FREQUENCY BIASES IN A BEAM TUBE CAUSED BY RAMSEY EXCITATION PHASE DIFFERENCES

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Abstract

A phase difference between the two interaction regions of a Ramsey excitation resonance structure results in a frequency bias in the measured beam resonance. A simple mathematical model is discussed which describes the dependence of this bias on the phase difference, the microwave power level, the modulation amplitude, and the resonance linewidth. This dependence results from the interaction of the modulated microwave excitation frequency with the asymmetric shape of the slightly shifted resonance line. In a first order approximation, no dependency on the frequency modulation amplitude is expected. Near-linear dependencies on the linewidth and microwave power level which are quite pronounced even at relatively small cavity phase differences are predicted.

The theoretical results are compared with one set of experimental data on the microwave power dependence as measured in 1969 with the primary cesium beam standard NBS-III. After a correction is applied to remove the power dependence due to spectral impurity of the microwave excitation, the remaining measured power dependence agrees quantitatively with that calculated using a cavity phase difference of about 3 milliradians.

Key Words: Cavity phase shift, Cesium beam tube, Frequency accuracy, Frequency standard, Power shift, Resonance line shape.

1. Introduction

In accordance with the international definition,[1] the unit of time interval, the second, is realized with a cesium beam apparatus. Several bias corrections have to be applied to the measured resonance frequency in order to obtain the resonance frequency of the unperturbed, free cesium atom.[2] In this paper we consider only those bias corrections which are associated with a phase difference between the two interaction regions of a Ramsey excitation resonance structure.[3]

In this paper we present a simple mathematical model which allows analytical solutions to the question of frequency shifts in the presence of a cavity phase difference. The results are intended to aid in the understanding of the physical mechanisms involved. Although our mathematical model is only an approximation to the physical situation, and is tailored to a particular set of conditions, we feel that it is close enough to the real conditions which were encountered in the National Bureau of Standards Frequency Standard,[2] NBS-III, to warrant a quantitative comparison with experimental data from this apparatus. This is done in Section V of this paper.

II. Calculation of Frequency Shifts

We will proceed on the basis of the following set of assumptions and approximations: (a) No frequency deviations from resonance exceed the linewidth; (b) the microwave interrogation power does not deviate greatly from optimum power; (c) the modulation of the microwave signal is sinusoidal; (d) the modulation frequency is less than the resonance linewidth; (e) the velocity distribution in the atomic beam is Maxwellian, and (f) $\ell << L$ where ℓ and L are the lengths of the interaction and drift regions, respectively, as defined by Ramsey.[3]

In the presence of a cavity phase difference the central peak of the Ramsey resonance pattern will be displaced from the resonance frequency of the unperturbed atom. Furthermore, the lineshape will display a certain asymmetry. We restrict ourselves to the conditions (a) and (b) above and can in first order approximate the central peak of the Ramsey pattern, $I\{\nu\}$, by

$$I\{\nu\} = A + B\phi \sin\left\{\frac{\pi(\nu - \nu_\phi)}{U}\right\} + C\cos\left\{\frac{\pi(\nu - \nu_\phi)}{W}\right\}. \quad (1)$$

In eq (1) A, B, and C are constants under given experimental conditions; W is the linewidth at half intensity; U has the characteristic of a linewidth: $U \equiv W$ for monovelocity atoms, U is the same order of magnitude as W for our assumption (e) above; and ν_ϕ is a parameter related to the cavity phase difference ϕ. Using condition (e) above, it can be shown that

$$\nu_\phi - \nu_o = \frac{3\phi}{4\pi} W. \quad (2)$$

In eq (2) ν_o is the resonance frequency in the absence of a cavity phase difference. Because of the asymmetry, the frequency of the central peak ν_p differs from ν_ϕ. By setting $dI\{\nu\}/d\nu = 0$, we obtain

$$\nu_p \approx \nu_\phi + \frac{BW^2\phi}{\pi CU}. \quad (3)$$

We now proceed to calculate the frequency to which the slave oscillator frequency servo will "tune-in." If we modulate ν in a sinusoidal fashion [condition (c) above] about some value ν_1 with the amplitude ν_m and the angular frequency ω we can write

$$\nu\{t\} = \nu_1 + \nu_m \sin\omega t. \quad (4)$$

We now make use of assumption (d) above and substitute eq (4) into eq (1), using eq (3) to express B. We obtain

$$I\{t\} = A + C\left[(\nu_p - \nu_\phi)\frac{\pi U}{W^2}\sin\left\{\frac{\pi}{U}(\nu_1 - \nu_\phi + \nu_m \sin\omega t)\right\} + \cos\left\{\frac{\pi}{W}(\nu_1 - \nu_\phi + \nu_m \sin\omega t)\right\}\right]. \quad (5)$$

We are interested in the coefficient of that term in $I\{t\}$ which varies as $\sin\omega t$, because the servo forces this term to be zero.

Now

$$\sin\left\{\frac{\pi}{U}(\nu_1 - \nu_\phi + \nu_m \sin\omega t)\right\} = 2\cos\left\{\frac{\pi}{U}(\nu_1 - \nu_\phi)\right\} J_1\left\{\frac{\pi\nu_m}{U}\right\}\sin\omega t + \text{harmonics}$$

and

$$\cos\left\{\frac{\pi}{W}(\nu_1 - \nu_\phi + \nu_m \sin\omega t)\right\} = -2\sin\left\{\frac{\pi}{W}(\nu_1 - \nu_\phi)\right\} J_1\left\{\frac{\pi\nu_m}{W}\right\}\sin\omega t + \text{harmonics}.$$

Then the coefficient of $\sin\omega t$ in $I\{t\}$ [eq (5)] vanishes when

$$\sin\frac{\pi}{W}(\nu_1 - \nu_\phi) = (\nu_p - \nu_\phi)\frac{\pi U}{W^2}\cos\frac{\pi}{U}(\nu_1 - \nu_\phi)\frac{J_1\left(\frac{\pi\nu_m}{U}\right)}{J_1\left(\frac{\pi\nu_m}{W}\right)}. \quad (6)$$

In eq (6), ν_1 can now be interpreted as the resonance frequency of the beam tube, since ν_1 is the frequency to which the servo will "tune-in." Note that ν_1 is not equal to either ν_p or ν_ϕ.

We have assumed that the offset is always small compared to the linewidth, i.e., $(\nu_1 - \nu_\phi) << W$, $(\nu_1 - \nu_\phi) << U$, and we can rewrite eq (6)

$$\nu_1 - \nu_\phi \approx (\nu_p - \nu_\phi)\frac{U J_1\left\{\frac{\pi}{U}\nu_m\right\}}{W J_1\left\{\frac{\pi}{W}\nu_m\right\}}. \quad (7)$$

Substituting eq (2) and eq (3) into eq (7) we obtain

$$\nu_1 - \nu_o \approx \frac{3\phi W}{4\pi}\left[1 + \frac{4B}{3C}\frac{J_1\left\{\frac{\pi}{U}\nu_m\right\}}{J_1\left\{\frac{\pi}{W}\nu_m\right\}}\right]. \quad (8)$$

Equation (8) gives us the frequency bias $\Delta\nu$ which we define

$$\nu_1 - \nu_o \equiv \Delta\nu. \quad (9)$$

In order to evaluate eq (8) we have to express B/C in terms of the operating conditions. Following eq (1) we choose a linear approximation based on the numerical results of Ref. 3, and we obtain $B\phi\pi/CU \approx 2\phi/3\pi W_o$. W_o is the linewidth at optimum power P_o. Equation (8) can now be written

$$\Delta\nu \approx \frac{3\phi W}{4\pi}\left[1 + \frac{8U}{9\pi^2 W_o}\frac{J_1\left\{\frac{\pi}{U}\nu_m\right\}}{J_1\left\{\frac{\pi}{W}\nu_m\right\}}\right]. \quad (10)$$

III. Frequency Modulation Amplitude Dependence

In eq (10) any dependence on the frequency modulation amplitude is solely due to the arguments of the Bessel functions which contain ν_m. The approximate nature of our discussion requires that we restrict our discussion to modulation amplitudes not exceeding the optimum modulation, that is to

$$\nu_m \le \nu_{mo} \equiv \frac{1}{2}W_o. \quad (11)$$

This and the fact that U and W can be expected not to differ greatly leads us in a first order approximation to

$$\Delta\nu \approx \frac{3\phi W}{4\pi}\left[1 + \frac{8W}{9\pi^2 W_o}\right]. \quad (12)$$

According to eq (12), $\Delta\nu$ has no functional dependence on ν_m. We conclude that in the limits of our approximation no frequency modulation amplitude dependent frequency bias is to be expected for $\nu_m \le \nu_{mo}$. This result is in accordance with our experimental observations on the cesium beam frequency standards at the National Bureau of Standards.

IV. Power Dependence

We can use eq (12) to obtain the dependence of the frequency bias $\Delta\nu$ on the microwave power P. Equation (12) shows that the frequency bias is a function of the phase difference and the resonance linewidth. The linewidth, moreover, is a function of the microwave power level. This can be understood intuitively if one realizes that with decreasing microwave power the slower molecules become relatively more effective which results in a corresponding line-narrowing.[3] The frequency bias $\Delta\nu$ thus becomes power dependent. A numerical calculation of the relationship between linewidth and microwave power is depicted in Fig. 1. In Fig. 2 we depict the frequency bias $\Delta\nu$ as a function of both linewidth [eq (12)] and microwave power [eq (12) and Fig. 1]. The ordinate is normalized to $\Delta\nu/\Delta\nu_o = 1$ at optimum power, $P = P_o$ and $W = W_o$, where $\Delta\nu_o$ denotes the frequency bias at optimum power.

Equation (12) and Fig. 2 show that the power dependence of the frequency bias is almost linear; however, we note that the extrapolation of this near-linear portion to $P \to 0$ does not yield a vanishing frequency bias. A reduction of power from P_o to $P_o/2$ will result in a frequency bias change of about 1/3 of the total bias at optimum power or more accurately

$$\Delta\nu\,\{P_o/2\} \approx 0.70\,\Delta\nu_o. \quad (13)$$

V. Experimental Results

The reported measurements were performed during the accuracy evaluation of the cesium beam frequency standard, NBS-III, in 1969.[2] Figure 3 depicts the experimental results. Plotted is the fractional frequency change as a function of the microwave power level for opposing beam directions (solid lines). The precision of these measurements is discussed in Ref. 2. The microwave power changes ranged from P_o (optimum power) to not less than one-fourth P_o. The actual value for P_o was $P_o = 1.4$ mW; the frequency modulation amplitude was adjusted to $\nu_m \approx \nu_{mo}$. The two slopes are $+22 \times 10^{-13}$ per mW and -11×10^{-13} per mW. The total fractional frequency change with beam reversal at $P = P_o$ is 78×10^{-13} which gives us $\Delta\nu_o/\nu_o = 39 \times 10^{-13}$.

We believe that the asymmetry in the two slopes of Fig. 3 is caused by asymmetries in the spectrum of the microwave signal.* Such effects may be expected to be unchanged under beam reversal and to depend linearly on the microwave power (for a small single sideband perturbation).[3] We obtain symmetric slopes if we subtract a slope of $+5.5 \times 10^{-13}$ per mW. Thus we can conclude that the spectrum causes a power shift of $+5.5 \times 10^{-13}$ per mW (dashed line in Fig. 3), and the cavity phase shift a power dependence of $\pm 16.5 \times 10^{-13}$ per mW (dashed-dotted lines in Fig. 3).

As we expected theoretically, we have a seemingly linear functional dependence. The measured biases at optimum power and half-optimum power are respectively, $\Delta\nu_o/\nu_o = 39 \times 10^{-13}$ and $\Delta\nu\{P_o/2\}/\nu_o = 27.5 \times 10^{-13}$ or

$$\Delta\nu\{P_o/2\} \approx 0.71\ \Delta\nu_o \qquad (14)$$

which is in agreement with the theoretical results of Fig. 2 and eq (13).

We can now calculate the cavity phase shift ϕ from eq (12). The numerical calculation yields

$$\Delta\nu_o = 0.26\ W_o \phi\ . \qquad (15)$$

The linewidth was measured to be $W_o = 45$ Hz and we obtain from eq (15) with the power dependent frequency change discussed before $\phi \approx 3 \times 10^{-3}$ radians.

VI. Conclusions

We presented an analytical discussion of frequency shifts in a beam tube which relate to the presence of a cavity phase difference between the two Ramsey excitation regions. The results are approximate and may be expected to be different for different experimental conditions (see Section II).*

*Other causes, e.g., the second-order Doppler effect which is totally ignored in this paper, must be taken into consideration when extreme accuracies (beyond the present state of the art) are the objective.

The results, as summarized in eqs (10) and (12) and in Fig. 2 show that frequency shifts caused by cavity phase differences, the microwave power level, the resonance linewidth (velocity of atoms), and the frequency modulation amplitude are closely interrelated.

Frequency changes due to changes in the microwave power level are quite pronounced, which is contrary to the result of a previous treatment of this subject.[4] In fact, the power dependence could be utilized to determine the frequency bias when other causes for frequency bias are known to be absent, e.g., microwave spectrum effects (see Section V). In this case the measurement of the frequency change with reduction of the microwave power to, for example, $P_o/2$ yields directly the frequency bias according to eq (13). We have used this method in recent experiments on new cesium beam tubes.

A cavity phase difference will, in general, also cause frequency shifts to occur if the frequency modulation amplitude is changed. The frequency changes are virtually absent if a frequency modulation amplitude of less than half the linewidth is chosen (Section III). However, if larger frequency modulation amplitudes are used or if the cavity phase difference is unusually large (large bias) this effect could become significant. An analytical treatment which is valid for these conditions would require a higher order approximation than was attempted in this paper.

In the presence of a spectrum-related frequency bias of unknown magnitude, methods like the one discussed above will not be adequate. Beam reversal presently appears to be the only method** which then allows the separation and individual measurement of the different frequency biases. Beam reversal changes the sign in eq (12); consequently, one should then obtain microwave power (and frequency modulation amplitude) dependencies which are identical for the two beam directions, except for the sign reversal. Any deviations from this symmetry would indicate the presence of additional effects, e.g., signal spectrum asymmetry. The frequency biases can then be obtained as was demonstrated in Section V (Fig. 3).

*The treatment of the power dependence due to cavity phase differences, as presented in this paper, leads to a rather simple physical picture: The chosen microwave power level acts like a velocity selector because only atoms within a rather narrow velocity range will have a significant transition probability. Atoms with velocities other than that selected by the given power level do contribute, but only in some minor fashion as may be seen from eq (12) and Figs. 1 and 2.

**Beam velocity changes,[5] as applied to the detection and correction of cavity phase differences, act in a way quite similar to the variation of the microwave power. Both act on W in eq (12). However, this method also should only be used with great caution if spectrum related frequency biases are present.

We summarize: In the absence of other biases, e.g., spectrum-related biases, variation of the microwave power offers a way to easily detect and correct for a cavity phase difference and the related frequency bias.

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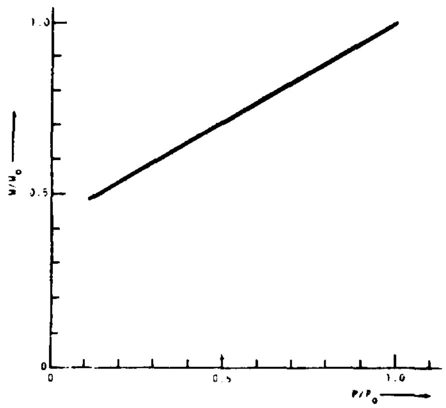

FIGURE 1 - Calculated dependence of the linewidth as a function of the microwave power. Linear approximation.

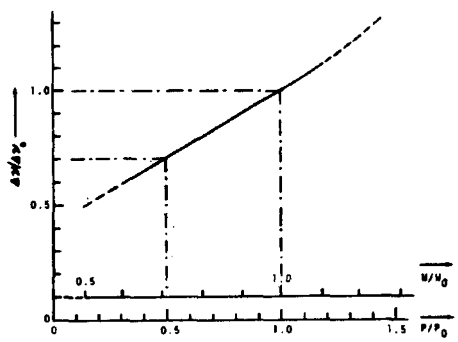

FIGURE 2 - Calculated Frequency bias (normalized) as a function of linewidth and microwave power.

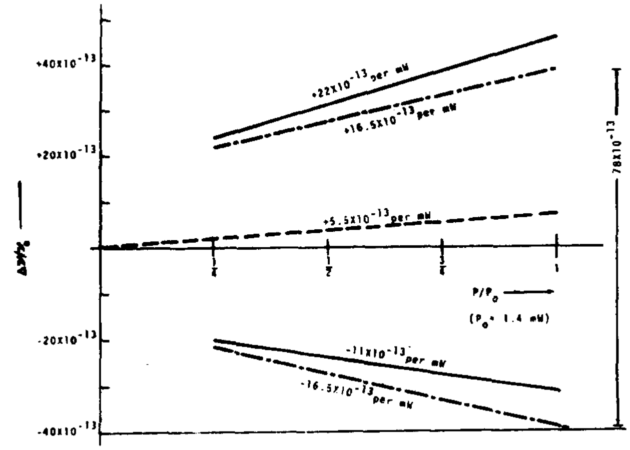

FIGURE 3 - Experimental results. Solid lines depict the fractional frequency shift measured as a function of the microwave power level (P_0 = 1.4 mW) for both directions of the atomic beam. The dashed line represents the bias attributed to spectral impurities of the microwave signal. The dashed-dotted lines are obtained by subtracting this bias from the measured results and represent the bias due to a cavity phase difference. For details on the experimental procedure and a discussion of the measurement precision and accuracy, see Ref. 2.