Abstract—A compact cold-atom clock based on coherent population trapping is being developed. The clock aims to ultimately achieve a timing uncertainty of a few nanoseconds per day. Here we present an initial evaluation of the three main systematic frequency shifts – the first-order Doppler shift, the light shift, and the Zeeman shift. Planned steps to reduce the size of these shifts will also be presented.

I. INTRODUCTION

A relatively compact, cold-atom clock is under development. The clock is interrogated with light by use of coherent population trapping (CPT) in a lin || lin polarization configuration [1]. This technique allows for excellent control of the phase of the interrogation fields on size scales much smaller than that of a microwave cavity, which is desirable for a small system. The use of cold atoms allows the system to achieve relatively long interrogation periods without requiring buffer gases, which introduce large, temperature-dependent shifts in clocks based on vapor cells.

This work presents an initial evaluation of the three main systematic frequency shifts that currently limit the clock’s long-term stability in our unshielded system – the first-order Doppler shift, the light shift, and the Zeeman shift.

II. EXPERIMENT

Our clock is based on a compact source of laser-cooled $^{87}$Rb atoms. The vacuum system has a two-chambered design, with a 2D MOT [2] loading a 3D MOT in a differentially-pumped configuration. The volume of the vacuum chamber, including the 2 l/s ion pump, is approximately 150 cm$^3$. The chamber design is very similar to the double-MOT design presented in Ref. [3].

To produce clean, coherent light for control and minimization of the light shift, we have developed a laser system based on an optical phase-lock loop built from two commercial megahertz-broad DFB/DBR laser diodes [4]. The use of phase locking results in a relatively clean, two-component spectrum that is free from zero- and higher-order modulation sidebands that cause light shifts without contributing to the CPT signal. The master laser is locked to the $F = 2 \rightarrow F' = 1$ transition on the D1 line at 795 nm via polarization-

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rotation spectroscopy [5], and the slave laser is locked to a frequency near the $F = 1 \rightarrow F' = 1$ transition by locking the beat-note frequency to an accurate frequency reference, which allows us to perform absolute frequency measurements and evaluate systematic shifts. So far, we have achieved a phase error variance of 0.35 rad$^2$ between the two lasers, which corresponds to ~75 % of the power in the coherent carrier.

We probe the clock by use of CPT in the lin || lin geometry, which is known to produce high-contrast CPT spectra owing to the absence of trap states [1]. The linear polarization of the CPT light can be represented as a sum of $\sigma^+$ and $\sigma^-$ circularly polarized light components, which results in probing the atoms with a superposition of two lambda systems. The relevant energy-level diagram for $^{87}$Rb with the two clock lambda systems is shown in Fig. 1. The frequencies of both of the resonances are sensitive to magnetic fields in first order, but the size of the first-order shift is equal and opposite. As long as the two lambda systems have the same signal strength, the linear Zeeman shift will not cause a net shift on the clock’s frequency.

To reduce the light shift [6] - [8] and eliminate power broadening [9] we use Ramsey interrogation. Typically, we prepare atoms in the dark state with a 400 $\mu$s CPT pulse with an intensity of 20 $\mu$W/cm$^2$ for each CPT frequency component. Then we let the dark state evolve for an 8 ms Ramsey period. We then probe the phase of the dark state with a 50 $\mu$s CPT pulse. We derive our signals from the ratio of the transmission of the second Ramsey pulse to that of the first Ramsey pulse.

Figure 1. The energy levels of $^{87}$Rb relevant to lin || lin CPT interrogation. The two lambda systems are shown in red. In blue is the field-sensitive transition that we use to measure the magnetic field. Note that there is also a second, symmetric field-sensitive transition at lower frequency that is not marked.

We limit our interrogation periods to less than 10 ms. The short interrogation periods allow us to efficiently recapture atoms from cycle to cycle with our 3 mm MOT beams, enabling a typical laser cooling-stage duration of 45 ms for loading $2 \times 10^6$ atoms.

We observe Fourier-limited Ramsey resonances with a transmission contrast of 55%, with a typical absorption of 7%. To lock the clock to the hyperfine ground state splitting, we alternately probe the central fringe on opposite sides of the line and steer the clock to the central fringe. An Allan deviation is shown in Fig. 2. The short-term stability is currently limited by laser frequency noise to $4 \times 10^{-11} \tau^{-1/2}$. The long-term stability is limited by magnetic field drift in our unshielded system, as discussed below.
III. Systematic Frequency Shifts

We have begun to evaluate the primary systematic effects that shift the clock frequency. The largest shifts are the Zeeman shift, the first-order Doppler shift, and the light shift.

A. Zeeman Shift

Our current vacuum system is not magnetically shielded, and the second-order Zeeman shift currently limits the long-term stability of the clock. We have quantified the shift by performing in-situ measurements of the magnetic field by jumping the Raman detuning every 5 minutes while the clock is operating to measure the frequency of the field-sensitive transition shown in Fig. 1. The field drift currently limits the long-term stability at the $2 \times 10^{-12}$ level (Fig. 2).

The two lambda systems probed by the clock have the same quadratic sensitivity to magnetic field, but the two CPT resonances also exhibit a linear shift that is equal and opposite. To second order, the Zeeman shift for the two transitions is given by

$$\Delta v_{\text{mag}} = 431 \text{ Hz/G}^2 \pm 2.8 \text{ kHz/G}. \quad (1)$$

We typically operate our clock at a bias field of about 100 mG, at which the second-order Zeeman shift is approximately 4 Hz. The clock drift that we observe corresponds to a drift in the earth’s field at the sub-milliGauss level.

Since the residual linear Zeeman shift has opposite sign for the two $\Lambda$ systems, only certain values of the magnetic field lead the Ramsey fringes for the two resonances to interfere constructively. This is illustrated in Fig. 3. The width of the envelope for the fringes in Fig. 3 is set by the average time period required to pump the atoms into the dark state. The first probe pulse is usually set to 400 $\mu$s to maximize the fringe contrast, but the average pumping period into the dark state is $\sim$100 $\mu$s, which sets the 10 kHz width of the fringe envelope. This width sets a lower limit of about 20 mG on the applied bias field, since the field must be large enough to separate the field-sensitive transitions from the clock fringes.
Because of magnetic-field gradients, arising mainly from the ion pump, we are forced to use a bias field that is significantly larger than this minimum field of 20 mG in order to maximize the fringe contrast. With our typical Ramsey period of 8 ms, we use a magnetic bias field of 88 mG, and the linear splitting exceeds the Ramsey linewidth by a factor of four.

Our next-generation apparatus will be shielded, so we should be able to reduce the magnetic bias field to the lowest, nonzero field where the fringe sets constructively interfere, which is about ¼ of the current value. At this level, the second-order Zeeman shift will be $1/16$ of the value that we have now.

Note that if the signal strengths of the two lambda systems are the same, then the first-order Zeeman shift averages to zero. If there is an imbalance, then a first-order shift is possible. In the future, we plan to do a systematic study of the line weights to put limits on any imbalance by comparing spectra in which the fringes are in and out of phase, like the data in Fig. 3.

B. 1st-order Doppler shift

The 1st-order Doppler shift is proportional to the atom’s average velocity and arises from the interaction of moving atoms with the travelling-wave component of our interrogation light. The largest contributor to the atoms’ velocity in our system is free fall. The 1st-order Doppler shift is analogous to the distributed-cavity phase shift in fountain clocks, which is convenient to think of in terms of a phase shift between the microwave CPT field during the second pulse and the atomic coherence that is generated with the first CPT pulse. The shift may be expressed as

$$\Delta v_D = k_{\text{CPT}} dx/(2\pi T_R).$$

Here, $k_{\text{CPT}}$ is the wave vector for the optically carried microwave beat-note of the CPT field, $dx$ is the change in position along the propagation direction of the CPT field, and $T_R$ is the Ramsey period. With $T_R = 10$ ms, the size of the 1st-order Doppler shift is $3\times10^{-10}$ when the atoms are probed with a pure travelling wave.

To initially measure the 1st-order Doppler shift in a case where the shift is large, we probe along the direction of gravity. Measurements of frequency shift versus atom velocity and probe direction agree well with Eq. 2 with no adjustable parameters. When the atoms are probed with a vertical standing wave (parallel to gravity), the shift is consistent with zero and has a current uncertainty of $1\times10^{-11}$. Ongoing
measurements will reduce this uncertainty, and probing horizontally will significantly reduce the size of any potential shift.

C. Light Shift
The light shift (AC Stark shift) is one of the most important systematic frequency shifts in clocks based on coherent population trapping, and it could ultimately limit the performance of such clocks. We have begun a systematic evaluation of the different contributions to the light shift. A measurement of clock frequency versus probe intensity is shown in Fig. 4 for several values of the detuning, $\delta$. The measurements are well explained by adding different light-shift contributions described briefly below (see also Ref. [7]).

![Figure 4. Measurements of the central fringe frequency for several common-mode CPT laser detunings versus intensity of the CPT light components. Here, the Ramsey period was 8 ms, and the atoms began the clock cycle in the $F = 2$ ground state.](image)

1) Resonant CPT generating couplings:
A detailed theory on light shifts from CPT-generating couplings was previously developed to explain resonant coherent shifts in a beam clock based on sodium atoms [6]. Resonant coherent shifts have also been explored more recently in the context of vapor-cell clocks [7],[8]. For optical detunings that are small compared to the optical linewidth, and when the two CPT frequency components have equal Rabi frequencies, the light-shift expressions in Ref. [6] can be simplified to

$$\Delta v_{LS} = \frac{\Delta \rho_0 \delta}{2 \pi \Gamma R^2} \frac{\Omega^2 \tau_p}{\exp(\Omega^2 \tau_p / \Gamma) - 1}. \tag{3}$$

$\delta$ is the optical detuning, $\Gamma$ is the optical linewidth, $\Omega$ is the Rabi frequency, $T_R$ is the Ramsey period, and $\tau_p$ is the duration of the first Ramsey (pumping) pulse. $\Delta \rho_0$ is the normalized population difference between the ground states at the start of a probe cycle, and varies between $\pm 1$ depending on the initial hyperfine ground state. Thus, the sign of the shift can be modulated by altering the initial population between ground states. Surprisingly, this resonant-coherent shift gets smaller when $\Omega^2 \tau_p$ gets larger.

The measurements in Fig. 4 confirm the general behavior predicted by Eq. 3. We have confirmed that the resonant CPT light shift practically vanishes above an intensity of 20 $\mu$W/cm$^2$ in our experiment. We have also performed optical pumping experiments that confirm that the shift
changes sign depending on the hyperfine state occupied by the atoms at the start of the clock cycle.

2) **Resonant non-CPT generating couplings:**
Because of the limited bandwidth of our optical phase lock loop, only ~75 % of the power measured in the beat-note spectrum is in the carrier. The rest of the power resides in a noise pedestal that arises mostly from the slave laser ($F = 1 \rightarrow F' = 1$). Because this noise pedestal is close to resonance, we believe that the resulting light shift is the main contributor to the total light shift at intensities above 20 µW/cm². It exhibits a dispersive lineshape with the optical detuning $\delta$. The resulting frequency-sensitivity around resonance ($\delta = 0$) is 1 Hz/MHz. Since this shift scales with the frequency noise power, it will get smaller if we can improve the percentage of power in the beat-note carrier in our optical phase-lock loop. It may be possible to achieve a higher percentage of power in the carrier if we can use a slave laser with a higher frequency-modulation bandwidth.

3) **Non-resonant couplings:**
There is also a light shift that arises from non-resonant coupling through excited states that are not part of the clock’s lambda systems. The largest contributor to such shifts is coupling through the $F' = 2$ excited state. The CPT light components are detuned by 817 MHz from this transition. At an intensity of 20 µW/cm², the total off-resonant shift is of order 0.5 Hz, but it is constant to within a fractional frequency of $1 \times 10^{-12}$ for detunings of ±2 MHz. The non-resonant light shift can be made smaller by tuning the intensity ratio of the two CPT light components, since the shifts from the two components have opposite sign and can be made to cancel.

The measured fractional frequency dependence on common-mode optical detuning under our current operating conditions is $1 \times 10^{-10}$/MHz, largely dominated by the resonant non-CPT contribution. We control the detuning of our CPT lasers to the 3 kHz level for 20,000 s integration periods, which leads to fractional frequency changes from the light shift of $3 \times 10^{-13}$, making this an order of magnitude smaller than the Zeeman-shift-induced drift in our system on the same time scale. The clock shift from intensity fluctuations is $1 \times 10^{-12}$ per 1 % intensity change. We control the intensity of the interrogation light to a level of < 0.1 % variation, so we anticipate no drift from intensity changes.

Because we can precisely control the CPT beam parameters and the atomic internal state, our experiment is a unique test-bed to evaluate the different light-shift contributions. There are many adjustable parameters, including detuning, total intensity, the ratio of the intensities of the two frequency components, and the initial hyperfine ground-state populations. Efforts are currently underway to find operating conditions that minimize the total light shift.

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**REFERENCES**


