Laser Cooling of Trapped Ions.

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1. – Introduction.

Trapping of ions is used to make measurements that are difficult or impossible to perform by other techniques, because the ions can be held for long periods in a well-controlled environment. Cooling of the ions has several beneficial effects for such experiments. First, it increases the time that the ions remain in the trap. Second, it can lead to increased precision for measurements of masses, magnetic moments and optical or microwave spectra. Third, some phenomena, such as the formation of spatially ordered structures of ions, can be observed only at low temperatures.

We use the term «cooling» to mean a reduction in the velocities of the ions. Use of the term does not imply that the ions are in thermal equilibrium with each other nor that they have a well-defined temperature. The motional modes that are cooled may be random or coherent. The secular motion in a Paul (r.f. quadrupole) trap and the axial motion in a Penning trap are examples of random modes. The micromotion in a Paul trap and the magnetron rotation in a Penning trap are examples of coherent modes. Laser cooling is very effective and has been used to obtain temperatures much less than 1 K[1,2]. However, since laser cooling depends on the energy level structure of the ions and on the availability of narrow-band radiation sources matched to the level spacings, it has been applied only to a few kinds of ions. Other cooling methods are more generally applicable[2-4]. Resistive cooling, active feedback cooling, or r.f. sideband cooling can be applied to all ions, since they depend only on the charge and mass of the ions. Collisional cooling is also widely applicable. Sympathetic cooling is the cooling of one ion species through Coulomb collisions with another, laser-cooled ion species. It has been demonstrated with ions in Penning traps[5,6] and for (a few) ions in Paul traps[7-10]. In this lecture, we concentrate on laser cooling. Other methods will be discussed elsewhere[11].

2. – Optical sideband cooling.

Laser cooling has been described from different viewpoints. Here we will describe it in terms of mode coupling by a parametric drive [11, 12]. This has the advantage that it also includes r.f. sideband cooling [11, 13, 14].

Figure 1 is a generalized diagram of the mode structure of the cooled system. The system contains two modes, represented by the subscripts a and b. We assume that $\omega_a \ll \omega_b$, so the energy levels separate into manifolds as shown. For simplicity, we assume the level separations in each manifold are the same, so that the energy is $\hbar(n_a\omega_a + n_b\omega_b)$, but this condition is not essential. The cooling relies on 1) parametric coupling of the two modes and 2) relaxation of the b mode by spontaneous decay at a rate γ_b. Spontaneous decay of the a mode is assumed to be negligible. Here, the a mode is a mode of oscillation of an ion, which is to be cooled. Figure 1 shows one part of the cooling process. The particle is initially in the state ($n_a = 1$, $n_b = 0$). It absorbs a photon of energy $\hbar(\omega_b - \omega_a)$ from the externally applied field and is driven to the state ($n_a = 0$, $n_b = 1$). The particle then spontaneously emits a photon of energy $\hbar\omega_b$ and makes a transition to the state ($n_a = 0$, $n_b = 0$). In this process, system's energy is reduced by $\hbar\omega_a$. This is the basic idea of sideband cooling. Heating occurs if the coupling causes absorption of photons of energy $\hbar(\omega_b + \omega_a)$.

In general the process is more complicated. Other absorption processes of the form $(n_a, n_b) \to (n_a - 1, n_b + 1)$ can be driven by the same applied coupling.

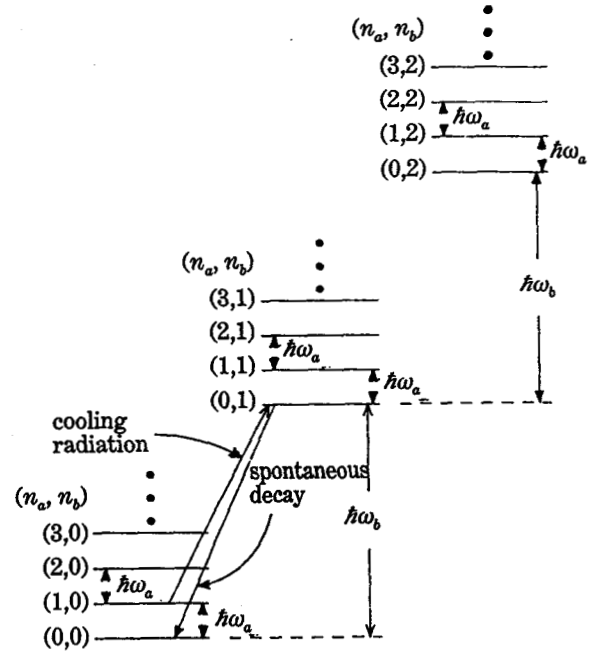

Fig. 1. – Energy level structure for a particle which is suitable for sideband cooling, having two modes, labeled a and b.

The entire distribution of n_a and n_b must be taken into account. Also, even when the coupling is tuned to resonance, other nonresonant transitions can occur, since the resonances are broadened by spontaneous emission.

For laser cooling, ω_b is an optical frequency. In this case, the b mode is just a two-level system ($n_b = 0$ or 1). The recoil of the cooled particle upon the spontaneous emission of a photon of frequency ω_b leads to heating. Let $\langle \Delta n_a \rangle$ be the average change in n_a per spontaneous emission. Then,

$$\hbar\omega_a \langle \Delta n_a \rangle = R \equiv (\hbar\omega_b)^2/2mc^2, \tag{1}$$

where R is the recoil energy (the kinetic energy of an initially stationary particle of mass m after emission of a photon of frequency ω_b). We assume that the photon is emitted into free space. That is, the emitted radiation is not influenced by cavity effects. Two cases of laser cooling will be treated separately: $\omega_a \ll \gamma_b$ (sidebands unresolved) and $\omega_a \gg \gamma_b$ (sidebands resolved).

Sideband cooling is closely related to methods of dynamic nuclear orientation [15]. The method of polarization by forbidden transitions is directly analogous to sideband cooling. In this method, an applied r.f. field drives transitions between states which differ in both the nuclear and electronic spin quantum numbers. This corresponds, for example, to the $(n_a = 1, n_b = 0) \rightarrow$ $\rightarrow (n_a = 0, n_b = 1)$ transition in fig. 1. The electronic spin relaxes quickly $[(n_a = 0, n_b = 1) \rightarrow (n_a = 0, n_b = 0)]$. The nuclear ($a$) mode reaches the same polarization as the electronic (b) mode.

VYATCHANIN [16] has discussed, in general terms, the cooling of a quantum oscillator (or two-level system) by parametric coupling to another quantum oscillator (or two-level system) which is more strongly damped. This treatment is directly applicable to sideband cooling.

2·1. *Optical sideband cooling (resolved sidebands).* – In optical sideband cooling, the b mode is an internal optical transition in a trapped particle. Generally, the transition is an electronic transition (at an optical frequency) which relaxes radiatively by spontaneous emission at the rate γ_b. The quantum number n_b takes only the values 0 and 1. Here we consider the resolved sideband case ($\gamma_b \ll \omega_a$) where ω_a is the oscillation frequency of the cooled mode of the bound particle. (The unresolved sideband case will be discussed in subsect. 2·3.)

The parametric coupling from the ground-state vibrational manifold ($n_b = 0$) to the excited-state vibrational manifold ($n_b = 1$) is done with narrow-band radiation. The width $\Delta\omega_b$ of the radiation should be less than or on the order of γ_b for optimal cooling. It should be resonant with the first lower sideband at frequency $\omega_b - \omega_a$. If the recoil energy R (defined in eq. (1)) is less than the vibrational energy $\hbar\omega_a$, then sideband cooling is possible. (Otherwise the cooling which results from the photon absorption is undone by the recoil from the photon emission.) An atom absorbs photons of energy $\hbar(\omega_b - \omega_a)$ and re-emits pho-

tons of average energy $\hbar\omega_b - R$. Hence, when $R \ll \hbar\omega_a$, on the average, each scattered photon reduces the atom's vibrational energy by $\hbar\omega_a$ and the atom's vibrational quantum number n_a by 1. In this way, it is possible to make the mean quantum number $\langle n_a \rangle$ much less than 1. If $\langle n_a \rangle \ll 1$ for all motional degrees of freedom, then the atom resides in the ground-state level of the confining potential most of the time.

Optical sideband cooling of the thermal degrees of freedom of many simultaneously trapped particles is possible in principle. However, the kinetic energy in coherent degrees of freedom usually limits the minimum attainable kinetic energy. The higher energies of the coherent motions in both the Penning trap and in the Paul trap are consequences of Coulomb repulsion. In a Penning trap, any ion that does not lie on the symmetry axis of the trap rotates about the axis, due to crossed electric and magnetic fields ($\boldsymbol{E} \times \boldsymbol{B}$ drift). This rotation gives rise to a kinetic energy that depends quadratically on the radial distance of the ion from the trap axis. It might be possible for a few ions to lie along the symmetry axis. However, if there are many ions, the combined effects of the trapping fields and the mutual Coulomb repulsion of the ions will force some of the ions to lie at a finite distance from the axis.

In a quadrupole Paul trap, the Coulomb repulsion between ions is balanced by a force from the trapping potential, directed toward the center of the trap. Only one ion can occupy the center of the trap, where the applied trap fields go to zero. Any offset of an ion from the trap center leads to micromotion of the particle at the frequency of the applied r.f. field. The kinetic energy in this nonthermal motion cannot be reduced by sideband cooling. So, although it is possible to reduce the kinetic energy of the secular motion for more than one ion, the kinetic energy in the micromotion remains undiminished and can be substantial. This limitation could be overcome in a Paul trap with linear geometry [8, 17-19]. In such a trap, the magnitude of the r.f. field approaches zero on a line, rather than at a point.

Even for a single ion, optical sideband cooling in a Penning trap is difficult. The magnetron, cyclotron and axial frequencies cannot be degenerate for stable trapping. This leads to a complicated sideband spectrum [20]. The spectrum contains sidebands not only at the fundamental motional frequencies (for example, $\omega_b \pm \omega_m$), but also all intermodulation products ($\omega_b + j\omega_c' + k\omega_z + l\omega_m$, where j, k, l are any integers). At high temperatures this spectrum is nearly continuous. Therefore, precooling to near the Dicke limit ($x_{r.m.s.} \ll \lambda/2\pi$) is required before sideband cooling is done. Here, $x_{r.m.s.}$ is the r.m.s. value of the x coordinate of the ion, and $\lambda = 2\pi c/\omega_b$.

Attaining the lowest vibrational energies (the minimum $\langle n_a \rangle$) for all degrees of freedom would require cooling with a laser tuned to an individual sideband for each of the three motional degrees of freedom. Finally, the condition that the recoil energy R be much less than any of the motional energies is more readily satisfied in the Paul trap than in the Penning trap.

For both Paul and Penning traps, the sideband cooling limit for a single ion is

$$\langle n \rangle = \frac{1}{4}(\alpha + 1/4)(\gamma_b/\omega_a)^2, \tag{2}$$

where α depends on the angular distribution for photon emission and is of order 1. Equation (2) is valid when the intensity of the cooling radiation is below saturation and when $\gamma_b/\omega_a \ll 1$. The limit results from a balance between cooling and heating. The most important cooling process is on-resonance absorption of a photon on the lower sideband ($n_a \rightarrow n_a - 1$), followed by emission on the carrier ($n_a \rightarrow n_a$). (In the Dicke limit, almost all emission is on the carrier.) The most important heating processes are 1) off-resonance $n_a \rightarrow n_a + 1$ transitions, followed by emission on the carrier, and 2) off-resonance $n_a \rightarrow n_a$ transitions, followed by emission on the lower sideband ($n_a \rightarrow n_a + 1$). This result was derived by NEUHAUSER *et al.* [21] and by others [12, 22]. WINELAND *et al.* [20] considered the effect of finite laser bandwidth and also explicitly treated resolved optical sideband cooling in a Penning trap.

LINDBERG [23] and JAVANAINEN *et al.* [24] have derived the steady-state energy of a single two-level atom, confined in a one-dimensional harmonic well of frequency ω_a, for arbitrary laser frequency, laser intensity and γ_b/ω_a:

$$\langle E_a \rangle \equiv \hbar\omega_a\left(\langle n_a \rangle + \frac{1}{2}\right) = \tag{3a}$$

$$= \frac{(A + B + C)\hbar}{-4\Delta}[\omega_a^2(\Delta^2 + \gamma_2^2 + 6\kappa^2) + 4\gamma_2^2(\Delta^2 + \gamma_2^2 + 2\kappa^2)]^{-1},$$

where

$$A = \alpha[\omega_a^2(\Delta^2 + 5\gamma_2^2 + 4\kappa^2 - \omega_a^2)^2 + 4\gamma_2^2(\Delta^2 + \gamma_2^2 + 2\kappa^2 - 2\omega_a^2)^2], \tag{3b}$$

$$B = \omega_a^2(\Delta^2 + \gamma_2^2 + 2\kappa^2)(\Delta^2 + 5\gamma_2^2 + 8\kappa^2 + \omega_a^2), \tag{3c}$$

and

$$C = 4\gamma_2^2[(\Delta^2 + \gamma_2^2 + 2\kappa^2)^2 + 2\kappa^2(\Delta^2 - 3\gamma_2^2 - 3\omega_a^2)]. \tag{3d}$$

This result holds in the limit that $R/\hbar\omega_a \ll 1$. Here, κ is the Rabi frequency, which is proportional to the square root of the laser intensity, and $\gamma_2 \equiv \gamma_b/2$. The optical field is a travelling wave. The detuning Δ is defined as $\Delta \equiv \omega - \omega_b$, where ω is the laser frequency. It must be negative (laser frequency below resonance), or else there is no steady state. Equation (3) reduces to eq. (2) in the limit $\kappa \rightarrow 0$, $\gamma_2/\omega_a \rightarrow 0$ and $\Delta \rightarrow -\omega_a$.

The value of $\langle n_a \rangle$ can be determined experimentally from the relative transition strengths for on-resonance absorption on the first lower and upper sidebands [20, 25]. When $\langle n_a \rangle \ll 1$, the strength of resonant absorption on the lower

sideband ($\omega_b - \omega_a$), which is proportional to $\langle n_a \rangle$, approaches zero. When the ion is in the lowest kinetic-energy state ($n_a = 0$), it is no longer possible to extract vibrational quanta from the ion. The strength of the upper sideband ($\omega_b + \omega_a$) is proportional to $\langle n_a \rangle + 1$. Thus, if the lower and upper sidebands are probed with saturating power, the ratio of their absorption strengths becomes independent of power and directly gives $\langle n_a \rangle$.

A single $^{198}Hg^+$ ion, confined in a small Paul trap, has been laser-cooled, in the resolved sideband limit, to near the zero-point energy of motion [25]. The electronic energy levels of Hg^+ are shown in fig. 2. The procedure was as follows: First, the ion was laser-cooled in the Doppler-cooling limit (discussed in subsect. 2·2.1), by radiation scattered from the strongly allowed 194 nm first resonance line. This reduced the temperature to about 1.7 mK and the mean vibrational quantum number n_a to about 12. The secular frequency ($\omega_a/2\pi$) was

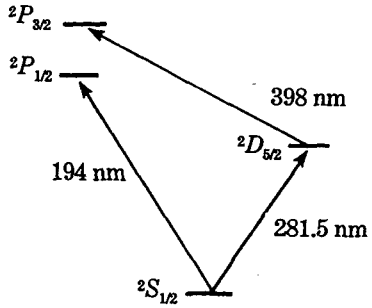

Fig. 2. – A simplified energy level diagram of $^{198}Hg^+$, showing the transitions used for optical sideband cooling.

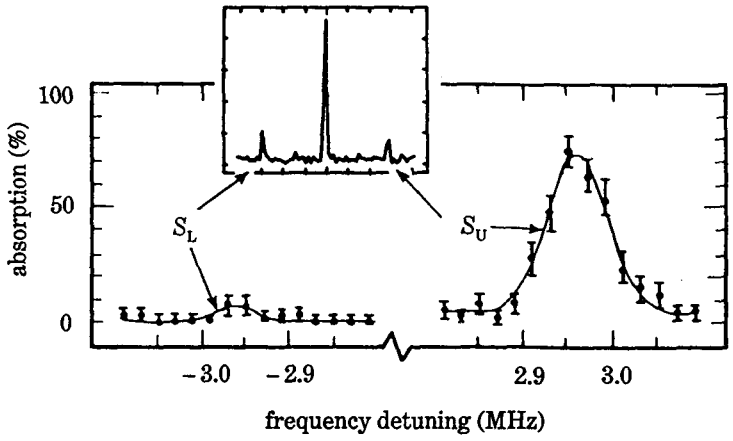

Fig. 3. – Absorption spectrum of the $^2S_{1/2}$-to-$^2D_{5/2}$ electric-quadrupole transition. The vertical scale is the probability that the ion makes a $^2S_{1/2}$-to-$^2D_{5/2}$ transition when the probing 282 nm radiation is applied. The spectrum in the small inset was taken before the sideband cooling was applied. S_L is the first lower sideband (frequency $\omega_b - \omega_a$), S_U is the first upper sideband (frequency $\omega_b + \omega_a$). The asymmetry between S_U and S_L indicates that the average quantum number for the oscillation of the ion is much less than 1.

about 3 MHz. Laser radiation, tuned to the first lower vibrational sideband of the narrow 282 nm $5d^{10}6s\ ^2S_{1/2}$-to-$5d^96s^2\ ^2D_{5/2}$ transition, was then applied to the ions. In order to speed the cooling process, 398 nm laser radiation was used to drive ions from the $^2D_{5/2}$ state to the $5d^{10}6p\ ^2P_{3/2}$ state, which decays rapidly to the ground $^2S_{1/2}$ state. This allowed the absorption of 282 nm photons to proceed faster than the 90 ms natural lifetime of the $^2D_{5/2}$ state would have allowed. After scattering enough photons to remove 12 vibrational quanta from the ion for each degree of freedom, laser radiation of saturating intensity probed the 282 nm sideband spectrum.

The results are shown in fig. 3. The fact that the intensity of the lower sideband (S_L) is much less than that of the upper sideband (S_U) indicates that $\langle n_a \rangle \ll 1$. From these data, it was deduced that the ion was in the lowest vibrational quantum state, for the y and z degrees of freedom, 95% of the time. This corresponds to a temperature of (47 ± 3) μK. Since the 282 nm probe beam was nearly orthogonal to the motion in the x direction, the measurement was not very sensitive to the energy in this degree of freedom. For this reason, a precise measurement of the x vibrational energy could not be made in this experiment. Nevertheless, cooling of all degrees of freedom could be accomplished in a straightforward manner by orienting the 282 nm beam at a different angle with respect to the trap axes.

2·2. *Optical sideband cooling (unresolved sidebands).* – In most experiments on optical sideband cooling, a strongly allowed optical transition, which has a natural linewidth γ_b of approximately 20 MHz or more, is used. This is much greater than the motional frequencies of the trapped ion (typically 3 MHz or less). The sideband structure of the optical resonance is not resolved. It is then natural to treat the motion of the ion classically. The scattering of laser light by the ion leads to a force on the ion. The scattering takes place in a time of about γ_b^{-1}, much less than the orbital period, so it can be assumed to give an instantaneous impulse to the ion. In the low-intensity limit, the average force on an atom is the product of the average momentum transfer per scattering (the photon momentum $\hbar k$) and the scattering rate. The size of the force is maximized when the laser frequency, in the frame moving with the ion, is in resonance with the optical transition. Fluctuations in the force arise from 1) the randomness of the direction of the scattered photon and 2) the random time distribution of the scattering events. As an alternative to this semi-classical approach, the cooling can also be explained with the general concepts of sideband cooling [12, 21, 23, 24]. It is necessary, in this case, to include a large number of sidebands in the analysis.

2·2.1. Paul traps. An ion in a Paul trap can be treated, to a good approximation, as a particle in a three-dimensional harmonic potential, undergoing simple harmonic motion at the secular frequencies. The micromotion

can be added by numerical simulation [26-28], but is hard to treat analytically.

For simplicity, consider an ion moving in a harmonic well in the x direction, interacting with a laser beam propagating in the $+x$ direction. If the ion has a velocity v and the laser detuning is Δ, the average force is

$$F_{av} \approx \frac{I\sigma_0}{\hbar\omega} \frac{\gamma_2^2}{(\Delta - kv)^2 + \gamma_2^2} \hbar k \approx \tag{4}$$

$$\approx \frac{I\sigma_0 \gamma_2^2}{\hbar\omega(\Delta^2 + \gamma_2^2)} [1 + 2\Delta kv/(\Delta^2 + \gamma_2^2)] \hbar k \equiv$$

$$\equiv F_0 - m\Gamma v,$$

where I is the laser intensity, σ_0 is the resonant scattering cross-section, and m is the mass of the ion. The approximation holds for low intensities ($I\sigma_0/\hbar\omega \ll \gamma_2$) and low velocities ($kv \ll \gamma_2$).

In the last line of eq. (4), F_0 is a velocity-independent force, which slightly displaces the equilibrium position of the ion, and $m\Gamma v$ is a damping force (for $\Delta \ll 0$), which leads to cooling. The fluctuations in the force, which are due to the discreteness of the photon scattering, have properties similar to electronic shot noise [29]. The equation of motion for the ion position is the same as that of a series RCL (resistance-capacitance-inductance) circuit driven by a fluctuating e.m.f. The steady-state energy of the ion can be calculated by methods analogous to those used to calculate noise in electronic circuits [30].

The steady-state energy is

$$\langle E_a \rangle = \frac{(1 + \alpha)\hbar(\gamma_2^2 + \Delta^2)}{-4\Delta}. \tag{5}$$

This agrees with eq. (3) in the limit where $\kappa \to 0$ and $\omega_a/\gamma_2 \to 0$. The lowest energy is obtained when $\Delta = -\gamma_2$:

$$\langle E_a \rangle = k_B T = \frac{(1 + \alpha)\hbar\gamma_2}{2}. \tag{6}$$

For $\alpha \approx 1$, this yields the Doppler-cooling limit, $k_B T \approx \hbar\gamma_2 = \hbar\gamma_b/2$, which also holds, under suitable conditions, for free two-level atoms [1,2,12,21,31]. For typical cases, $T \approx 1$ mK.

Optical sideband cooling in a Paul trap was first demonstrated experimentally by NEUHAUSER *et al.* [21]. They cooled Ba^+ ions, using the 493 nm, $6s\ ^2S_{1/2}$-to-$6p\ ^2P_{1/2}$ transition. One complication in the experiment was that the $6p\ ^2P_{1/2}$ state can decay to the metastable $5d\ ^2D_{3/2}$ state. A 650 nm laser beam drove the ions from the $5d\ ^2D_{3/2}$ state back to the $6p\ ^2P_{1/2}$ state, so that the cooling could continue. The effect of cooling was to increase the storage time of the ions in the trap. Figure 4 shows the effect of laser cooling when the 493 nm laser

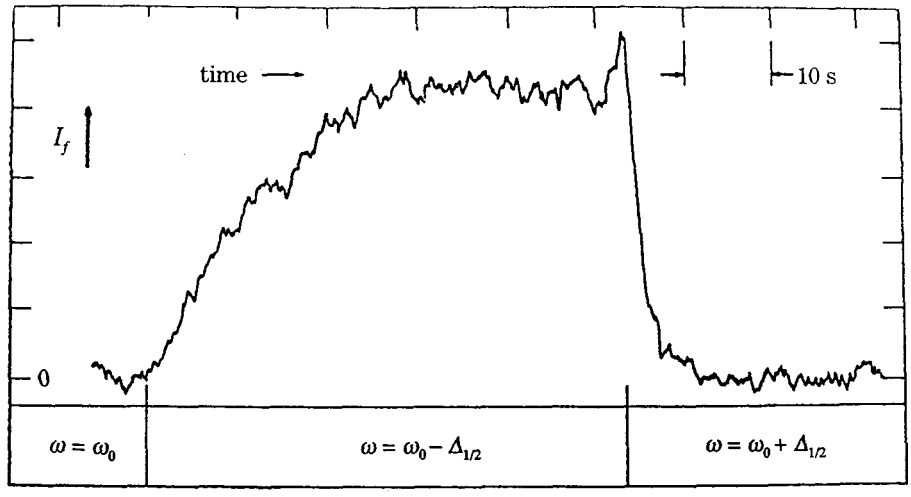

Fig. 4. – The first optical sideband cooling of ions in a Paul trap. The Ba^+ fluorescence intensity is plotted as a function of time. The trap is initially empty when the laser frequency ω is equal to the atomic resonance ω_0. The fluorescence increases when ω is tuned below the atomic resonance to $\omega_0 - \Delta_{1/2}$. It decreases rapidly when $\omega = \omega_0 + \Delta_{1/2}$. Here, $\Delta_{1/2}$ is the resonance half-width in the discharge tube used as a reference.

is tuned below the $6s\,^2S_{1/2}$-to-$6p\,^2P_{1/2}$ transition. Later, NEUHAUSER *et al.* estimated the temperature of a single ion to be less than 36 mK from the size of its photographed image [32].

BERGQUIST *et al.* [33] cooled a single Hg^+ ion, confined in a Paul trap, using the 194 nm $5d^{10}6s\,^2S_{1/2}$-to-$5d^{10}6p\,^2P_{1/2}$ transition. The narrow 282 nm $5d^{10}6s\,^2S_{1/2}$-to-$5d^9 6s^2\,^2D_{5/2}$ transition was used to determine the temperature. The relative intensities (transition probabilities) at the frequencies of the carrier (ω_b) and at the first sidebands ($\omega_b \pm \omega_a$) were measured. Equation (44) of ref. [12] relates the relative intensities of the carrier and sidebands to the temperature. The measured temperature was (1.6 ± 0.5) mK, in good agreement with the Doppler-cooling limit of 1.7 mK. In later experiments with a single $^{199}Hg^+$ ion, the 282 nm carrier was observed with a linewidth of under 200 Hz ($Q \approx 5 \cdot 10^{12}$). A typical resonance curve is shown in fig. 5. Similar experiments have been done with a single Ba^+ ion by NAGOURNEY *et al.* [34]. They observed the 1.8 μm $6s\,^2S_{1/2}$-to-$5d\,^2D_{5/2}$ transition with a linewidth of about 40 kHz.

Optical sideband cooling of ions in Paul traps has been applied to other areas of physics. Radiative lifetimes have been measured in single, laser-cooled ions of Mg^+ [35], Ba^+ [36, 37], Hg^+ [38] and Sr^+ [39]. Rates for quenching of a metastable state by various gases have been measured in single, laser-cooled Ba^+ ions [37]. The fluorescence from a single cooled ion can be easily observed. This has made it possible to observe certain inherently quantum properties of the electromagnetic field, such as photon antibunching [40-42] and quantum

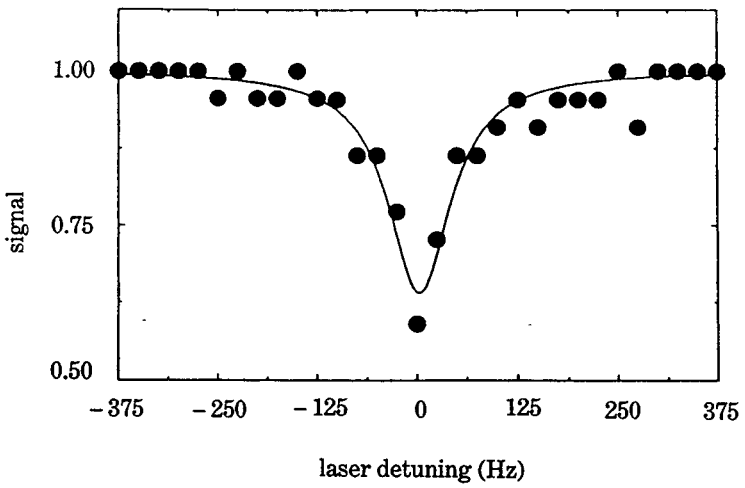

Fig. 5. – Absorption resonance for a single Zeeman hyperfine component of the $5d^{10}6s\ ^2S_{1/2}$-to-$5d^9 6s^2\ ^2D_{5/2}$ transition in a single $^{199}Hg^+$ ion. The probability that the ion did not make a transition to the $^2D_{5/2}$ state is plotted on the vertical axis. The relative frequency of the 563 nm laser, which was frequency doubled to 282 nm, is plotted on the horizontal axis. The curve is a least-squares fit to a Lorentzian. The width of the Lorentzian is about 90 Hz.

jumps [36, 41, 43]. Yb^+ ions have been laser cooled in a Paul trap for potential application as an optical frequency standard [44].

If several ions in a Paul trap are cooled to a low enough temperature, they arrange themselves into spatial configurations which minimize the potential energy. Such ordered patterns have been observed with laser-cooled Hg^+ [7, 10], Mg^+ [45] and Ba^+ [9, 26]. Phase transitions of these ions between ordered and disordered states have been studied, especially with regard to classical chaos [26, 27, 45, 46].

2·2.2. Penning traps. In some respects, optical sideband cooling of ions in Penning traps and Paul traps are similar. The axial and cyclotron modes are cooled if the laser is detuned below resonance. The differences arise from the $\boldsymbol{E} \times \boldsymbol{B}$ rotation of the ions around the trap axis. The rotation frequency of a single ion is the magnetron frequency ω_m. The rotation frequency of many ions is increased by electric fields caused by space charge. Cooling of the rotation requires control of L_z, the z component of the canonical angular momentum of the ions [47]. For a single ion of charge q and mass m, the canonical angular momentum is $\boldsymbol{l} = \boldsymbol{r} \times \boldsymbol{p}$, where $\boldsymbol{r}$ and $\boldsymbol{p}$ are the position and the canonical momentum of the ion. The canonical momentum is defined as $\boldsymbol{p} \equiv m\boldsymbol{v} + (q/c)\boldsymbol{A}(\boldsymbol{r})$, where $\boldsymbol{v}$ is the velocity of the ion and $\boldsymbol{A}(\boldsymbol{r})$ is the vector potential at $\boldsymbol{r}$. For convenience, we choose the symmetric gauge, where $\boldsymbol{A}(\boldsymbol{r}) = \frac{1}{2}\boldsymbol{B} \times \boldsymbol{r}$ and $\boldsymbol{B}$ is the magnetic field.

The z component of $\boldsymbol{l}$ is

$$l_z \equiv m v_\theta \rho + q B \rho^2 / 2c , \tag{7}$$

where v_θ and $\rho = (x^2 + y^2)^{1/2}$ are the azimuthal component of the velocity and the cylindrical radius of the ion. For N identical ions of charge q and mass m,

$$L_z \equiv \sum_{i=1}^{N} \left(m v_{\theta_i} \rho_i + \frac{q B \rho_i^2}{2c} \right), \tag{8}$$

where v_θ and ρ_i are the azimuthal component of the velocity and the cylindrical radius of the i-th ion. If the trap has perfect axial symmetry and there are no interactions with the outside world, L_z is conserved. For the typical case, where the rotation frequency is much less than the cyclotron frequency qB/mc, the second term of eq. (8) dominates. Conservation of L_z is then equivalent to conservation of the mean squared radius of the ion plasma. A more general proof of the fact that conservation of L_z leads to radial confinement has been given by O'NEIL [48].

There are confined thermal equilibria, whose properties are determined by the three conserved quantities: N, L_z and the energy [49]. One property of these thermal equilibria is uniform rotation. That is, the velocity distribution, with respect to a rotating frame, is Maxwellian. In a continuous-fluid model, the zero-temperature equilibria are uniformly rotating spheroids of uniform density [47].

Asymmetries of the trap fields apply torques to the ions [50], so that L_z is not perfectly conserved. This leads to radial expansion. The light scattering force from a laser beam can be used to apply a torque to the ions. This torque can be used to cancel the torques from the trap fields and establish radial confinement. Such a torque could also be applied with a neutral atomic beam.

The first experimental demonstration of optical sideband cooling in a Penning trap was made by WINELAND *et al.* [51]. Approximately $5 \cdot 10^4$ Mg^+ ions were cooled by a 280 nm laser beam tuned slightly lower in frequency than one Zeeman component of the $3s\ ^2S_{1/2}$-to-$3p\ ^2P_{3/2}$ transition. The temperature of the ions was determined from noise current induced on the electrodes. The ions were cooled below 40 K. This was the measurement limit set by the electronic noise. Figure 6 shows the decrease in temperature when the laser is turned on, with a frequency below the resonance frequency. Collisions with neutral gas molecules cause the ions to rethermalize after the laser is turned off. Optical sideband cooling of Mg^+ ions in Penning traps has also been observed by PLUMELLE *et al.* [52] and by THOMPSON *et al.* [53]. IMAJO *et al.* have also observed optical sideband cooling of Be^+ ions in a Penning trap [54].

DRULLINGER *et al.* [5] laser-cooled Mg^+ ions to approximately 0.5 K. The temperature was determined from the Doppler width of the transition. They also demonstrated the connection between the plasma radius and the canonical

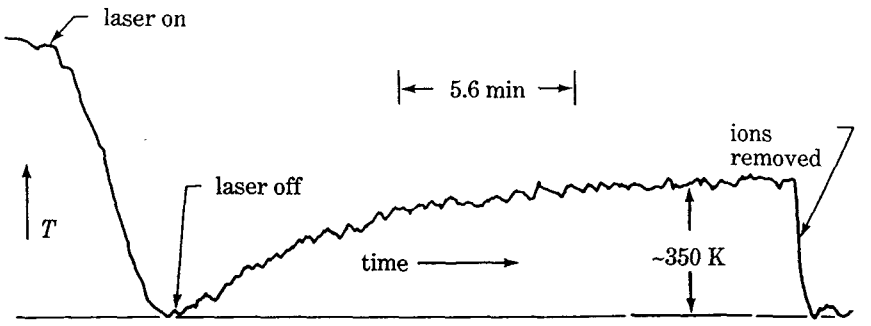

Fig. 6. – The first optical sideband cooling of ions in a Penning trap. The vertical scale is the axial temperature of the Mg^+ ions as measured from the currents induced in the trap electrodes by the ions' motion. The ions were heated by the laser before the beginning of the data. The laser was turned on with a frequency below the resonance frequency, cooling the ions to below 40 K. After the laser is turned off, the ions thermalize to the ambient temperature of 350 K.

angular momentum. Placing the laser beam on one side or the other of the axis exerted a torque on the ions, which caused radial expansion or contraction of the plasma.

If the cooling laser beam is perpendicular to the rotation axis, the steady-state temperature can be calculated from the rotation frequency and the laser frequency and intensity profile [55]. The temperature is calculated from the requirement that the work done on the ions by the laser beam, averaged over the velocity distribution, be zero. The temperature increases with the rotation frequency, the distance between the axis and the laser beam, and the laser intensity. ITANO *et al.* [55] obtained good agreement between the calculated and observed temperatures of Be^+ ions, over a wide range of experimental parameters.

In general, lower temperatures are obtained when the cooling laser beam is not perpendicular to the magnetic field. BREWER *et al.* [56] observed temperatures of Be^+ ions as low as 2 mK, when cooling beams were directed simultaneously at 90° and at 55° with respect to the magnetic field. They also confirmed experimentally the relationship between the rotation frequency and the shape of the ion plasma that was predicted in ref. [47].

The reduction of Doppler shifts and the spatial confinement which results from the cooling has been useful for several experiments. Some examples are in mass spectroscopy [57] and in the observation of single-ion quantum jumps [58]. High-resolution optical spectra of Mg^+ [5] and Be^+ [59] have yielded hyperfine constants, fine-structure separations and isotope shifts. R.f.-optical double-resonance experiments have yielded ground-state hyperfine constants and g-factors of $^{25}Mg^+$ [60] and $^9Be^+$ [57].

A frequency standard, based on a 303 MHz ground-state hyperfine transi-

tion in laser-cooled $^9Be^+$ ions, has been demonstrated [61]. The estimated error of this standard, about 1 part in 10^{13}, is comparable to that of primary cesium atomic-beam standards. A search was made for any dependence of the frequency on the orientation of the magnetic field, with null results [62]. This is one of the most precise tests of local Lorentz invariance.

Ion plasmas which have high enough densities and low enough temperatures are called strongly coupled. In such plasmas, the positions of neighboring ions are correlated with each other, as in a liquid or a solid. Strongly coupled plasmas have been produced by optical sideband cooling of ions in Penning traps [63]. Spatial ordering, in the form of concentric shells, has been directly observed by imaging of the resonance fluorescence [64]. Numerical simulations predict the existence of such shells [65, 66].

3. – Sympathetic laser cooling.

Because laser cooling depends on having a favorable energy level structure, it has been demonstrated on only a handful of different ion species (Be^+, Mg^+, Ba^+, Hg^+, Sr^+ and Yb^+). However, the technique of sympathetic laser cooling [6, 67] can be used to cool any ion species to sub-kelvin temperatures. In sympathetic laser cooling, two different ion species are loaded into a trap. One of the ion species is laser-cooled. The other species is sympathetically cooled by its Coulomb interaction with the laser-cooled species. In a Penning trap, the sympathetically cooled species must have the same sign of charge as the laser-cooled ion species. Sympathetic laser cooling may enable the formation of cold, strongly coupled plasmas of positrons [67].

In principle, sympathetic laser cooling can be done in either a Paul or a Penning trap. In a Paul trap, ordered, crystallike structures in which at least one of the «lattice» sites was occupied by an impurity (perhaps another isotope) that was not directly laser-cooled have been observed [7-10]. These ions did not fluoresce, but their presence was detected by their effect on the positions of the other ions. This shows that at least a small number of ions can be sympathetically laser-cooled to sub-kelvin temperatures in a Paul trap. (If the nonfluorescing ions had not been cold, the ordered structures could not have formed.) In general, r.f. heating and the available cooling laser power limits the number of ions that can be sympathetically laser-cooled in a Paul trap. In the Penning trap, which is free of r.f. heating, experiments have demonstrated sympathetic laser cooling on much larger numbers of ions [6, 68, 69]. Sympathetic laser cooling is a form of collisional cooling between charged particles. However, the low temperatures that can be achieved with laser cooling produce some new features.

For simplicity, consider two ion species with like charges but different masses confined in the same Penning trap. The rotation of the ions about the magnetic-field axis of the trap produces a centrifugal separation of the ion species [70]. For typical experimental conditions, where the rotation frequency is much less than the cyclotron frequency, the rotation frequency is nearly independent of ion mass. This is because the rotation is a circular $\boldsymbol{E} \times \boldsymbol{B}$ drift. The velocity for linear $\boldsymbol{E} \times \boldsymbol{B}$ drift is independent of mass. However, a more exact calculation shows that the higher-mass ions rotate at a higher frequency than the lower-mass ions, if they occupy the same volume in the trap, so that they are subjected to the same electric fields.

The equal and opposite momentum transfers due to the different rotation frequencies will cause the lighter ions to move in (toward the trap axis) and the heavier ions to move out. This separation of the ions continues until both species are rotating at the same frequency, as a result of the change in the space charge induced electric fields. Uniform rotation is one characteristic of thermal equilibrium of a nonneutral plasma [70]. In the low-temperature limit, the separation is complete, and a gap forms between the ion species [6, 67, 70]. The higher-mass ions form a «doughnut» around the lower-mass ions. Although the separation of the ion species limits the thermal coupling between the species, sympathetic laser cooling has been demonstrated experimentally in a number of cases [5, 6, 68].

The first temperature and density measurements on sympathetically cooled ions were done with Hg^+ ions, sympathetically cooled by laser-cooled Be^+ ions. The ions were confined in a Penning trap. Up to 12000 Be^+ ions, with temperatures less than 0.2 K, sympathetically cooled a larger number of Hg^+ ions to temperatures less than 1.8 K. As expected, Hg^+ ions were observed at larger distances from the trap axis than the Be^+. However, since the laser beams which probed the spatial distribution of the ions were perpendicular to the trap axis, not along the axis, complete separation could not be experimentally verified. The sympathetic cooling also prevented the radial diffusion of the Hg^+ ions. The Be^+ cooling laser applied a torque to the Be^+ ions, and this torque was transmitted by the Coulomb interaction to the Hg^+ ions. If the Be^+ cooling laser was blocked or tuned off resonance, the Hg^+ ions left the trap in several minutes. With sympathetic cooling, the Hg^+ ions were confined indefinitely.

In another Penning trap experiment, laser-cooled Mg^+ ions were used to sympathetically cool and radially confine Be^+ ions [68, 69]. The sympathetic cooling was used to improve the performance of the $^9Be^+$ frequency standard discussed in subsect. 2·2.2. In order to avoid a.c. Stark shifts, the laser used to cool and detect the Be^+ ions was turned off when the hyperfine transition was driven. With this laser off, the Be^+plasma expanded radially, due to torques originating from trap asymmetries. The radial expansion releases electrostatic potential energy, leading to heating of the ions [61]. The expansion and subsequent heating were eliminated by using approximately 10^5 Mg^+ ions to sympa-

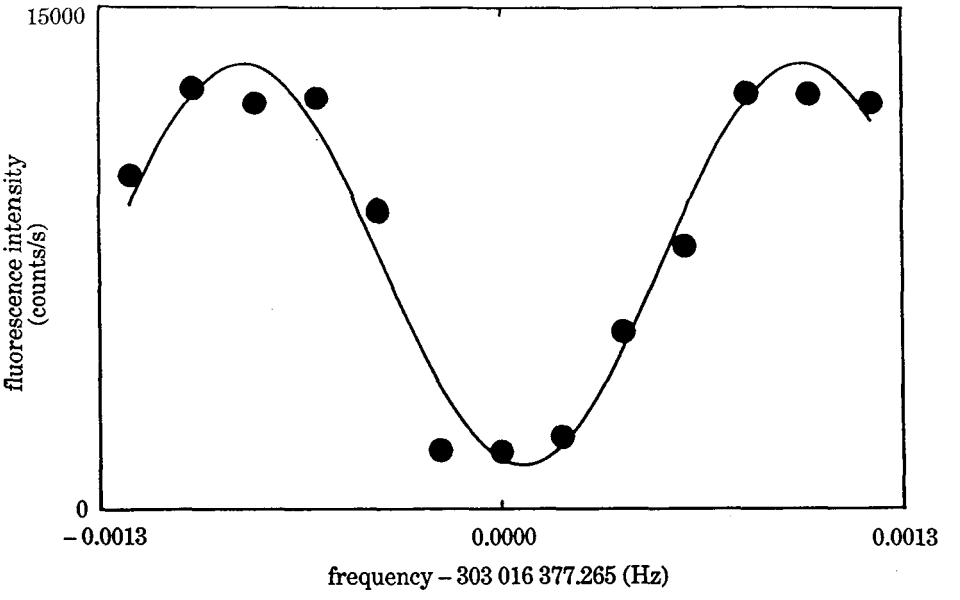

Fig. 7. – Resonance signal of the 303 MHz hyperfine transition of $^9Be^+$ at a magnetic field of 0.8194 T. A decrease in the fluorescence intensity corresponds to an increase in the transition probability. The width of the resonance is 900 μHz.

thetically cool about 5000 Be^+ ions to temperatures less than 0.25 K. The a.c. Stark shift on the Be^+ hyperfine transition, due to the Mg^+ cooling laser, was estimated to be less than 1 part in 10^{15}. This could be neglected relative to other systematic shifts. The Mg^+ ions were observed, with an imaging photomultiplier tube, to form a doughnut around the Be^+ ions. The size of the gap between the two species could not be determined, because the optical system could not be focused at both wavelengths at the same time. In this way a cold, steady-state cloud of Be^+ ions was obtained, independent of the Be^+ cooling laser. The sympathetic cooling enabled long measurement periods (up to 550 s), free from the perturbing effects of the Be^+ cooling laser. The observed resonance linewidth is inversely proportional to the measurement time. For a measurement time of 550 s, a 900 μHz linewidth was observed on the 303 MHz hyperfine transition. A typical resonance curve is shown in fig. 7.

The frequency standard based on this hyperfine transition was used to test the linearity of quantum mechanics [69]. A nonlinearity would have resulted in a dependence of the resonance frequency on the degree of excitation of the transition, analogous to the frequency shift of an anharmonic oscillator with amplitude. An upper limit of 6 μHz was placed on a nonlinear correction to the Hamiltonian of the Be^+ nucleus.

4. – Other cooling methods for ions.

Cooling based on optical dipole forces in combination with optical pumping, such as Sisyphus cooling, has been used with great success on neutral atoms [71]. Because these new cooling mechanisms lead to temperatures substantially below the Doppler-cooling limit, it is natural to investigate how these mechanisms might be employed in trapped-ion experiments. In the limit that the amplitude of the trapped-ion motion is greater than the optical wavelength λ responsible for the cooling, the cooling rates and cooling limits should be about the same as for free neutral atoms. It is interesting to consider Sisyphus cooling in the limit where the ion is confined to the Lamb-Dicke regime (amplitude of motion less than $\lambda/2\pi$). Assume the ion is harmonically bound in one dimension with oscillation frequency ω_a. For the condition $\omega_a \ll \gamma_b$, the limiting energy is theoretically found to be $(n_a + \frac{1}{2})\hbar\omega_a$, where $n_a \simeq 1$ [72]. This is considerably smaller than the limit $\hbar\gamma_b$ for Doppler cooling.

Other types of sympathetic laser cooling are being considered. At NIST we are investigating the possibility of cooling ions in one trap by electronically coupling them to laser-cooled ions contained in a second trap [73].

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