

TIME AND FREQUENCY

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FREQUENCY STANDARDS AND ACCURACY

In 1967 the General Conference on Weights and Measures adopted the cesium resonance frequency for the definition of the second. Universal Coordinated Time (UTC) has used a close approximation to the atomic seconds since 1972 (1). Time scales which refer to the rotation of the earth such as UTC are generated by inserting or leaving out seconds (leap seconds) at certain specified dates during the year, as necessary. This process is coordinated worldwide by the Bureau International de l'Heure (BIH). UTC is the de-facto basis for civil or legal time in most countries of the world (2). In addition to cesium beam standards, the atomic hydrogen maser has found use as primary frequency reference and clock.

Other promising techniques have been developed. The following table depicts a summary of all those techniques which promise accuracies better than 1×10^{-13} together with their currently reported accuracies. Accuracy is to be understood here in a very special meaning: It is a measure of the degree to which--in an experimental evaluation--one can deduce the unperturbed frequency of the respective transition by subtracting environment and apparatus related effects from the measured frequency (3).

FREQUENCY STABILITY

More important than accuracy to most frequency & time metrologists is probably the stability of a standard. Stability can be characterized in the frequency domain or in the time domain. The instantaneous fractional frequency deviation $y(t)$ from the nominal frequency ν_0 is related to the instantaneous phase deviation $\phi(t)$ from the nominal phase $2\pi\nu_0 t$ by definition

$$y(t) = \frac{\dot{\phi}(t)}{2\pi\nu_0} \quad (1)$$

CLASS OF DEVICES	DOCUMENTED ACCURACY (ORDER OF MAGNITUDE)	COMMENTS
MICROWAVE BEAMS	(Cs) 10^{-13} (3)	Cs BEST DOCUMENTED, REALIZATION OF THE DEFINITION OF THE SECOND
ATOM STORAGE (H)	(H) 10^{-12} (4)	COATED STORAGE VESSEL ONLY USABLE FOR FEW, SELECTED ATOMS
ION STORAGE (Mg, Ba, Ne, Tr, ETC)	(He) 10^{-9} (5)	SIGNAL-TO-NOISE PROBLEMS WITH PRESENT TECHNIQUES
SATURATED ABSORPTION (CH ₄ , I ₂ , SF ₆ , ETC)	(CH ₄) 10^{-12} (6)	USEFULNESS AS "TRUE" TIME/FREQUENCY STANDARDS CRITICALLY DEPENDENT ON PRECISION MICRO-WAVE/INFRARED/VISIBLE FREQUENCY SYNTHESIS
TWO PHOTON TRANSITIONS (Mg, H, ETC)		
INFRARED & OPTICAL BEAMS (Mg, Ca, I ₂ , ETC.)	(I ₂) 10^{-11} (7)	

I. frequency domain:

In the frequency domain, frequency stability is conveniently defined (8) as the one sided spectral density $S_y(f)$ of $y(t)$. More directly measurable in an experiment is the phase noise or, more precisely, the spectral density of phase fluctuations $S_\phi(f)$ which is related to $S_y(f)$ by

$$S_y(f) = \frac{f^2}{v_0^2} S_\phi(f) \quad (2)$$

For the above, f is defined as the Fourier frequency offset from v_0 . Of course, $S_\phi(f)$ cannot be perfectly measured; however, useful estimates of $S_\phi(f)$ can easily be obtained. One useful experimental arrangement to measure $S_\phi(f)$ is given in Fig. 1. If the measured oscillator and the reference oscillator are equal in their total performance, and if the phase fluctuations are small, i.e., much less than a radian, then, for one oscillator (9):

$$S_\phi(f) \approx 2 \frac{V^2(f)}{V_R^2} \quad (3)$$

where $V(f)$ is the rms voltage at the mixer output due to the phase fluctuations within a 1 Hz bandwidth at Fourier frequency f , V_R is a reference voltage which describes the mixer sensitivity and is equivalent to the sinusoidal peak-to-peak voltage of the two oscillators, unlocked and beating. The phase lock loop is only necessary to keep the signals in phase-quadrature at the mixer; it must be a sufficiently loose lock, i.e., its attack time (corresponding to the unity-gain condition) is long enough to not affect all (faster) fluctuations to be measured. Fig. 2 depicts the measured phase noise performance of a system as in Fig. 1 which uses available state-of-the-art electronic components. Also shown is the measured phase noise of one of the best available crystal oscillators; its corresponding time domain performance is approximately shown in Fig. 4. Fig. 2 clearly shows that present measurement capabilities are not yet taxed by available oscillators.

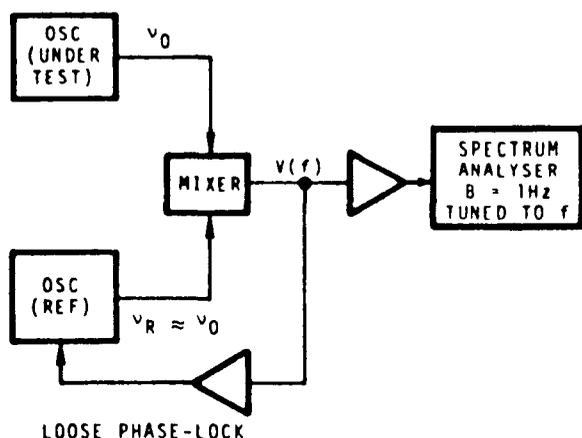


Fig. 1. Phase Noise Measurement System

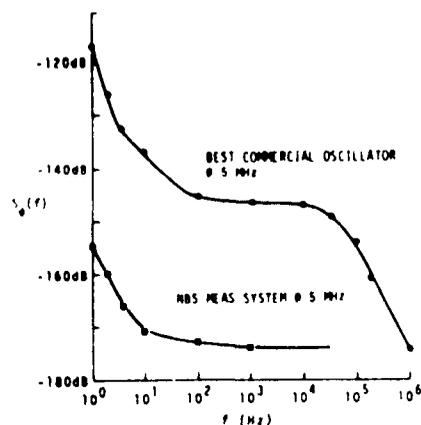


Fig. 2. Phase Noise Plots

II. time domain

The relationship between the frequency and time domain, essentially a Fourier transformation, is extensively covered in Ref. (8). In the time domain, frequency stability is defined by the sample variance:

$$\sigma_y^2(N, T, \tau, B) \equiv \left\langle \frac{1}{N-1} \left(\sum_{n=1}^N \bar{y}_n - \frac{1}{N} \sum_{k=1}^N \bar{y}_k \right)^2 \right\rangle \quad (4)$$

where $\langle \rangle$ denotes an infinite time average, N is the number of frequency readings in measurements of duration τ and repetition interval T , B is the bandwidth of the measurement system. Some noise processes contain increasing fractions of the total noise power at lower Fourier frequencies; e.g., for flicker or frequency noise the above variance approaches infinity as $N \rightarrow \infty$. This, together with the practical difficulty to obtain experimentally large values of N led to the convention of using always a particular value of N (10). In recent years, frequency stability has become almost universally understood as meaning the squareroot of the two-sample or Allan Variance (8) $\sigma_y^2(\tau)$ defined as in Eq. (4) for $N = 2$, $T = \tau$.

$$\sigma_y^2(\tau) = \left\langle \frac{(\bar{y}_{k+1} - \bar{y}_k)^2}{2} \right\rangle \quad (5)$$

$\sigma_y(\tau)$ is convergent for all noise processes commonly found in oscillators. It should be noted that even for Eq. (5) B remains an important parameter which must be taken into consideration. Fig. 3 depicts three different measurement systems (11) which may be used to determine $\sigma_y(\tau)$.

In Fig. 4 a typical measurement capability (at 5 MHz) using Schottky barrier diode mixers is depicted. As in the case of frequency domain measurements, it is obvious that the existing measurement system capability is fully adequate to measure any existing oscillator. Figure 4 is adapted from Ref. (12). It includes crys-

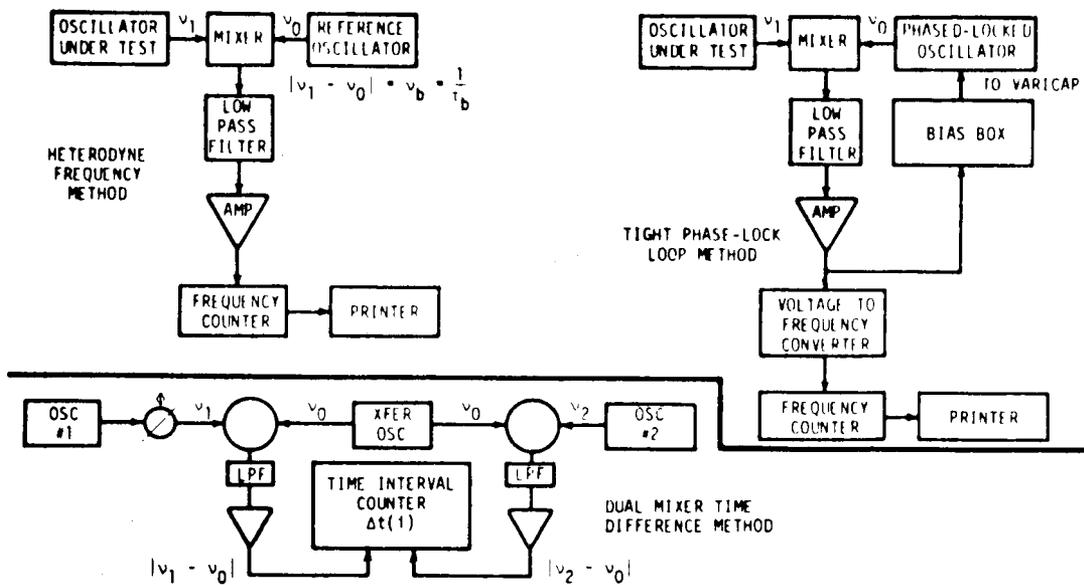


Fig. 3. Time Domain Measurement Systems

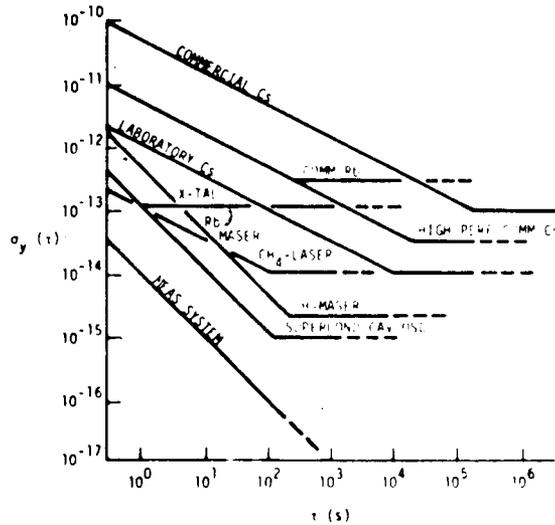


Fig. 4. Measured Frequency Stability of Various Frequency Standards

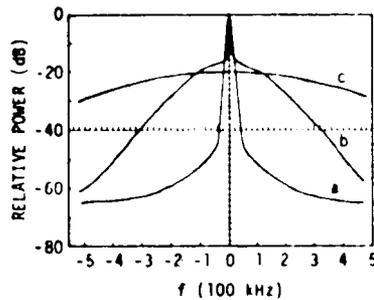


Fig. 5. Spectral Line Profile Under Multiplication. Resolution is Limited to 10 KHz Spectrum Analyzer Bandwidth

tal and superconducting cavity oscillators and various types of laboratory and commercial atomic frequency standards. Figure 4 shows that for short sampling times quartz crystal oscillators, superconducting cavity oscillators, and rubidium masers are the oscillators of choice. For medium-term stability, the hydrogen maser and superconducting cavity oscillator are superior to any other standard which is available today. For very long-term stability or clock performance, cesium standards are presently the devices of choice. Rubidium standards are not superior in any region of averaging times, however, they excel in the combination of good performance, cost and size.

FREQUENCY SYNTHESIS ABOVE THE MICROWAVE REGION

We will later return to the importance of long-term stability in time generation. We shall now consider the importance of high short-term stability (or low-phase noise) in the area of frequency synthesis into the infrared. Precision synthesis is a crucial prerequisite to high resolution absolute frequency measurements, to the use of frequency standards in the infrared and visible regions (comp. Table 1), and to the concept of a unified standard for time and length via the definition of the speed of light (13). Present successes with frequency synthesis to the 88 THz transition of methane realized only 6×10^{-10} measurement precision (14). One of the critical limitations is the need for a series of intermediate oscillators (klystrons, lasers) of inferior stability not only to compensate for inefficient multipliers but also to serve as spectral "purifiers". Let us examine the phase noise requirements on an oscillator for single step harmonic generation up to a certain harmonic number. If a state-of-the-art crystal oscillator at 5 MHz with a phase noise performance as in Fig. 2 is assumed, then Fig. 5 shows its spectral line profile at 9.2 GHz (curve a), 150 GHz (curve b), and 1.5 THz (curve c). Although there still is power available, the carrier has now totally disappeared, the linewidth has increased a factor of 10^6 from .006 Hz to 600 KHz. Clearly curve c can no longer be used as a precision reference signal.

The relative power in the carrier P_c , and the relative power in the pedestal P_p , (15) are shown in Fig.6.

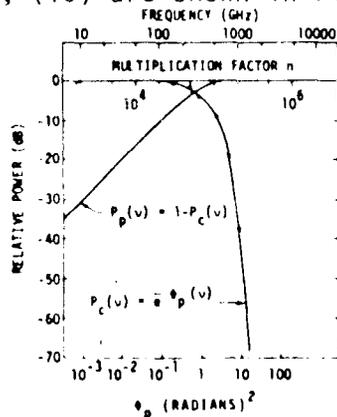


Fig. 6. Power in Carrier and in the Pedestal Under Multiplication

The exponential loss of power from the carrier when the mean square phase fluctuations from the pedestal $\phi_p(\nu)$ exceed 1 radian is the most serious effect. For the present state-of-the-art 5 MHz crystal oscillators, the power is evenly divided between carrier and pedestal at ~300 GHz while at 1 THz the carrier has only -50 dB of the total power. Therefore, the only way to extend the useful working range of the present 5 MHz oscillators above 1 THz is to reduce the phase modulation due to the pedestal. This can be done either by reducing the white phase modulation level of the pedestal or reducing the pedestal bandwidth B_0 , either in the oscillator or somewhere along the multiplier chain. Of course, changing the pedestal height or width affect the spectrum in different ways.

It appears possible (16) to construct crystal oscillators with at least 40 dB reduction in the white phase level of the pedestal. This would provide an oscillator with a possible working range extending to 30 THz without the use of intermediate oscillators or filters. The use of a passive filter with a bandwidth of 6 Hz at 9.2 GHz such as a superconducting cavity filter (15) would make it possible to multiply the present oscillators to 100 THz without the need for intermediate oscillators. The linewidth would be approximately 70 Hz. The realization of oscillators of higher spectral purity than presently available, a prerequisite for precise infrared frequency synthesis, thus is within today's technical possibilities.

TIME AND CLOCKS

One of the principal applications of frequency standards is their use as clocks. In a very real sense, any long-term frequency measurement and astronomical distance measurement is a time measurement. Astronomy has in a de-facto sense relied on a unified time-length standard by measuring distances in units of time using an adopted value for the speed of light.

The time error T at the elapsed time t after synchronization can be written as

$$T(t) = T_0 + R_0 t + \frac{1}{2} D t^2 + \dots + \epsilon(t) \quad (6)$$

where T is the time of the clock minus the time of the reference (ideal "true" time), T_0 is the synchronization error at $t = 0$ and R_0 the rate (fractional frequency) difference between the two clocks under comparison averaged around $t = 0$. D is the linear (fractional) frequency drift term and $\epsilon(t)$ contains all other fluctuations; e.g., those due to white noise, flicker noise, etc. The time dependence of $\epsilon(t)$ can be calculated or estimated statistically (17), knowing the power laws of the noise processes that model the clocks involved.

In addition to the above considerations it must be noted that time (date) in contrast to time-interval (frequency), cannot be reproduced. This is a very significant difference because accuracy and stability, which are well-defined quantities in discussing frequency, become more complex and elusive with regard to the passing of time (dates) as can be seen from Eq. (6). For example the question of a time standard and its long-term stability and accuracy is

not trivial. Our time is generated today by an ensemble of coordinated clocks of worldwide distribution (18) which establish the de-facto time (date) standard. Time (date) accuracy is loosely used as the degree of conformity of a clock to this de-facto reference which in reality may not be accurate nor stable but only uniform, the uniformity being assessed by internal comparisons and evaluations of the ensemble. The ensemble as a whole may have an undetected or unaccounted offset from the unit of time, a frequency drift term, etc. Thus internal estimates of "accuracy" (really uniformity) which today are of the order of microseconds per year, may be much too optimistic with regard to a hypothetical, ideal clock running unchangingly on the exact unit of time. Undetected frequency offsets of parts in 10^{13} and drifts of parts in 10^{13} per year are likely. The actual time errors then may be more like tens of microseconds per year. Such errors can have critical importance in long-term and frequency measurements in experiments such as a determination of a possible change in the fine structure constant with time. Only the availability of primary standards of sufficient accuracy can reduce such errors. Preferably, several primary standards which are based on different physical principles, e.g., kind of atom, design of apparatus, etc., should be available to minimize the probability of undetected effects common to one particular type of device.

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