A NEW ALGORITHM FOR CLOCK WEIGHTS FOR THE SIM TIME SCALE

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Abstract: The Sistema Interamericano de Metrologia (SIM) time scale, known as SIMT, was developed by combining the local time scales maintained by national metrology institutes (NMIs). SIMT became operational in January 2010 and is disseminated every hour via the Internet. SIMT is an ensemble time scale that treats the local time scales, SIMT(*k*), as individual clocks. This paper discusses a new weighting algorithm that was implemented in February 2012. The algorithm features an accuracy factor that proportionally limits the clock weights, based on the inverse of their relative frequency offset with respect to SIMT.

1. Introduction

It is a well-known fact that the base unit of time, the second, can be realized with smaller uncertainties than the other base units of the International System (SI). Time scales act as a bridge between the realization of the unit of time and the time measurement necessities in science, industry and society. Thus, producing a stable and accurate time scale that is accessible to a wide range of users is often a primary responsibility of an NMI.

The Bureau International des Poids et Mesures (BIPM) recently began a pilot experiment to compute Coordinated Universal Time (UTC) every week as opposed to every month, with the aim of increasing accessibility to this international time reference [1]. This SIM region has emphasized time scale accessibility for a number of years, beginning in 2005 when a network was developed to automate comparisons of national time references in real-time [2]. The SIM time scale that followed, SIMT, has been operational since January 2010, and its values are disseminated every hour via the Internet. SIMT is an ensemble time scale that treats the local SIM time scales, SIMT(*k*), as individual clocks [3, 4].

This paper discusses a new weighting algorithm for SIMT that limits the contribution of SIMT(*k*) scales to SIMT by an amount proportional to the inverse of their relative frequency offset with respect to SIMT. By doing so, the new algorithm has increased both the stability and the accuracy of SIMT.

Section 2 briefly describes the SIM time network (SIMTN) that produces the raw data for the SIMT computations. Section 3 provides an overview of the SIMT algorithm. The new weighting algorithm for SIMT is presented in Section 4. Section 5 compares the performance of SIMT before and after the implementation of the new weighting algorithm. Finally, we present conclusions in Section 6.

2. The SIM Time Network (SIMTN)

The SIMTN continuously compares the local SIM time scales to each other, and produces measurement results in near real-time. The comparisons are performed via the Global Positioning System (GPS) common-view and all-in-view techniques with multi-channel, single frequency (L1 band) receivers. The measurement data are exchanged and published via the Internet [2]. The SIMTN has operated continuously since 2005, and 18 nations have joined the network as of May 2012. SIMTN servers located at NRC in Canada, CENAM in Mexico, and at NIST in the United States host software that processes and displays measurement data whenever requested by a user. All three servers are linked from the SIM time and frequency working group website: http://tf.nist.gov/sim

Each server publishes web pages that display a grid containing the most recent time differences between SIM NMIs. The grids are updated every 10 minutes. When a user “clicks” a time difference value displayed on the grid, a phase plot of the comparison will appear. The phase plots can include up to 200 days of data, and the Time deviation and Allan deviation values for the selected data are automatically calculated and displayed. The SIMTN also generates a “data feed” that

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provides the clock comparison data that SIMT needs for its time scale computations.

The SIMTN allows all SIM NMIs to check the performance of their local time scales whenever necessary. The rapid publication of data has made it easy to quickly identify local time scale fluctuations and failures, a key benefit to NMIs that disseminate time or frequency within their countries, or who use their time scale as a reference for calibrations. A detailed description of the SIMTN, including an uncertainty analysis of its comparisons (typically less than 15 ns, $k = 2$), is provided in [2]. A discussion of benefits that the SIMTN provides to SIM NMIs is given in [5].

Approximately 25 cesium clocks and 9 hydrogen masers contribute data, in one way or another, to the SIMTN. This large number of atomic clocks, along with the data collection and processing capabilities of the SIMTN, made it possible to construct SIMT.

3. The SIM Time Scale (SIMT)

Time keeping requires reliability, accessibility, stability, and accuracy. To prevent time scales from being susceptible to a single point of failure, they are often designed to use more than one clock. The set of multiple clocks employed by a time scale is known as the clock ensemble. The time scale output is generated by combining time differences obtained from measurements of the individual clocks in the ensemble. The process of combining clock data usually involves a weighted average, and the best performing clocks are given the most "weight" in the time scale computation. When appropriately designed, and with some restrictions, a time scale can have metrological characteristics that are superior to any single clock within the ensemble.

There are a variety of applications for time scales, and thus there is no "best" algorithm for combining the time differences among the clocks of the ensemble. This is evident when examining the UTC and UTC(k) time scales. UTC is a virtual time scale, meaning that it produces no physical signal, and is generated with a post-processing scheme that introduces delays in its calculations. The goal in generating UTC is to make it as stable as possible in the long term, as it serves as an international reference. In contrast, the UTC(k) scales generated in the laboratories of the NMIs produce physical signals that support real work. For example, they can provide the reference for time signal broadcasts or for frequency and time calibrations. Because time scales can have different applications, it makes sense that different criteria should be used to build them. Therefore, the models that predict clock behavior, the weighting procedures, the periodicity of computing the time scale, the methods used to steer a time scale, and so on, often differ.

SIMT was constructed with specific requirements: i) to be produced in near real time, ii) to be a virtual time scale without a physical signal, iii) to use the local time scales of the SIM NMIs, SIMT(k), as the "clocks" of its ensemble, and iv) to avoid dependence on any single clock. In short, SIMT was designed to be an instantly accessible reference that could monitor and support the performance of local SIM time scales and operational timing systems.

To understand how SIMT works, consider that at the current time t, the prediction $\hat{x}_k(t+\tau)$ for the time difference $\hat{x}$ of the clock k with respect to SIMT at time $t+\tau$ can be written in terms of a known set of parameters. These parameters include: i) the time difference $x_k(t)$ of clock k with respect to the SIMT at time t, $x_k(t)$, ii) the fractional frequency difference $y_k(t)$ of clock k with respect to the SIMT at time t, and iii) the parameter D_k, which takes into account the drift of $y_k(t)$ during the time interval (t, $t+\tau$), as follows

$$\hat{x}_k(t+\tau) = x_k(t) + \left[y_k(t) + D_k\tau\right]\tau + \ldots \qquad (1)$$

Eq. (1) can be easily accepted because it can be seen as a Taylor expansion of the function x_k around the value $x_k(t)$ for a time interval of τ. Note that the frequency (rate) of SIMT is a free parameter that will drift over time due to measurement noise, and might eventually require steering.

Once the (future) time $t+\tau$ is reached, the time differences between clocks can be obtained from the SIMTN data feed via the Internet, so it is possible to compute the value of the time scale SIMT for that $t+\tau$ time. Of course, the predicted value of SIMT, computed at time t for the time $t+\tau$, will not necessarily be equal to the computation of SIMT at

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time $t+\tau$. However, the SIMT value, predicted at time t for $t+\tau$, can be corrected by the time difference measurements at $t+\tau$ by using the relation

$$x_k(t+\tau) = \sum_{j=1}^{N} \omega_j \left[\hat{x}_j(t+\tau) - x_{jk}(t+\tau) \right] \quad , \quad (2)$$

where x_{jk} is the measured time difference between clock *j* and clock *k* at time $t+\tau$.

The prediction $\hat{y}_k(t+\tau)$ of the fractional frequency deviation of clock *k* with respect to SIMT, at time $t+\tau$, is made according to

$$\hat{y}_k(t+\tau) = \frac{\hat{x}_k(t+\tau) - x_k(t)}{\tau}. \quad (3)$$

When the (future) time $t+\tau$ is reached, the correction for the frequency prediction can again be made through an exponential filtering defined by

$$y_i(t+\tau) = \frac{1}{1+m_i}\left[\hat{y}_i(t+\tau) + m_i(\tau) y_i(t) \right], \quad (4)$$

where m_i is a parameter that contains information about the averaging period when reaching the floor noise in clock *i*, given by the relation

$$m_i(\tau) = \frac{1}{2}\left[\sqrt{\frac{1}{3} + \frac{4}{3}\frac{\tau_{\min,i}^2}{\tau^2}} - 1 \right] . \quad (5)$$

Here, $\tau_{\min,i}$ is the integration period at which the noise floor of clock *i* is reached.

4. The new algorithm for SIMT weights

For weights ωi , the condition of normalization is, of course, kept at the SIMT computation algorithm as

$$\sum_{i=1}^{N} \omega_i = 1. \quad (6)$$

In order to have a mechanism to increase or decrease the weight for a single clock according to its frequency stability, the weights can be defined as inversely proportional to the frequency stability, which, in turn, can be estimated by use of the Allan deviation. Thus, the basic SIMT criteria for clock weighting were originally defined (from January 1, 2010 to February 8, 2012) as:

$$\omega_i \propto \frac{1}{\sigma_i(\tau)} \quad , \quad (7)$$

where $\sigma_i(\tau)$ is the Allan deviation of the clock *i* for τ = 10 days, computed from the previous 50 days of SIMTN data. This long integration period was selected to minimize the influence of the common-view time transfer noise that is inherent in the SIMTN data, and thus to provide a truer picture of the actual performance of the clocks.

With the aim to improve the stability of SIMT, the weighting algorithm was modified on February 9, 2012, to include an "accuracy factor" as follows:

$$\omega_i \propto \frac{1}{\sigma_i(\tau)} \times \frac{1}{\Delta f_i}, \quad (8)$$

where Δf_i is the relative frequency offset of the contributing clock with respect to the SIMT frequency. The weighting computation is made every 24 hours, at 0 hours, 0 minutes UTC. Thus, the weighting factor assigned to a clock remains constant throughout the UTC day.

The NMIs that operate ensemble time scales are allowed to have a weight as large as 40 %. The laboratories that operate time scales consisting of a single cesium clock have a weight limit of 10 %. Finally, NMIs without a cesium clock are not allowed to contribute to SIMT (their weight is 0). However, a time difference for all NMI time scales, SIMT – SIMT(*k*), is computed and published every hour.

If an NMI contributes to both SIMT and UTC, then SIMT(*k*) and the UTC(*k*) scales can be considered as equivalent at their source, because they are generated with the same clocks, algorithms, and equipment. However, there can be a time offset between SIMT(*k*) and UTC(*k*), due to the physical point within the laboratory where the time scale output is defined, and also due to delays introduced by cables. It is also important to emphasize that the laboratories that contribute to both SIMT and UTC operate two separate time transfer systems, and a lower-noise system is often used for the UTC contribution.

5. Frequency stability of SIMT

This section compares results before and after the new weighting algorithm was implemented on February 9th, 2012.

Figure 1 shows the weights assigned to the eight laboratories contributing to SIMT during the first three months that the new algorithm was in use (February 9, 2012, to May 9, 2012). Note that, due to its high stability and accuracy, the weight assigned to SIMT(NIST) is usually near the 40 % limit. Note also that the weight of SIMT(ICE) in Costa Rica was dropped to zero on April 15th, due to a cesium clock failure.

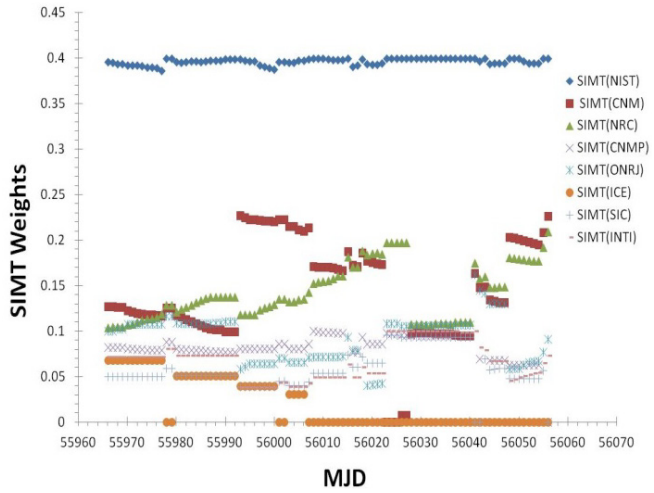

Figure 1. *Weight assignments for the SIMT computation as determined by the new algorithm.*

Figure 2 shows the frequency stabilities for the SIMT(*k*) "clocks" that contribute to SIMT. Figures 3 through 6 show the SIMT – SIMT(*k*) time differences for the February 9, 2012, to May 9, 2012 period. Here we show the weights (on the right vertical axis) as a function of the Modified Julian Date (MJD). The graph inset shows the frequency stability of the SIMT – SIMT(*k*) comparison. In figure 4 can be noticed zero values for CENAM weights around 56025 MJD due to a large frequency shift on the CENAM time scale. Actually this is a good example of how the new weight factor works. In figure 6, there is a point of weight zero for ONRJ that could be caused also by a large frequency shift on SIMT(ONRJ).

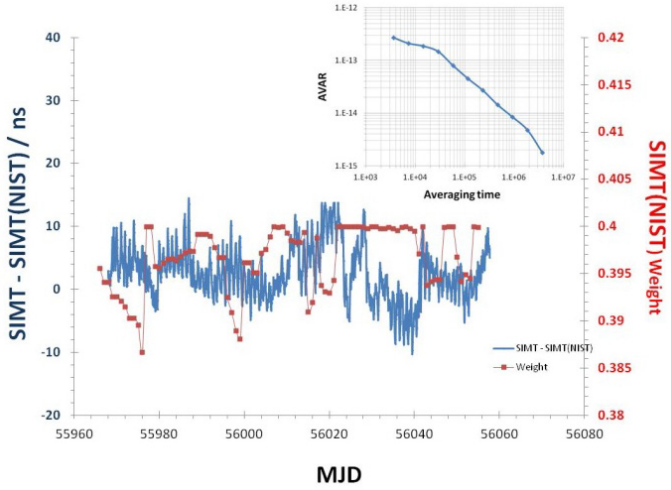

Figure 3. *SIMT(NIST) in the United States compared to SIMT. The inset shows the stability of the comparison.*

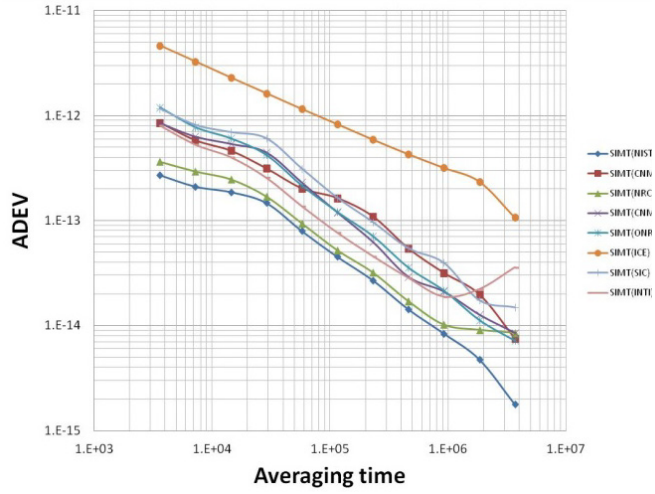

Figure 2*. Frequency stability of the SIMT- SIMT(k) differences for the SIM NMIs contributing to SIMT.*

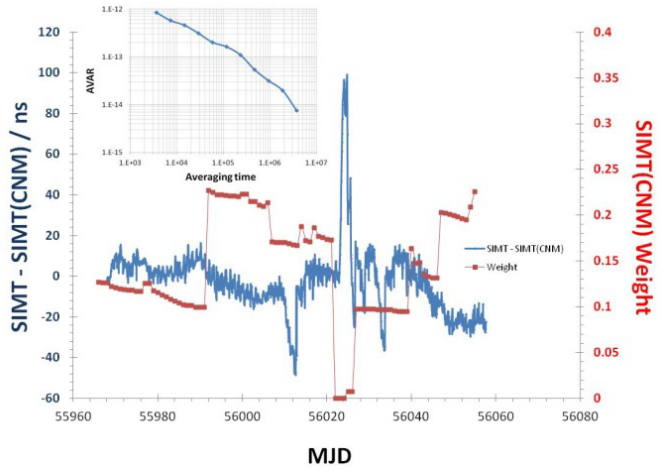

Figure 4*. SIMT(CNM) in Mexico compared to SIMT. The inset shows the stability of the comparison.*

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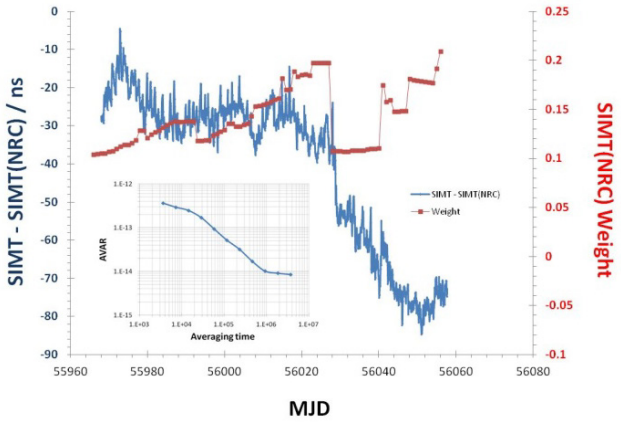

***Figure 5**. SIMT(NRC) in Canada compared to SIMT. The inset shows the stability of the comparison.*

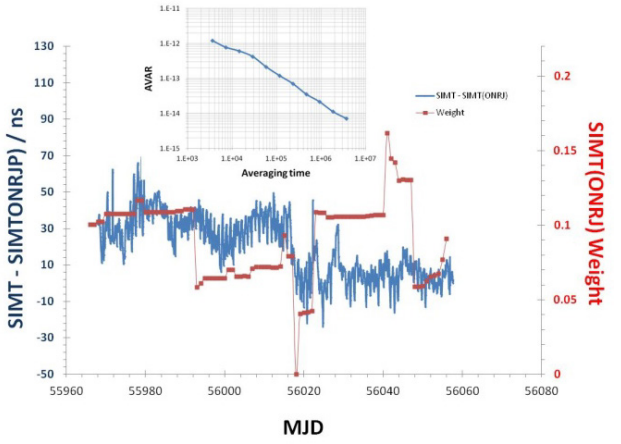

***Figure 6**. SIMT(ONRJ) in Brazil compared to SIMT. The inset shows the stability of the comparison.*

Figure 7 shows two measurements, each covering an interval of about 90 days, of SIMT - SIMT(NIST). The first measurement (blue color graph) corresponds to the period from February 23, 2011, to May 23, 2011, when the original weighting algorithm (Eq. 7) was in use. The second measurement (red color graph) corresponds to the period from February 9, 2012, to May 9, 2012, when the new weighting algorithm (Eq. 8) was in use. The results show that the dispersion of values was reduced with the new algorithm.

Figure 8 shows the Allan deviation of SIMT – SIMT(NIST) before and after the new algorithm was implemented, showing visible improvement at all averaging intervals. We assume that the SIMT(NIST) scale had the same frequency stability in 2011 and 2012, and that the improvement shown with the new algorithm in 2012 is due to an improvement in the stability of SIMT.

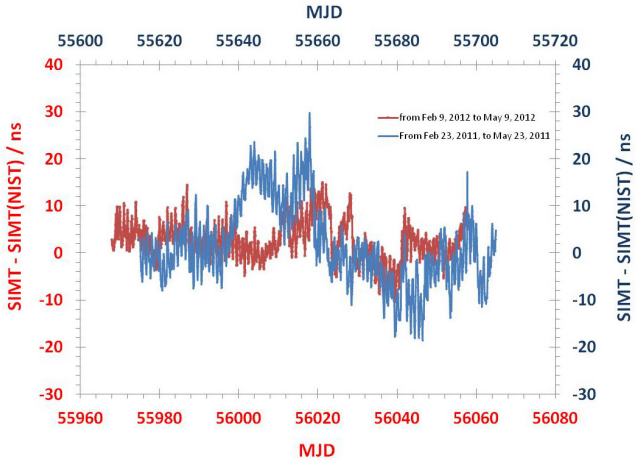

***Figure 7**. SIMT – SIMT(NIST) time differences. The blue trace was recorded with the original weighting algorithm, the red trace with the new algorithm.*

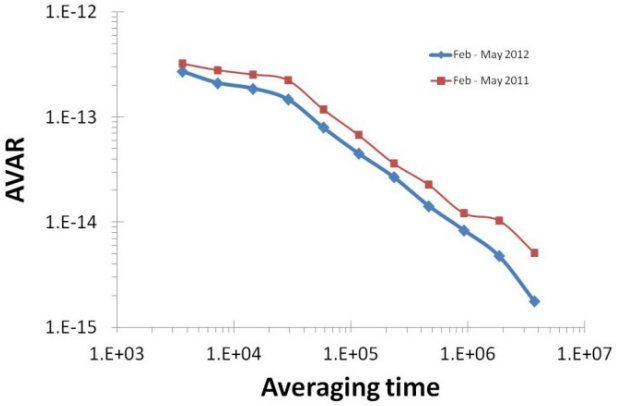

***Figure 8.** SIMT – SIMT(NIST) frequency stability. The red trace was recorded with the original weighting algorithm, the blue trace with the new weighting algorithm.*

6. Conclusions

We have presented a new algorithm for the computation of SIMT that limits the clock weights with an accuracy factor that is inversely proportional to the relative frequency offset between the contributing clock and SIMT. The new weighting

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algorithm has improved the frequency stability of SIMT in the short, medium and long term.

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