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LETTER TO THE EDITOR

Influence of ion dynamics on Hα and Hβ at low densities

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Abstract. Calculations for the Stark broadening of Hα and Hβ using a unified theory for both ions and electrons are presented in the density range 10^{14} to 10^{15} cm^{-3} at a temperature of 10^4 K. Although the theory is of marginal validity at these densities, it shows quite conclusively the influence of ion dynamics in reducing the dip at the centre of the Hβ profile and the peak in the centre of the Hα profile.

Recent experiments (Wiese *et al* 1972, Burgess and Mahon 1972) on the broadening of hydrogen Balmer lines have shown discrepancies with theoretical calculations (Kepple and Griem 1968, Vidal *et al* 1973) close to line centre, although good agreement is obtained with Vidal *et al* (1973) in the line wings. In particular, for Hβ, at electron densities $n_e \sim 8\times10^{16}$ cm^{-3}, the calculations yield central minima about 35 % down from the peak intensities, in contrast to a measured dip of only 15 %, (Wiese *et al* 1972) and at densities around 10^{15} cm^{-3} the central dip appears to be absent (Burgess and Mahon 1972). Similarly, for Hα, the unshifted component in the line centre shows up more strongly in the calculation than it does in the experiments (Wiese *et al* 1972). By approximate inclusion of effects due to inelastic collisions by electrons, Hill *et al* (1971) have found a relatively small reduction in structure at line centre. The calculations of Kepple and Griem (1968 and Vidal *et al* (1973) both treat the ions as stationary in the quasi-static approximation. However, it is known that the quasi-static approximation breaks down in the line centre and the dynamic properties of the ions could be responsible for some of the discrepancy. Ion–ion correlations have been shown to be important in microfield calculations (Mozer and Baranger 1960) and therefore should be included in any theory for ion dynamics. In fact, the theory of Dufty (1969) includes many-body effects but calculations (Lee 1973) thus far have been restricted to a second order evaluation of the desired resolvent operator. While the validity of these second order calculations is open to question, they nonetheless produce an observable effect on Hβ reducing (at 10^{15} cm^{-3}) the dip at line centre from about 64 % to about 43 % (before Doppler folding).

The purpose of this note is to present calculations of dynamic ion broadening effects using a unified theory for both ions and electrons. The ion–ion correlations are approximated by considering the ions and electrons as independent shielded quasiparticles, with a shielding length of order of the Debye length ρ_D. This procedure is certainly consistent to second order in the electric field (see Chappell *et al* 1970). In addition, it is found by Mozer and Baranger (1960) that the low frequency microfield, as required by a static theory, can also be closely approximated by considering the ions interacting only via Debye-shielded fields (with shielding length in vicinity of $\rho_D/\sqrt{2}$ or $\rho_D/\sqrt{1 \cdot 5}$) and neglecting other ion–ion correlations.

Since the unified theory (Smith *et al* 1969, Voslamber 1969, Vidal *et al* 1970, 1971) is essentially a binary collision theory, a *necessary* condition for its validity is that strong collisions be separated in time. Using the following formula for the strong collision radius $\rho_s \simeq (n^2 - n'^2)\, \hbar/m_e v_i$ (Kepple and Griem 1968) (where v_i is the relative ion velocity, m_e is the electron mass, and n an n' are the principal quantum numbers for upper and lower states respectively), this condition is

$$n_e \pi v_i \rho_s^2 < v_i \rho_s^{-1} \tag{1}$$

which gives $n_e \lesssim 7 \times 10^{14}\ \text{cm}^{-3}$ for Hβ for hydrogen ion perturbers at 10^4K (and a factor of about two less for helium perturbers). Since the time between strong electron collisions is greater than $n_e \pi v_i \rho_s^2$ by a factor of $\mu' (= (m_r/m_e)^{1/2}$ where m_r is the reduced ion mass and m_e the electron mass) and the duration of a strong electron collision is less than $t_s = v/\rho_s$ by the same factor μ', all strong collisions are separated in time when inequality (1) is satisfied. Therefore, the unified theory is used for both ions and electrons. It has been argued, however, by Capes and Voslamber (1972) that the condition (1) is not sufficient and that the cumulative effects of weak collisions also have to be taken into account. The unified theory neglects overlapping (in time) weak collisions beyond second order in the electric field (Smith *et al* 1973). The condition obtained by Capes and Voslamber (1972) (their condition (*c*)) may be rewritten as

$$\frac{\rho_{max}}{v_i} w_W < 1 \tag{2}$$

with $w_W = n_e v_i \pi \rho_s^2 \ln (\rho_{max}/\rho_s)$ and ρ_{max} is the maximum impact parameter of order of the Debye length ρ_D (Chappell *et al* 1970 show $\rho_{max} = 0 \cdot 68\ \rho_D$). Since w_W is essentially the effective frequency of weak collisions and ρ_{max}/v_i is the maximum duration of a collision, this condition (2) is obviously sufficient, since it implies *all* collisions are separated in time. For electron perturbers it is almost always satisfied. At low frequencies (less than the ion plasma frequency $\omega_{pi} (= v_i/\rho_D)$) shielding by the ions becomes important as well as by the electrons and a shielding length of about $\rho_D/\sqrt{2}$ is appropriate (Mozer and Baranger 1960). In this case condition (2) requires for Hβ for hydrogen ion perturbers $n_e \lesssim 10^{14}\ \text{cm}^{-3}$ at 10^4K. In the density range of interest for Hβ from $10^{14}\ \text{cm}^{-3}$ to $10^{15}\ \text{cm}^{-3}$ at 10^4K this condition is obviously too restrictive. At $10^{14}\ \text{cm}^{-3}$ the Doppler width, $\Delta\omega_D$, is over ten times ω_{pi}. Therefore, if we take $1/\Delta\omega_D$ as the time of interest (since convolution with the Doppler profile smears out all structure within $\Delta\omega_D$), then the distant collisions at about ρ_D are no longer effective and the maximum effective impact parameter is $\rho_{max} \sim v_i/\Delta\omega_D$ (see Lewis 1961). Since now, ρ_{max} and ρ_s are comparable, condition (2) is no longer more restrictive than condition (1).

It is also of interest to note that numerical calculations for helium lines (Barnard *et al* 1974) (where, however, distant collisions are less important) indicate that condition (1) is, within a factor of two or so, a reasonable criterion.

In spite of the uncertain (but admittedly marginal) validity it was decided that the unified results were worth presenting for comparison with experiment. To modify the calculations of Vidal *et al* (1970, 1971, 1973), it is only necessary to add a quantity $i(\Delta\omega_R)_{ions}$ for the ions to the similar quantity for the electrons and omit the ion field integration. In $i(\Delta\omega_R)_{ions}$ the quantity p_1 is unchanged, p_2 is changed to

$$p_2 = -(4\mu')\pi^{1/2}n_e D^3 C^2[B - \ln(4C^2\mu'^2)] \tag{3}$$

where $\mu' = (m_r/m_e)^{1/2}$ (with m_r the reduced mass for interacting ion–hydrogen atom pair) and

$$z = 3(nq-n'q')(\lambda\!\!\!^{-}/\mu+\tfrac{3}{2}(nq-n'q')a_0)^{-1} \tag{4}$$

with $\mu = (m_i/m_e)^{1/2}$; all other quantities refer to electrons as in Vidal *et al* (1970, 1971, 1973). It should be noted that this relation (equation (3)) for p_2 is only correct as long as $C\mu' \ll 1$. This is generally well fulfilled for the electrons but not necessarily for the ions, in particular for the outermost Stark components and with increasing electron density. When this inequality is not completely fulfilled the constant p_2 has been calculated according to (Vidal *et al* 1970)

$$p_2 = -8\sqrt{\pi}n_e D^3 C\int_0^\infty du u^2 \,e^{-u^2}\int_0^\infty \sin\left(\frac{2C\mu'}{u}z\right)(1+z^2)^{-1}\,dz. \tag{5}$$

It should be stressed however when $C\mu' \sim 1$, the condition of equation (1) can be violated since $\rho_s \simeq C\mu'\rho_D$ and $\rho_D/\rho_0 \gtrsim 1$ (with $4\pi/3\rho_0{}^3 n_e = 1$). In addition, in the calculations the electron Debye length ρ_D was used, rather than $\rho_D/\sqrt{2}$ which is appropriate for shielding by both ions and electrons, since at frequencies of interest (greater than ω_{pi}) shielding by ions is small (ie the Lewis cutoff is appropriate).

Figure 1 compares the Hβ results for hydrogen ion perturbers and for helium ion perturbers with the static results at $n_e = 10^{14}\,\text{cm}^{-3}$ and $T = 10^4$K. $\Delta\alpha$ is the usual reduced wavelength (Kepple and Griem 1968, Vidal *et al* 1973). Notice the dramatic

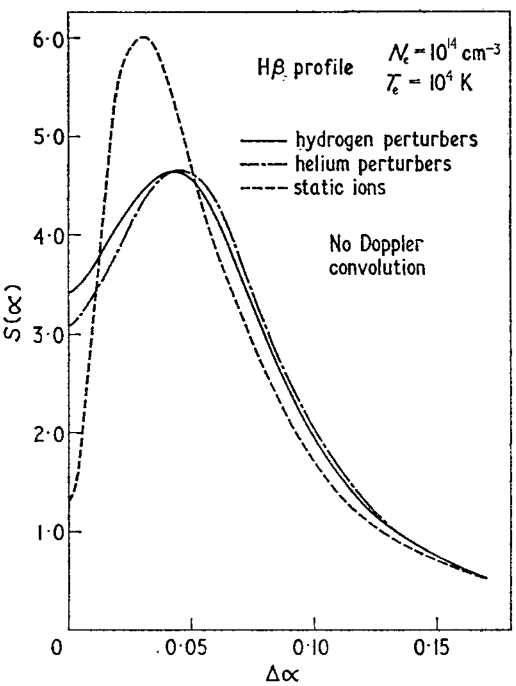

Figure 1. Comparison of static and unified theories for ion broadening of Hβ at 10^{14} cm^{-3}. Profile not convolved with Doppler profile.

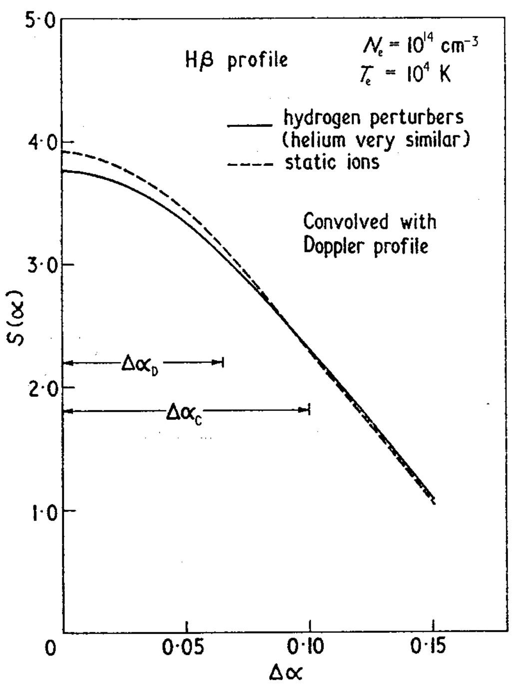

Figure 2. Comparison of static and unified theories for ion broadening of Hβ at 10^{14} cm^{-3}, with profile convolved with Doppler profile corresponding to 10^4 K. $\Delta\alpha_D$ and $\Delta\alpha_C$ represent Doppler width and Weisskopf frequencies.

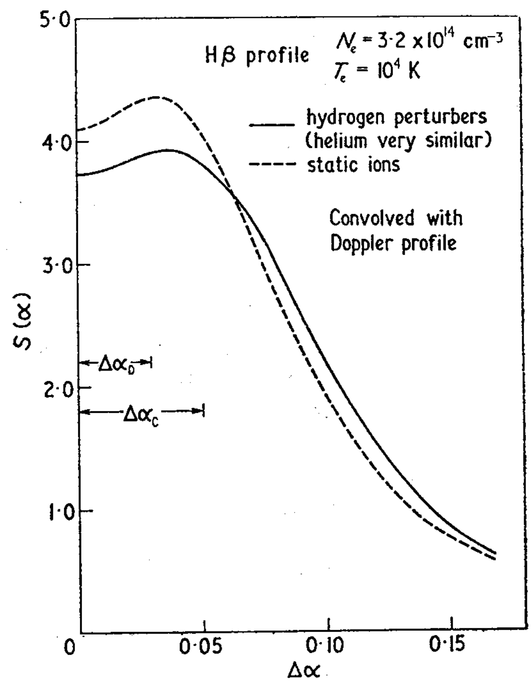

Figure 3. As in figure 2, but at $3{\cdot}2\times10^{14}$ cm^{-3}.

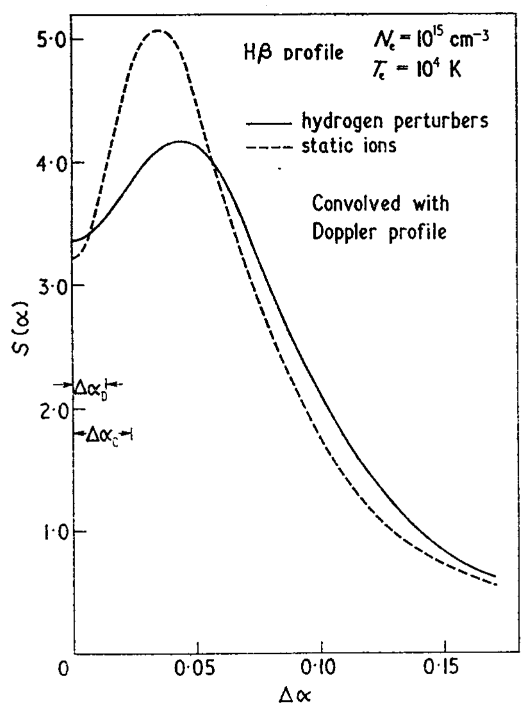

Figure 4. As in figure 2, but at 10^{15} cm^{-3}.

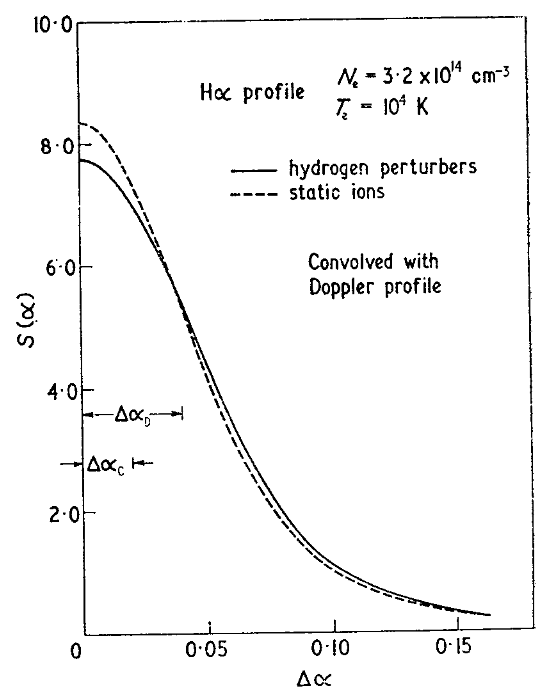

Figure 5. Comparison of static and unified theories for ion broadening of Hα at $3{\cdot}2\times10^{14}$ cm^{-3} with profile convolved with Doppler profile corresponding to 10^4 K.

filling in of the dip at line centre. This filling in of the dip is also apparent at lower densities where the condition (2) (corresponding to $n_e \lesssim 10^{14}\ cm^{-3}$) is better satisfied. The differences are much reduced after a convolution with the Doppler profile is performed, and results (after convolution with a profile corresponding to 10^4K) at densities of $10^{14}\ cm^{-3}$, $3{\cdot}2\times10^{14}\ cm^{-3}$, and $10^{15}\ cm^{-3}$ are shown in figures 2, 3 and 4 respectively. In addition, profiles with helium ion perturbers were calculated; they show a slightly more pronounced dip at line centre (roughly 10% more at $10^{14}\ cm^{-3}$), but after convolution the profiles are essentially indistinguishable. Figure 5 shows the corresponding line profiles for Hα at an electron density of $3{\cdot}2\times10^{14}\ cm^{-3}$ where the peak in the line centre due to the unshifted Stark component is noticeably reduced. The quantities $\Delta\alpha_D$ and $\Delta\alpha_C$ in figures 2 to 5 correspond to the Doppler width $\Delta\omega_D$ and Weisskopf frequency ($= v_i/\rho_s$) respectively. We do not expect a large variation with perturber mass since μ' only changes by a factor of $\sqrt{2}$ in going from hydrogen perturbers to very massive perturbers. Figures 2 to 5 are for equal ion and electron temperatures (10 000 K); calculations with electron temperature of 10 000 K and an ion temperature of 5 000 K yielded only minor differences. Furthermore, collisional effects on the Doppler profile (ie Doppler narrowing) will be unimportant at the densities considered because the mean free path for 90° deflection is much greater than the wavelength of the radiation (Rautian and Sobel'man 1967).

We conclude from these results that ion dynamics will considerably reduce the dip at the centre of Hβ and that these dynamic effects appear to explain part of the discrepancies between the theory and the results of Burgess and Mahon (1972) (although at about $10^{15}\ cm^{-3}$ they are in a region where this theory is particularly of marginal validity). Although Lee (1973) also takes into account ion dynamics, his second order (Dufty 1969) operator $H^{(2)}(\Delta\omega)$ actually gets quite large as $\Delta\omega\to0$ (and diverges if natural damping is not included). This divergence is a manifestation of the breakdown of the second-order approximation in the Lee–Dufty theory (Barnard *et al* 1974) and could account for too large a dip at line centre in his computations (since $H^{(2)}(\Delta\omega)$ occurs in the denominator of the expression determining the intensity).

Further work on ion dynamics will of course be necessary before our tentative conclusions relating problems with hydrogen profiles to ion dynamics are confirmed.

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