EXPERIMENTAL RESULTS ON NORMAL MODES IN COLD, PURE ION PLASMAS*

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Experimental measurements of some of the low order electrostatic modes of a nonneutral plasma of $^9Be^+$ ions stored in a Penning trap are discussed. The ions are laser-cooled and typically are strongly coupled, with couplings as large as 200 to 300. The measured mode frequencies agree well with a cold fluid calculation. The modes provide a nondestructive method for obtaining information on the plasma density and shape. In addition, mode excitation by field asymmetries may provide a practical limit to the density and number of charged particles that can be stored in a Penning trap. The damping of the modes should provide information on the plasma viscosity in a strongly correlated plasma.

1. INTRODUCTION

Penning traps[1-4] use a uniform magnetic field to confine ions in a direction perpendicular to the magnetic field. Confinement in the direction parallel to the magnetic field is provided by an electrostatic field. Figure 1 shows the Penning trap used in this work to confine up to 15 000 $^9Be^+$ ions and study some of the low order modes of a cold, pure ion plasma. Lasers were used to detect and laser-cool the $^9Be^+$ ions.[5,6] Some of the ion fluorescence was collected by a lens system and imaged onto the photocathode of a photon-counting, imaging detector as shown in Figure 1. Laser-cooling routinely produced ion temperatures less than 100 mK, and with some care temperatures less than 5 mK. The lasers also applied a torque which could either compress or expand the plasma. Typical ion densities ranged from 10^7 to $10^8/cm^3$ for a 0.8 T magnetic field, but reached as high as $10^{10}/cm^3$ for a 6 T magnetic field.

The ion confinement was sufficiently long for the ions to evolve to thermal equilibrium. Due to the axial magnetic field and radial forces, ions in a Penning trap undergo a rotation about the symmetry axis of the trap. In thermal equilibrium the rotation of the ions is rigid or uniform; that is, the rotation frequency is independent of ion radius.[7,8] A cloud of ions stored in a Penning trap can be considered a plasma, in particular

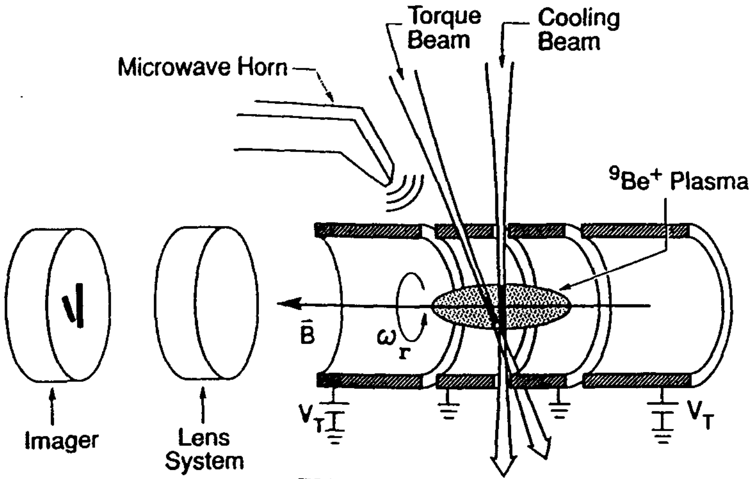

FIGURE 1

Sketch of a Penning trap used to make mode measurements on a plasma of $^9Be^+$ ions. The size of the plasma is exaggerated. The trap electrodes (shown in cross section) are right circular cylinders. They provide a quadratic trap potential near the trap center. The laser beams, microwave horn, and imaging system are used in the measurement of the plasma modes as described in Ref. 18.

**Present address: Physics Dept., Univ. of Texas, Austin, TX 78712 a nonneutral plasma, under conditions where the Debye length λ_D is small compared to the plasma dimensions.[4,7] The Debye length is given by (SI units are used in the following equations)

$$\lambda_D^2 = \frac{\epsilon_o k_B T}{n_o q^2}, \tag{1}$$

where k_B is Boltzmann's constant, T is the plasma temperature, n_o is the plasma density, q is the ion charge, and ϵ_o is the permittivity of vacuum. In the limit that λ_D is much less than the plasma dimensions, the ion density is constant in the plasma interior and drops to zero at the plasma boundary over a distance on the order of a few Debye lengths.[7,9] The density is related to the rotation frequency ω_r of the plasma by

$$n_o = \frac{m 2\omega_r(\Omega-\omega_r)}{q^2/\epsilon_o}. \tag{2}$$

Here m is the mass of a particle and $\Omega \equiv qB/m$ is the cyclotron frequency where B is the magnetic field. In a frame of reference rotating with the plasma rotation frequency ω_r, the magnetically confined nonneutral plasma of the Penning trap behaves like a one component plasma.[10] In particular, the static thermodynamic properties are identical and depend only on the dimensionless Coulomb coupling constant $\Gamma=(q^2/4\pi\epsilon_o)/(ak_BT)$ where $4\pi a^3 n_o/3=1$. A one component plasma consists of a single species of charge immersed in a uniform, neutralizing background of opposite charge.[11] Qualitatively, the trapping fields of the Penning trap can be thought of as providing the neutralizing background of opposite charge. With temperatures less than 5 mK and densities greater than $10^7/\text{cm}^3$, the ions are strongly coupled with couplings Γ greater than 120.

If the plasma dimensions are small compared to the trap dimensions, the electrostatic potential ϕ_T of the trap is, to a good approximation, quadratic over the region of the plasma,

$$\phi_T = \frac{m\omega_z^2}{4q}(2z^2-r^2), \tag{3}$$

and the effect of image charges in the trap electrodes may be neglected. Here r and z are cylindrical coordinates and ω_z is the axial frequency of a single particle in the trap. When all of the above conditions are satisfied ($\lambda_D \ll$ plasma dimensions, ϕ_T given by Eq. (3), and image charges negligible), the plasma has the simple shape of a spheroid (an ellipsoid of revolution).[8] The aspect ratio of the spheroid (the ratio of the plasma's axial extent to its diameter) is also related to the plasma rotation frequency ω_r.[8] Constant density equilibria exist for $\omega_m < \omega_r < \Omega-\omega_m$ where ω_m, the single-particle magnetron frequency, is given by

$$\omega_m = \frac{\Omega}{2} - \left(\frac{\Omega^2}{4} - \frac{\omega_z^2}{2}\right)^{1/2}. \tag{4}$$

When ω_r is slightly greater than ω_m, the plasma is shaped like a pancake. As ω_r increases, the plasma density increases and the plasma's aspect ratio increases by decreasing the plasma's radius and increasing the plasma's axial extent. At $\omega_r = \Omega/2$, the plasma obtains its maximum density and maximum aspect ratio. The condition is often called Brillouin flow. At Brillouin flow the plasma behaves in many ways like an unmagnetized plasma.[12] As ω_r increases beyond $\Omega/2$, the plasma's aspect ratio and density decrease.

A surprising result is that there are analytic solutions for all of the electrostatic modes of a cold, spheroidal nonneutral plasma in a uniform magnetic field. The calculation assumes a cold fluid description[12] of the plasma and ignores correlations and strong coupling effects. However, for the work described here, the $^9Be^+$ plasmas were strongly correlated and exhibited liquid-like and solid-like behavior. Due to the finite size of the plasma, the spatial correlations took the form of shell structure.[5,13-15] Although the plasmas are correlated, as long as the wavelength of the modes is long compared to the interparticle separation, the cold fluid model should provide a good description of the mode frequencies.[16] Correlations and strong coupling effects should be important in describing the mode damping.

This paper gives a brief description of the low order modes and measurements on spheroidal plasmas of $^9Be^+$ ions in a Penning trap. More details can be obtained in the Refs. 17-19. These modes may be important for a number of Penning trap experiments. For example, observation of the modes can provide a nondestructive method for obtaining information on the plasma density and shape when direct imaging techniques are not available. This is the case for antimatter[20,21] and electron plasmas. In addition, some of the plasma modes are zero-frequency modes (in the laboratory frame) for particular plasma densities. Field errors in the trapping potential can excite these modes and enhance radial transport.[22,23] Excitation of these modes may therefore set a practical limit on the density and number of particles that can be stored in a Penning trap.

2. DESCRIPTION OF THE MODES

The electrostatic modes of a Penning trap plasma can be described with the use of spheroidal coordinates by two integers (ℓ,m) with $\ell \geq 1$ and $m \geq 0$.[17] (Negative values of m are allowed, but do not give rise to new modes). The index m indicates that the plasma mode displays an $e^{im\phi}$ dependence where ϕ is the azimuthal angle. The index ℓ describes the mode dependence along a spheroidal surface (for example, the plasma boundary) in a direction perpendicular to $\hat{\phi}$. The $\ell = 1$ modes are the familiar center-of-mass modes where the ion plasma keeps its shape, but the center-of-mass of the plasma executes one of the three motions of a single ion in a Penning trap. For example, the (1,0) mode is the axial center-of-mass mode at frequency ω_z. There are two (1,1) modes which correspond to the perturbed cyclotron and magnetron center-of-mass modes at frequencies Ω-ω_m and ω_m. These center-of-mass frequencies can, in general, be measured or calculated very precisely.

Excitation of an $\ell = 2$ mode produces a quadrupole deformation of a plasma in a Penning trap. For example, when a (2,0) mode is excited, the plasma always stays spheroidal, but the aspect ratio of the spheroid oscillates in time at the (2,0) mode frequency ω_{20}. There are two (2,0) modes: a low frequency mode with a frequency ω_{20}^- on the order of the trap axial frequency ω_z and a high frequency mode with a frequency ω_{20}^+ on the order of the cyclotron frequency Ω. The low frequency mode is a plasma mode. In this mode, the axial and radial excursions of the plasma are 180° out of phase. The high frequency mode is an upper hybrid mode. In this mode the axial and radial excursions of the plasma are in phase. There are three (2,1) modes. In each of these modes the plasma keeps its equilibrium shape (for small amplitudes of excitation) but is tilted (or rotated) with respect to the magnetic field axis (z axis) of the trap. The tilted plasma then processes about the magnetic field axis at the mode frequency. In a frame of reference rotating with the plasma, one of these modes processes in a direction opposite to the plasma rotation and for a particular rotation frequency can be a zero frequency mode in the lab frame. This zero-frequency mode can be excited by a static field error.[22,23] For example, it can be excited by a tilt of the magnetic field with respect to the electrode symmetry axis. There are two (2,2) modes. In both of these modes, the plasma becomes an ellipsoid with unequal principal axes in the z = 0 plane. The ellipsoid rotates about the z-axis at the (2,2) mode frequency.

3. EXPERIMENTAL MEASUREMENTS

Most Penning trap experiments have been done with the plasma rotation frequency $\omega_r \ll \Omega$. In the experiment of Ref. 18, we used radiation pressure from two lasers to remove energy and control the angular momentum of a plasma of $^9Be^+$ ions.[18,24] This enabled us to obtain rotation frequencies throughout the allowed range $\omega_m < \omega_r < \Omega$-$\omega_m$. One laser, called the cooling laser (see Fig. 1), was directed near the center of the $^9Be^+$ plasma and had its frequency tuned below the cooling transition frequency of Be^+ (in the rest frame of the Be^+ ion). This laser removed energy and cooled the Be^+ ions. A second laser, called the torque laser, was directed at the radial edge of the plasma which receded the laser beam due to the plasma rotation. Its frequency was tuned above the Be^+ cooling transition frequency in order to interact with the blue shifted ions on the radial edge of the plasma. This laser beam applied a torque which enabled us to increase the plasma rotation frequency.

The static (2,1) mode was excited on a plasma of 2000 $^9Be^+$ions with B = 0.82 T [$\Omega(^9Be^+)/(2\pi)$ = 1.4 MHz]. This mode was excited by a tilt of the magnetic field when the plasma rotation frequency was increased to the rotation frequency where the (2,1) mode is static in the lab frame. Excitation of this mode was detected by a change in the ion fluorescence produced by an increase in the ion energy. Figure 2 shows the measured rotation frequency where heating was observed for misalignments of the trap magnetic and electric field axes of greater than 0.01°. We have also been able to increase the rotation frequency of plasmas consisting of 40 000 $^9Be^+$ ions at B = 6 T [$\Omega(^9Be^+)/(2\pi)$ = 10.2 MHz]. With care we could obtain ion densities near $10^{10}/cm^3$ when $\omega_r \simeq \Omega/2$. For this larger number of ions with B = 6 T, the (2,1) heating resonance was much stronger. In addition, we observed other heating resonances at rotation frequencies less than the (2,1) heating resonance. These additional resonances presumably correspond to higher order static resonances which have recently been calculated by Dan Dubin of the University of California at San Diego.[19] They tend to limit the plasma density to even lower values than the static (2,1) resonance. This has important implications for work where high

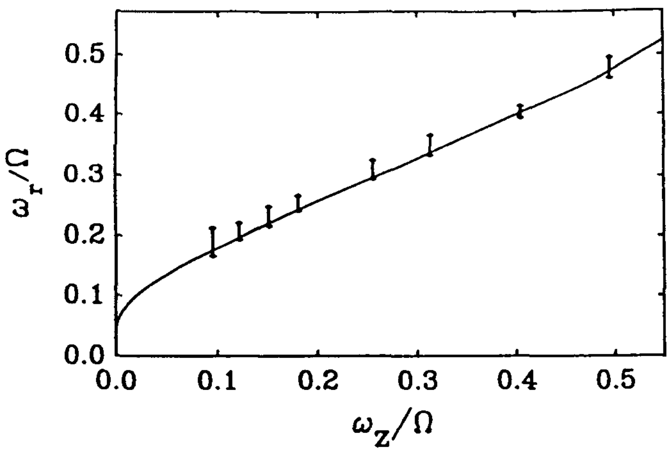

FIGURE 2
Plasma rotation frequency ω_r at which heating was observed as a function of the single-particle axial frequency ω_z. Both frequencies are expressed in units of the cyclotron frequency Ω. The solid line indicates the calculated rotation frequency ω_r at which $\omega_{21} = 0$ in the laboratory frame. This is a universal curve for any charged particle species involving no adjustable parameters.

densities and large numbers of charged particles are to be stored in a Penning trap.

We have also detected and measured the (2,0) plasma and upper hybrid modes on plasmas of a few thousand $^9Be^+$ ions at B = 0.82 T. These modes were excited by applying a potential with a sinusoidal time dependence between the ring and endcap electrodes of the trap. They were detected by a change in the ion fluorescence when the frequency of the applied rf was resonant with the mode frequency. Measurements of the plasma mode frequency as a function of the plasma's rotation frequency are shown in Fig. 3 for two different trap axial frequencies. The solid line shows the calculated plasma mode frequency. Good agreement between the predicted and observed modes is obtained with no adjustable parameters. Figure 3 shows two additional calculations. In the first, shown as dashed lines, the magnetic field is assumed to be effectively infinite; that is, the ions are not allowed to move radially, and the mode frequency is calculated assuming a simple axial stretch of the charged spheroid. In the second calculation, shown as dotted lines, the magnetic field is assumed to be

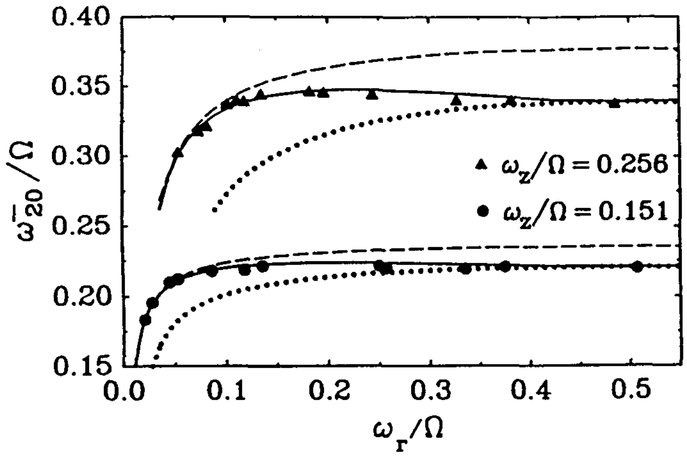

FIGURE 3
(2,0) plasma mode frequency $\omega_{\bar{2}0}$ as a function of the rotation frequency ω_r for $\omega_z/\Omega = 0.151$ and $\omega_z/\Omega = 0.256$. All frequencies are expressed in units of the cyclotron frequency Ω. The theoretical curves are therefore independent of the particle charge-to-mass ratio. The circles and triangles give the experimental data. The solid lines give the cold fluid model predictions for $\omega_{\bar{2}0}$. The dashed and dotted lines give the high- and low-magnetic-field calculations for $\omega_{\bar{2}0}$, respectively.

effectively zero. The (2,0) plasma mode behaves like a mode of a strongly magnetized plasma at low rotation frequencies and an unmagnetized plasma near the Brillouin limit. Figure 4 shows measurements of the upper hybrid (2,0) mode along with calculations of this mode frequency. In fact, on the basis of frequency measurements it is difficult for us to distinguish between the (2,0) upper hybrid mode and other, higher-order, upper hybrid modes [for example, a (4,0) upper hybrid mode]. However, the data of Fig. 4 was taken with a weak drive, so it was likely that we were exciting only the (2,0) upper hybrid mode. We could not excite this mode near the Brillouin limit because the mode becomes difficult to couple to with external fields there.[19] Measurement of either the plasma or the upper hybrid (2,0) mode determines the plasma rotation frequency and aspect ratio. Therefore measurement of these modes may be useful in experiments with trapped positrons,[20] antiprotons,[21] and ions where other nondestructive means of obtaining this information are not available. We have done some calculations which provide the plasma rotation frequency (and therefore the density and aspect ratio) for a measured (2,0) mode frequency. These will be included in a future publication.[19]

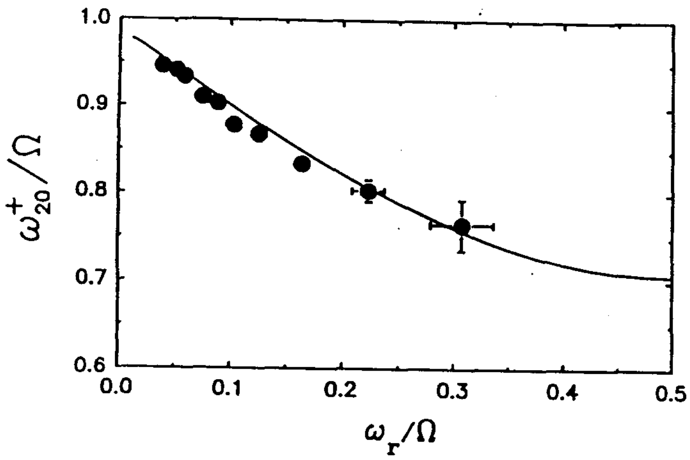

FIGURE 4
Upper hybrid mode frequency ω_{20}^{+} as a function of the rotation frequency ω_r for $\omega_z/\Omega = 0.151$. All frequencies are expressed in units of the cyclotron frequency. The circles give the experimental data. The uncertainty for measurements done at low rotation frequencies is approximately the size of the circles. At high rotation frequencies the uncertainties increase because the mode becomes difficult to excite. The solid line gives the cold fluid model predictions.

4. CONCLUSION

We have measured the frequencies of some of the quadrupole modes on cold, spheroidal plasmas of $^{9}Be^{+}$ ions in a Penning trap. Measurements were made between low rotation frequencies $\omega_r \simeq \omega_m << \Omega/2$, where the plasma behaves as if it is strongly magnetized, and high rotation frequencies $\omega_r \simeq \Omega/2$, where the plasma behaves as if it is unmagnetized. Excellent agreement is obtained between the measured mode frequencies and calculations of these frequencies using a cold fluid model. The modes provide a nondestructive method for obtaining information on the plasma density and shape. In addition, the modes may provide a practical limit to the density and number of charged particles that can be stored in a Penning trap. In the future we will try to measure the damping of the modes. Measurement of the mode damping should provide information on the viscosity of a strongly correlated plasma. To our knowledge, very little experimental work has been done in this area.

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