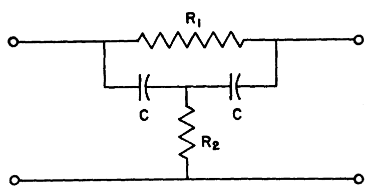
A Note on a Bridged-T Network*

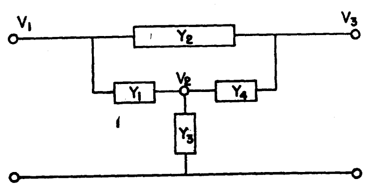
PETER G. SULZER†, ASSOCIATE, IRE

Summary—A bridged-T network previously employed for the measurement of resistance is analyzed, and is shown to be useful as the frequency-determining element of a resistance-tuned oscillator. The resistance-capacitance form of the network permits a wide tuning range and simple switching between ranges, while the resistance-inductance network suggests the possibility of building an oscillator having decade frequency dials. With both forms of the network a moderate value of equivalent Q can be obtained, which is desirable for accuracy and stability of frequency calibration.

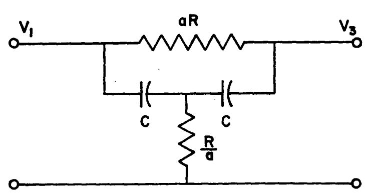
THE NETWORK of Fig. 1(a) has been used for the measurement of resistance at radio frequencies.[1] It is the purpose of this note to show that the network also has application in a dual-feedback RC sinusoidal oscillator.



(a)



(b)



(c)

Fig. 1—Bridged-T networks.

Considering the general bridged T of Fig. 1(b), and writing the equations for the node voltages V_2 and V_3 in terms of the component admittances Y_1, Y_2, Y_3, Y_4,

$$V_1Y_1 - V_2(Y_1 + Y_3 + Y_4) + V_3Y_4 = 0, \tag{1}$$

$$V_1Y_2 + V_2Y_4 - V_3(Y_2 + Y_4) = 0. \tag{2}$$

Solving for the voltage ratio V_3/V_1,

$$\frac{V_3}{V_1} = \frac{Y_1 + Y_2 + \dfrac{Y_1Y_2}{Y_4} + \dfrac{Y_2Y_3}{Y_4}}{Y_1 + Y_2 + \dfrac{Y_1Y_2}{Y_4} + \dfrac{Y_2Y_3}{Y_4} + Y_3}. \tag{3}$$

Employing the circuit of Fig. 1(c) for convenience, and substituting the appropriate admittances in (3),

$$\frac{V_3}{V_1} = \frac{\dfrac{2}{a} + j\omega RC + \dfrac{1}{j\omega RC}}{\left(\dfrac{2}{a} + a\right) + j\omega RC + \dfrac{1}{j\omega RC}}, \tag{4}$$

$$= \frac{\dfrac{2}{a} + ju}{\dfrac{2}{a} + a + ju}, \tag{5}$$

where $u = (\omega/\omega_0) - (\omega_0/\omega)$, and $\omega_0 = 1/RC$. Thus a voltage-ratio minimum, accompanied by zero phase shift, is obtained at ω_0. Fig. 2 is a plot of the magnitude and phase angle of (5), which shows that the sharpness and depth of the minimum increase with a.

It is of interest to obtain the equivalent Q of the network, which is done by defining $Q = (\omega_0/\Delta\omega) = 1/u$, where the limits of $\Delta\omega$ occur when the network response is 3 decibels higher than its minimum value.

Equating the expression for the magnitude of the right-hand side of (5) to $\sqrt{2}$-times its minimum value, and solving for u,

$$u = \frac{2}{a\sqrt{1 - \dfrac{8}{(2 + a^2)^2}}};$$

and therefore,

$$Q_e = \frac{a}{2}\sqrt{1 - \frac{8}{(2 + a^2)^2}} \tag{6}$$

$$\approx \frac{a}{2} \quad \text{for} \quad a \gg 1. \tag{7}$$

Thus a fairly selective network can be obtained with this RC bridged T. Although the sharpness of the response is not comparable to that obtained with the parallel T[2] a smaller number of circuit elements is required, while variable-frequency operation is conveniently accomplished with a dual-section variable capacitor. A simple audio-frequency oscillator is produced[3] if the network is placed in the negative-feedback path of an amplifier having both positive and negative feedback. In this application the possibility of obtaining a moderate value of Q indicates superiority over a previously used network,[4] which has a Q of about 1/3.

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† National Bureau of Standards, Washington, D. C.

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[2] H. H. Scott, "A new type of selective circuit and some applications," Proc. I.R.E., vol. 26, pp. 226–235; February, 1938.

[3] P. G. Sulzer, "Wide-range *RC* oscillator," *Electronics*, vol. 23, pp. 88–89; September, 1950.

[4] F. E. Terman, R. R. Buss, W. R. Hewlett, and F. C. Cahill, "Some applications of negative feedback with particular reference to laboratory equipment," Proc. I.R.E., vol. 27, pp. 649–655; October, 1939.

It is of interest to determine the effect of adding a small capacitance bC across R/a, as shown in Fig. 3(a). Substituting the appropriate admittances in (3),

$$\frac{V_3}{V_1} = \frac{\frac{1}{a}(2+b) + ju}{\frac{1}{a}(2+b) + a + j\left(u + b\frac{\omega}{\omega_0}\right)}. \tag{8}$$

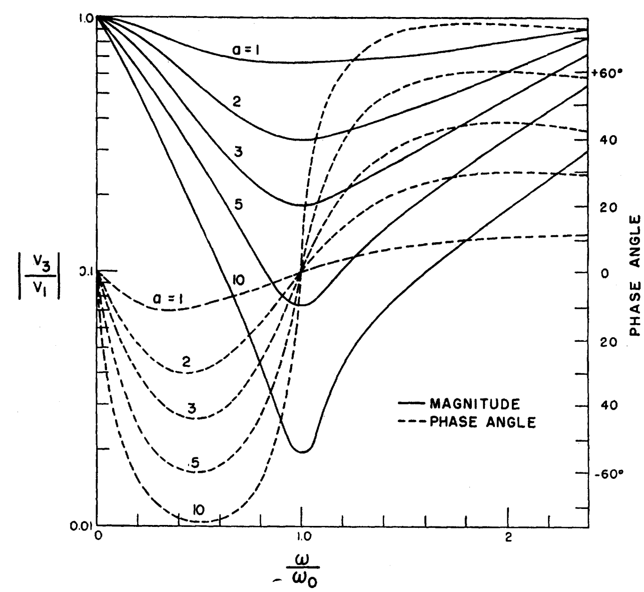

Fig. 2—Amplitude and phase characteristics of the bridged-T network of Fig. 1(c).

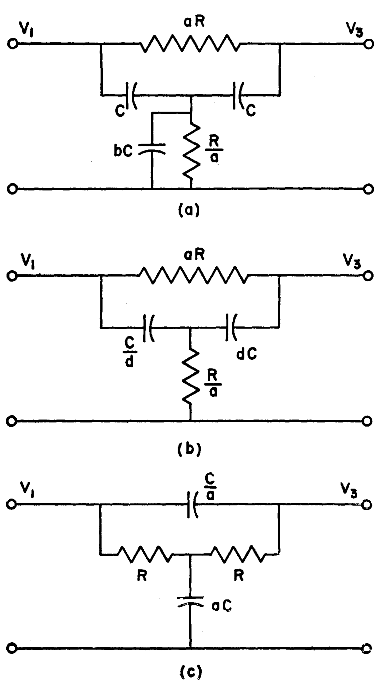

Fig. 3.—Modified bridged-T networks.

A new resonant frequency ω_1 is defined when the phase angle of (8) becomes zero, which occurs when

$$\frac{u}{\frac{1}{a}(2+b)} = \frac{u + b\frac{\omega}{\omega_0}}{\frac{1}{a}(2+b) + a}. \tag{9}$$

Solving for the angular-frequency ratio,

$$\left(\frac{\omega_1}{\omega_0}\right)^2 = \frac{1}{1 - b\left(\frac{2+b}{a^2}\right)}. \tag{10}$$

It is seen from (10) that the effect of the additional capacitor is to increase the resonant frequency of the network. Furthermore, the amount of increase depends upon the capacitance ratio b. Consequently, if variable-capacitance tuning is employed, the additional capacitor will have its greatest effect at the low-capacitance end of a tuning range, which is of considerable practical value when one dial calibration must suffice for two or more decade ranges.

One might suspect that such a trimming process would alter the attenuation of the network. Substituting (10) in (8) and obtaining the magnitude, the new voltage ratio at resonance is

$$\left|\frac{V_3}{V_1}\right| = \frac{2+b}{a^2+2+b}. \tag{11}$$

To consider some practical values, suppose $a=4$, and $u=b=0$. From (5), $|V_3/V_1| = 1/9 \approx 0.111$. Letting $a=4$ $u=0$, and $b=0.1$, $|V_3/V_1| \approx 0.116$ (from 11), while, from (10), $(\omega_1/\omega_0) \approx 1.0065$. Thus with $b=0.1$ the resonant frequency is increased approximately 0.65 per cent, while the attenuation is decreased about 5 per cent.

A variation of the network is obtained if the capacitors C are unequal, as shown in Fig. 3(b). Employing (3) as before,

$$\frac{V_3}{V_1} = \frac{\frac{1}{a}\left(d + \frac{1}{d}\right) + ju}{\frac{1}{a}\left(d + \frac{1}{d}\right) + ad + ju}, \tag{12}$$

while the equivalent Q is given by

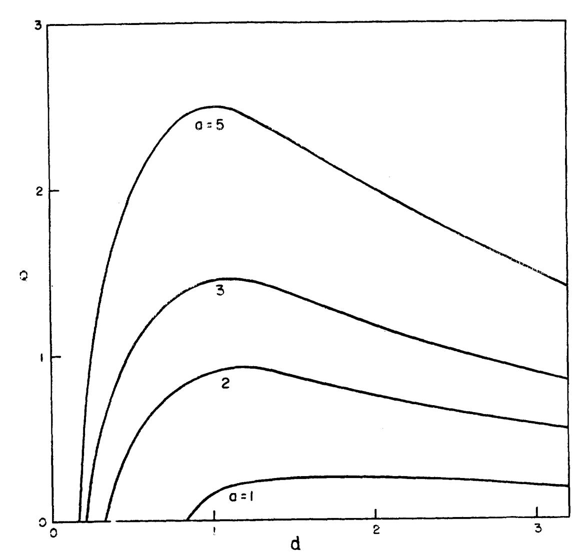

Fig. 4—Q_e versus d for the unsymmetrical bridged-T network.

$$Q = \frac{a\sqrt{d^2(a^4+2a^2-1)+2(a^2-1)-\frac{1}{d^2}}}{d^2(a^2+1)+(a^2+2)+\frac{1}{d^2}}. \quad (13)$$

Fig. 4 contains a plot of (13) showing that a slight increase in Q for low values of a is obtained by using the unsymmetrical bridged T.

A symmetrical variation is obtained if the capacitors and resistors of Fig. 1(c) are interchanged, as shown in Fig. 3(c). It can be shown that the properties of this network are the same as those of Fig. 1(b). The use of equal resistances is convenient when resistance tuning is to be obtained with a dual potentiometer.

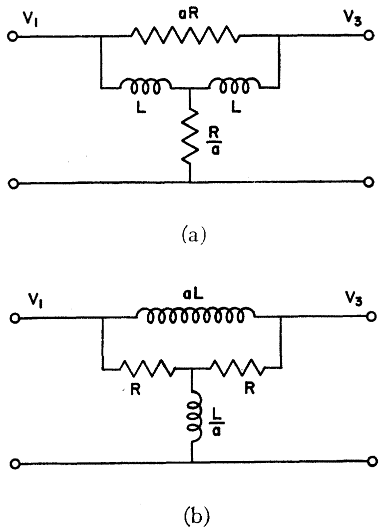

Fig. 5—*RL* bridged-T networks.

The two additional networks of Fig. 5 are obtained by using resistors and inductors. Considering Fig. 5(a) and employing (3),

$$\frac{V_3}{V_1} = \frac{\frac{2}{a}+ju}{\frac{2}{a}+a+ju}, \quad (14)$$

where V_3, V_1, and u are defined as before, and $\omega_0 = R/L$. Thus ω_0 is proportional to R, which presents the very interesting possibility of a decade-frequency oscillator, with frequency selected by decade dials, as are resistance or capacitance in the familiar "decade boxes." For (14) to apply, it is required that the magnitude of L be independent of frequency, and that L be lossless. This first requirement can be met if L is operated well below its self-resonant frequency; however, the second requires some consideration.

Substituting in (3) as before, and allowing for a finite inductor Q,

$$\frac{V_3}{V_1} = \frac{\dfrac{\frac{1}{Q}-j1}{\omega L\left(\frac{1}{Q^2}+1\right)}+\dfrac{2}{aR}+\dfrac{j\omega L}{R^2}+\dfrac{\omega L}{QR^2}}{\dfrac{\frac{1}{Q}-j1}{\omega L\left(\frac{1}{Q^2}+1\right)}+\dfrac{2}{aR}+\dfrac{j\omega L}{R^2}+\dfrac{\omega L}{QR^2}+\dfrac{a}{R}}. \quad (15)$$

If, as before, ω_1 is defined as the angular frequency producing zero phase angle,

$$\left(\frac{\omega_1}{\omega_0}\right)^2 = \frac{1}{1+\frac{1}{Q_2}}. \quad (16)$$

With an inductor Q of 10 the resonant frequency is decreased approximately $\frac{1}{2}$ per cent, which is not serious in most applications.

The Effects of Anisotropy in a Three-Dimensional Array of Conducting Disks*

GERALD ESTRIN†, STUDENT MEMBER, IRE

***Summary*—This microwave delay lens medium is shown to have both magnetic and electric anisotropy, which necessitates an analysis describing its refractive properties for obliquely incident waves.**

A simple linear transformation is applied to the field equations such that the transformed system is magnetically isotropic. Classical solutions from studies in optics provide the ray velocity surfaces in that system. An inverse transformation yields the ray velocity surfaces in the original medium.

Huyghens construction is employed, for two particular arrays, to determine the refracted wave direction after oblique incidence.

Introduction

The use of an array of conducting elements to focus microwaves has received increasing attention since its introduction by Kock[1] in 1948.

When the conducting elements are spherical particles, their symmetry permits the evaluation[2] of averaged dielectric and permeability coefficients which include the effects of interaction between the spheres. That same symmetry makes the coefficients independent of

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† The Institute for Advanced Study, Princeton, N. J.

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