}\frac{dE}{dt} \cong \frac{1}{E}P_e.$$

For the example conditions, the maximum fractional rate of change of B_0 is equal to 10^3/sec, which is easily satisfied. Similar stability requirements are imposed on the trap electric potentials, but the requirements are relaxed by the ratio $\sim\nu_m/\nu_c$.

D. Determination of k

The division factor k may be determined two ways: (1) by a prior knowledge of ν_l to an accuracy of better than $1/k$ and a measurement of ν_c', or (2) a measurement of ν_c' for two different values of k. It is interesting to ask if a given geometrical arrangement can support different values of k. This is true if the cyclotron orbit passes through the laser spot for different k values. From the expression $\omega_c' = v/r_c$, it follows that

$$\frac{dr_c}{r_c} = -\frac{d\omega_c'}{\omega_c'}\left(\frac{c}{v}\right)^2.$$

In the above example, for k to change by 1, $d\omega_c'/\omega_c' = 10^{-4}$ and we have $dr_c = 119$ nm $< S_0$. Therefore the same geometrical configuration will support division by a few values of k.

E. Radiation pressure

In addition to the force on the electron along the x direction as it passes through the laser field, there are forces along the other directions as well. For example, the distortion of the laser fields for tight focusing[3] causes a force along the z direction. A larger force in the z direction is due to radiation pressure; this is the force due to the $\mathbf{v}\times\mathbf{B}$ term experienced by the electron passing through the laser spot. The corresponding energy gained per pass, ΔW_z, is much smaller than ΔW because the electron axial velocity is extremely small. For the example above, $\Delta W_z \ll \hbar\omega_z$; moreover, this force is periodic with frequency ν_c', not ν_z, and therefore its effect is negligible.

F. Background gas pressure

Clearly, any collisions with background gas will cause the electron to either be scattered out of the trap or, for example, receive sufficient axial energy that ΔW averages to zero over many passes and phase lock is lost. In an apparatus at liquid-helium temperatures, pressures of $\lesssim 1.3\times10^{-11}$ Pa (10^{-13} Torr) should be obtainable, and the primary background gas constituent should be He. For $v/c = 0.8$, the electron-He total scattering cross section (σ) is estimated to be[10] 2.4×10^{-19} cm², yielding a total "destructive" collision rate (R) of

$$R = n(\text{He})v\sigma \lesssim 2.2\times10^{-5}/\text{sec},$$

where n(He) is the He density. Therefore, continuous operation for a time $\gtrsim 4\times10^4$ sec could be expected, which is long enough to complete a precise frequency measurement.

V. SCENARIO OF OPERATION

A single electron could be provided in the Penning trap by using established procedures.[4,5] The relativistic cyclotron orbit could be initialized by using an external drive (across the split ring electrodes) at frequency ν_c'. Note that this external drive frequency must be swept down as the electron energy increases due to the relativistic mass shift; for the example above $\nu_c'(\text{final}) = 0.6\nu_c'(\text{initial})$. With microwave electric field strengths of 1V/cm at the cyclotron orbit, this "run-up" time can be less than 1 msec. With the external cyclotron drive held at $\nu_c'(\text{final})$, the inhomogeneous excitation at frequency $\nu_z + \nu_m$ must then be applied in order to "freeze out" the magnetron oscillations. The laser can then be turned on and the ν_c' drive turned off and phase locking should occur. Note that the inhomogeneous excitation at $\nu_z + \nu_m$ must be continuously applied or the magnetron motion will be excited by the background thermal radiation. The focused laser beam could be projected through the trap if, for example, the endcaps are made of mesh with hole size $\gg\lambda$.

VI. OPERATION AT LONGER LASER WAVELENGTHS

Experimental conditions (traps size, etc.) are considerably relaxed by using lower laser frequencies and therefore reducing confinement constraints. For example, if $\nu_c = 25$ GHz, $R_0 = 1.6Z_0 = 0.2$ cm, $V_0 = 500$ V, $v/c = 0.8$, $S_0 = \lambda$, and $T_z = 4$ K, then $\nu_z = 866$ MHz, $n = 96$, $z_{96} = 1.57\ \mu$m, and $r_m = 200$ nm, which is suitable for CO_2 laser wavelengths (10 μm).

VII. DISCUSSION

The use of a type of frequency divider as discussed here potentially has important advantages over presently used techniques. First, the divider could replace in a single device the elements in a frequency synthesis chain which may use several multipliers and intermediate oscillators. It should be able to divide over a continuous range of laser frequencies being limited only by the frequency sensitivity of the focusing lens. Also, we note that in frequency multipliers of order k, the noise spectral density increases as k^2 relative to the carrier. For high-order multiplication this may mean that the carrier becomes completely lost in the noise unless an oscillator with very high spectral purity is used as the input oscillator. With a type of frequency divider as discussed here, this problem does not occur. The noise spectral density decreases relative to the carrier, being limited by the added instrumental noise in the ν_c' detection electronics.

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APPENDIX

Doppler-effect-generated optical sideband cooling[5,11] and magnetron cooling[5,6] are formally identical, but for the sake of conciseness only the specific case of magnetron cooling is examined. Briefly, the technique works as follows: To reduce the magnetron oscillation amplitude, the axial motion can be driven by an inhomogeneous rf field at the sideband frequency $\nu_z + \nu_m$; however, the axial motion predominantly reradiates at frequency ν_z. The energy difference per scattered microwave photon $h[\nu_z - (\nu_z + \nu_m)] = -h\nu_m$ must come from the magnetron energy; therefore, the oscillation amplitude decreases. Qualitatively, the cooling is limited because the magnetron motion is excited by the noise generated from the thermal axial motion via the inhomogeneous rf field.[12] This noise has the same spectral shape as that of the axial oscillation, but is centered around frequency ν_m.

To estimate the cooling limit classically, assume that the electron in the Penning trap is subjected to an inhomogeneous rf field

$$\mathbf{E}(t) = \mathbf{E}(\mathbf{r}) \cos(\omega_z + \omega_m)t, \tag{A1}$$

where

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 + \hat{i}\left(\frac{\partial E_x}{\partial x}x + \frac{\partial E_x}{\partial z}z\right) + \hat{k}\left(\frac{\partial E_z}{\partial x}x + \frac{\partial E_z}{\partial z}z\right),$$

and the x coordinate of the electron motion can be written

$$x = r_c \cos(\omega_c' t + \phi_c) + r_m \cos(\omega_m t + \phi). \tag{A2}$$

Substituting Eq. (A2) into (A1) we find that the axial motion is driven by a resonant rf electric field

$$E_z(t) = \frac{\partial E_z}{\partial x}\frac{r_m}{2}\cos(\omega_z t - \phi_m),$$

and the driven axial resonance has the form[13]

$$z_d(t) = b_z \cos(\omega_z t - \phi_m - \tfrac{1}{2}\pi), \tag{A3}$$

where $b_z = eE_z\tau_z/(2m\omega_z)$, and τ_z is the amplitude-damping time constant. The power delivered to the external damping resistance is[13]

$$\frac{dE_z}{dt} = \frac{mb_z^2\omega_z^2}{\tau_z} = \frac{(eE_z)^2\tau_z}{(4m)} = \left(e\frac{\partial E_z}{\partial x}r_m\right)^2 \tau_z(16m)^{-1}.$$

The corresponding rate of magnetron energy change is

$$\frac{dE_m}{dt} = \frac{\omega_m}{\omega_z}\frac{dE_z}{dt} = \frac{\omega_m}{\omega_z}\left(e\frac{\partial E_z}{\partial x}r_m\right)^2 \tau_z(16m)^{-1}. \tag{A4}$$

Note that for the magnetron motion, r_m becomes smaller as E_m increases since most of the magnetron energy is electric field potential energy.

The axial motion can be described by

$$z(t) = z_d(t) + z_n, \tag{A5}$$

where $z_d(t)$ is given by Eq. (A3) and z_n is the thermally excited noise oscillation. Substituting Eq. (A5) into Eq. (A1) we find that the magnetron motion is driven by the rf electric field

$$E_x(t) = \frac{\partial E_x}{\partial z}\,(z_d(t) + z_n)\cos(\omega_z + \omega_m)\,t. \tag{A6}$$

The first term is the "damping" term which resulted in the magnetron energy change given by Eq. (A4). The second term is responsible for the magnetron noise excitation. To estimate the effect of this term we consider the following: If the undamped magnetron motion is driven by a resonant electric rf field the resulting amplitude contains a term which grows according to[14]

$$r_m(t)\cos(\omega_m t + \zeta) = \frac{eE_x}{2m}\frac{\omega_m t}{(\frac{1}{2}\omega_z^2 - \omega_m^2)}\cos(\omega_m t + \zeta).$$

When excited by noise, the amplitude grows in this fashion only for a time τ_c, where τ_c is the coherence time of the noise. In this case the coherence time is determined by the bandwidth of the axial oscillation, and we have $\tau_c = 2\tau_z$. After t/τ_c coherence times the amplitude has grown in a random-walk fashion and we have

$$\langle r_m^2\rangle = \left(\frac{eE_x\omega_m\tau_c}{2m(\frac{1}{2}\omega_z^2 - \omega_m^2)}\right)^2\frac{t}{\tau_c}. \tag{A7}$$

The total magnetron energy can be written as

$$E_m = \tfrac{1}{2}mr_m^2(\omega_m^2 - \tfrac{1}{2}\omega_z^2). \tag{A8}$$

Using Eqs. (A6)–(A8) we obtain

$$\frac{dE_m}{dt} = -\frac{\tau_z[e(\partial E_x/\partial z)]^2\langle z^2\rangle\omega_m^2}{16m(\frac{1}{2}\omega_z^2 - \omega_m^2)}. \tag{A9}$$

Balance between "cooling" [Eq. (A4)] and "heating" [Eq. (A9)] is obtained when

$$r_m^2 = \frac{(\omega_z\omega_m)}{(\frac{1}{2}\omega_z^2 - \omega_m^2)}\langle z^2\rangle, \tag{A10}$$

where we have assumed that the rf magnetic field is negligible and have used Faraday's law. When $\omega_m \ll \omega_z$ we have

$$r_m \cong [(2\omega_m/\omega_z)\langle z^2\rangle]^{1/2}. \tag{A11}$$

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