

Quantum information processing and quantum control with trapped atomic ions

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Abstract

The role of trapped atomic ions in the field of quantum information processing is briefly reviewed. We discuss some of the historical developments that enabled ions to enter the field and then summarize the basic mechanisms required for logic gates and the use of the gates in demonstrating simple algorithms. We describe potential pathways to reach fault-tolerant error levels and large-scale devices, and highlight some of the main problems that will be faced in achieving these goals. Possible near-term applications in applied and basic science, such as in metrology and quantum simulation, are discussed.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

Following Shor's development of a quantum-mechanical algorithm for efficient number factoring and its recognized potential practical applications [1], there was a dramatic increase of activity in the field of quantum information science. The possible realization of general-purpose quantum information processing (QIP) is now explored in many settings, including condensed-matter, atomic and optical systems. Trapped atomic ions have proven to be a useful system in which to study the required elements [2] for such a device. Ions are attractive, in part, because qubits based on their internal states are also useful for atomic clocks and have very long coherence times, in some cases exceeding ten minutes [3, 4]. In addition, due to their mutual Coulomb repulsion, trapped ions naturally form into arrays of spatially separated qubits. With the use of focused laser beams, this enables selective qubit addressing, coherent manipulations, and high-fidelity qubit-state readout with state-dependent laser scattering [5, 6]. With these tools, simple algorithms have been demonstrated [6]. However, current operation fidelities are significantly below those required for fault tolerance, and efforts towards scaling to a large system are only beginning. Solving these problems will involve significant technical challenges, but straightforward

solutions are being explored. In the meantime, quantum communication systems that utilize trapped ions will likely soon be implemented [7, 8] and simple elements of QIP are already being used in metrology [6].

This paper briefly reviews a bit of the history and the ideas behind QIP with trapped ions, and summarizes current capabilities, problems and topics for future consideration. Comprehensive treatments including technical details are given elsewhere [6, 9–15]. Of course, all elements required for universal quantum computation are not needed for some applications, but it is likely that the technical hurdles will be the same; therefore, we focus here on the requirements for general, large-scale QIP. Many experimental research groups throughout the world are now working on various aspects of trapped-ion QIP; currently, the list includes groups at Garching (MPQ), Georgia Tech, Griffiths University, ICFO (Barcelona), Imperial College, Innsbruck University, Los Alamos National Laboratory, MIT, National University of Singapore, NIST (USA), NPL (UK), Osaka University, Oxford University, PTB and University of Hannover (Germany), Sandia National Laboratory (USA), Siegen University, Simon Fraser University, Sussex University, University of Aarhus, University of California Berkeley, University of Maryland, Université de Paris, University of Ulm, University of Washington, Wabash College and the Weizmann Institute.

1.1. Background

In a general context, QIP with atoms can be traced back quite far. The Stern–Gerlach experiment is probably the classic example, where an atom’s internal angular-momentum states are entangled with its possible trajectories through an inhomogeneous magnetic field and subsequently projected onto one of the possible correlated outcomes. A more recent example is the single electron ‘AC Stern–Gerlach’ g -2 experiments of Hans Dehmelt *et al* [16], where an entangling radio frequency (RF) transition caused the electron’s cyclotron-motion quantum state to change conditioned on its spin state. This example has a close relation to the current trapped-ion QIP experiments [10, 17].

By 1990, trapped-ion experiments had reached the point where the mechanical motion of the ions could be cooled to the ground state [18]; this could be verified by an excitation of the ion’s internal state conditioned on its motional quantum state. When coupled with optical pumping to a given internal state, this would provide an internal/motional fiducial state from which other quantum states could be produced. At the very low kinetic energies produced by laser cooling, the interaction of laser beams with the ions would be in the Lamb–Dicke regime (where the extent of ion motion is much less than the wavelength). This, coupled with the relatively small dissipation in the ion system, would enable the exploration of the analogue of cavity quantum electrodynamics (QED) experiments in the strong-coupling limit [9, 19–25]. Here, the harmonic oscillator associated with a cavity electromagnetic field mode would be replaced by the harmonic oscillator associated with an ion’s mechanical motion. Various techniques for generating nonclassical motional states were examined, including those for producing Fock states, squeezed states, Schrödinger cat states and ‘spin-squeezed’ states, with some initial experimental demonstrations realized in [26–29].

At the 1994 International Conference on Atomic Physics (Boulder, Colorado), Artur Ekert presented a lecture outlining the ideas of quantum computation [30], a subject new to most of the audience. This inspired Ignacio Cirac and Peter Zoller, who attended that conference and were very familiar with the capabilities of the trapped-ion experiments, to propose a basic layout for a quantum computer utilizing trapped ions in their seminal paper of the following year [5]. In this scheme (figure 1), the motion of the ions is strongly coupled and best described by the normal modes of a kind of pseudo-molecule. In general, the motion of each mode is shared among all the ions and can act as a data bus for transferring information between ions. A single-qubit rotation (the relatively easy part) is implemented by applying a focused laser beam or beams onto that ion. The harder part is to perform a logic gate between two selected ions. This can be accomplished by first freezing out the motion of the ions (putting all modes in the ground state) with laser cooling. The internal qubit state of one ion is then transferred onto the qubit formed from the ground and first excited state of a particular mode of motion (figure 1, laser beam 1). Laser beam 2 then performs a logic gate between the (shared) motion qubit and a second selected ion. Finally, the initial transfer step on the first ion is reversed, restoring the motion to the ground state and effectively having performed the logic gate between the internal qubit states of

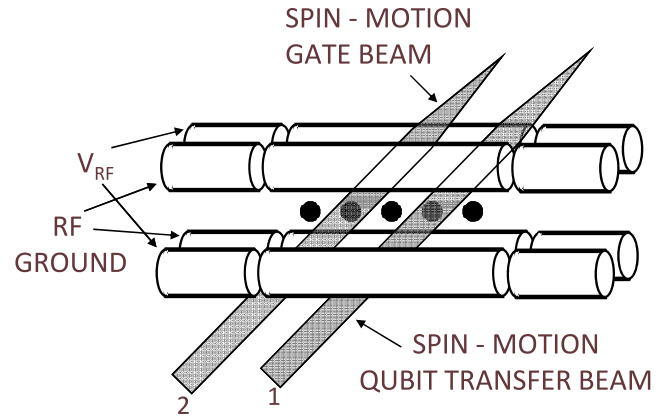


Figure 1. Scheme for quantum computation proposed by Cirac and Zoller [5]. Quadrupolar electrodes are configured to produce a linear array of trapped ion qubits (filled black circles). Two diagonally opposite rods support an RF potential to realize a Ponderomotive pseudopotential transverse to the trap’s (horizontal) axis. Static potentials applied to the end segments confine ions along the axis. Ideally, all motional modes are laser-cooled to the ground state before logic operations. The quantized modes of motion can be used as a data bus to share information between the internal state qubits of ions that are selected by focused laser beams (see text).

the two selected ions. The logic gate between the motion qubit and internal-state qubit was demonstrated in [31] (a variation of this gate that depends on the motional wavepacket size [32] was implemented in [33]). The complete Cirac/Zoller gate between two selected qubits was demonstrated in [34]. More streamlined versions of such deterministic multiqubit logic gates have now been realized (below), but they all employ the basic idea that the ions’ collective motion provides the data bus for sharing quantum information between qubits.

2. Ingredients for trapped-ion QIP

2.1. Traps

The workhorse devices for trapped-ion QIP have been linear RF quadrupole Paul traps, shown schematically in figure 1 [35–38]. These traps are basically RF quadrupole mass analyzers that are plugged on the ends with appropriate static potentials [12]. To a first approximation, they can be viewed as providing a three-dimensional (3D) harmonic well with the strength of the well along the trap axis made relatively weak compared with the transverse directions. In this case, at low temperatures the ions arrange into a linear array, where the ion spacings are determined by a balance between the external confining potential and the ions’ mutual Coulomb repulsion. Typical ion spacings range from approximately 2 to 10 μm . A single ion’s transverse oscillation frequencies are given by

$$\omega_{x,y} \simeq \frac{qV_{\text{RF}}}{\sqrt{2}\Omega_{\text{RF}}mR^2}, \quad (1)$$

where q and m are the ion’s charge and mass, V_{RF} and Ω_{RF} are the RF (peak) potential and frequency, and R is the distance from the trap axis to the nearest electrode surface. For $V_{\text{RF}} = 150 \text{ V}$, $\Omega_{\text{RF}}/2\pi = 50 \text{ MHz}$ and $R = 150 \mu\text{m}$, $^{25}\text{Mg}^+$ ions have a transverse oscillation frequency of

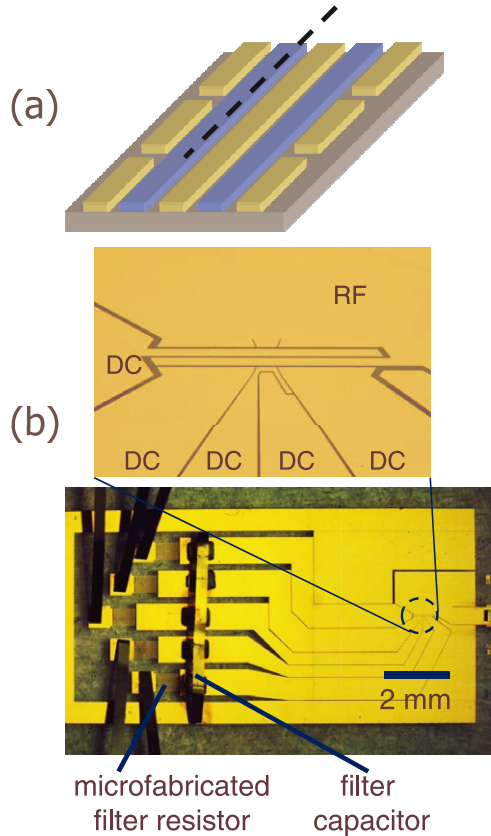


Figure 2. Surface-electrode trap. The electrode geometry shown in figure 1 can be transformed so that all electrodes lie in a plane as shown schematically in (a). The axis of the trap lies above the plane as indicated by the dashed line. (b) Photographs of a single-zone surface-electrode trap in which gold electrodes are deposited on a quartz substrate with the use of lithographic techniques (from [40]). Other examples are described in [39]. Advantages of such a geometry include the ability to fabricate many traps in the same number of steps required to fabricate a single trap, and the precise electrode alignment provided by lithography.

$\omega_{x,y}/2\pi \simeq 12.3$ MHz. Since two-qubit gate speeds scale with the motional frequencies (limited by $\omega_{x,y}$) small traps are desired. Progress from several groups towards making small traps is summarized in [6, 39]. For simplicity of construction, the 3D trap electrode structure shown in figure 1 can be transformed into a planar structure where the ions are trapped above the plane as indicated in figure 2.

2.2. Qubit detection

In trapped-ion (and neutral atom) experiments, the primary qubits are formed from two internal atomic states. For detection purposes, it is very useful to have one of these states form a ‘cycling’ transition with a particular optically excited state. This qubit state is chosen to scatter many photons before optical pumping relaxation occurs, while scattering from the other qubit state is minimal. Even if only a small fraction of the photons are detected, overall state-detection efficiency can be quite high [41–44] and nearly unit detection efficiency is possible. With the use of this technique, detection fidelities in trapped-ion QIP experiments have now reached 0.9999 [45].

2.3. Qubit gates

For the basic gate scheme outlined above, we need to efficiently couple an ion’s internal states to its motion. Laser beams provide a good means for this because the gradient of the laser beam’s field has a length scale given by the laser wavelength λ . For simplicity, we consider here only one of the simplest forms of coupling (and the one employed in the Cirac/Zoller scheme [5]). More comprehensive discussions of the various couplings and multiqubit gates are given elsewhere [6, 9–15]. Consider a single trapped ion that has an optical transition (frequency ω_0) excited by means of a single-electron electric-dipole interaction by a laser beam of frequency ω_L propagating along the z -axis. The interaction is given by

$$H_I = -e\vec{r} \cdot \hat{\epsilon} E_0 \cos(kz - \omega_L t + \phi) \\ = \hbar\Omega(S_+ + S_-)(e^{i(kz - \omega_L t + \phi)} + e^{-i(kz - \omega_L t + \phi)}), \quad (2)$$

where \vec{r} is the electron coordinate relative to the ion’s core, e is the electron charge, $\hat{\epsilon}$, E_0 and k are respectively the laser beam’s electric field polarization, amplitude and wave vector, z is the ion’s position, ϕ is the electric field phase at the mean position of the ion, $S_+ (= |e\rangle\langle g|)$ and $S_- (= |g\rangle\langle e|)$ are the internal-state raising and lowering operators, and $\Omega \equiv -\langle e|\vec{r} \cdot \hat{\epsilon}|g\rangle/2\hbar$, with $|g\rangle$ and $|e\rangle$ denoting the ion’s ground and optically excited states. It is convenient to view the qubit states as ‘spin’ states in analogy with the two states of a spin-1/2 particle; therefore, we will make the identification $|g\rangle \leftrightarrow |\downarrow\rangle$, $|e\rangle \leftrightarrow |\uparrow\rangle$. Thus, single-qubit gates correspond to rotations of the spin states on the Bloch sphere. To treat the ion’s motion quantum-mechanically, we write $z = Z + z_0(a + a^\dagger)$, where Z is the ion’s mean position, $z_0 = \sqrt{\hbar/2m\omega_z}$ is the spread of the ground-state wavefunction, with m and ω_z the ion’s mass and oscillation frequency, and a and a^\dagger are the lowering and raising operators for the ion motion. If we go to the interaction pictures for the ion’s internal states ($S_+ \rightarrow S_+ e^{i\omega_0 t}$) and motion states ($a^\dagger \rightarrow a^\dagger e^{i\omega_z t}$) and assume $\omega_L \simeq \omega_0$, then neglecting terms that oscillate near $2\omega_0$ (rotating wave approximation), equation (2) becomes

$$H_I \simeq \hbar\Omega S_+ e^{i(kz - (\omega_L - \omega_0)t + \phi)} + \text{H.C.} \\ \simeq \hbar\Omega S_+ e^{-i((\omega_L - \omega_0)t - \phi)} (1 + i\eta(ae^{-i\omega_z t} + a^\dagger e^{i\omega_z t})) + \text{H.C.}, \quad (3)$$

where H.C. stands for Hermitian conjugate and $\eta \equiv kz_0 = 2\pi z_0/\lambda$ is the Lamb–Dicke parameter, which we assume here to be much less than 1 (for an ion of mass 40 u (e.g. $^{40}\text{Ca}^+$) in a well with $\omega_z/2\pi = 3$ MHz and $\lambda = 729$ nm, we find $z_0 = 6.5$ nm and $\eta = 0.056$). For $\omega_L = \omega_0$ and $\eta\Omega \ll \omega_z$, to a good approximation we can neglect the second term in equation (3) and obtain $H_I \simeq \hbar\Omega e^{i\phi} S_+ + \text{H.C.}$ This gives the Hamiltonian for ‘carrier’ transitions or single-qubit rotations about a vector in the x – y plane of the Bloch sphere. (Rotations about the z -axis can be implemented with two carrier ‘ π ’ transitions ($\Omega t = \pi/2$) of different phase or AC Stark shifts from nonresonant beams [46–48]). If we assume $\omega_L = \omega_0 - \omega_z$ (laser tuned to the ‘red sideband’), the resonant term gives $H_I \simeq \hbar\eta\Omega e^{i(\phi + \pi/2)} S_+ a + \text{H.C.}$, which is the Jaynes–Cummings Hamiltonian from cavity QED [49]. This interaction and

the ‘blue sideband’ interaction (for $\omega_L = \omega_0 + \omega_z$) provide a simple form of coupling between internal states and motion. When addressing a single ion among multiple ions confined in the same well, the qualitative features are as described above, but the motion corresponds to a particular motional mode that is selected by tuning the laser frequency ω_L appropriately [6, 9–15]. As example applications of these interactions, the red-sideband interaction (applied for a duration $t = \pi/(2\eta\Omega)$) provides the internal-state qubit to motion-state transfer $(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)|0\rangle \rightarrow |\downarrow\rangle(\alpha|0\rangle + \beta|1\rangle)$ in the Cirac/Zoller scheme above, where $|0\rangle$ and $|1\rangle$ are the ground and first excited Fock states for the selected motional mode. The subsequent gate between the motion qubit and the internal-state qubit of the second selected ion is accomplished by implementing a 2π rotation ($\eta\Omega t = \pi$) on a $|\uparrow\rangle|1\rangle \leftrightarrow |aux\rangle|0\rangle$ transition where $|aux\rangle$ is a third ‘auxiliary’ internal state. This operation applies a $e^{i\pi} = -1$ phase factor on the $|\uparrow\rangle|1\rangle$ component of the wavefunction thereby implementing a ‘ π -phase gate’ between the internal-state and motional-state qubits.

Such single-photon optical interactions have been used with great success, particularly by the Innsbruck group, for carrying out QIP operations on Ca^+ ions. For qubits based on hyperfine or Zeeman ground state sublevels, transitions can be driven by coherent two-photon stimulated-Raman transitions [10, 31] with the use of two laser beams of frequency ω_a and ω_b and phases ϕ_a and ϕ_b at the mean position of the ion. The above expressions hold with the replacements $k \rightarrow k_b - k_a$, $\omega_L \rightarrow \omega_b - \omega_a$ and $\phi \rightarrow \phi_b - \phi_a$.

Somewhat more streamlined gates can be realized in which multiple ions are addressed simultaneously by the same laser beam(s) [50–54]. These ‘geometric’ gates can be viewed as arising from quantum phases that are acquired when a mode of the ions’ motion is displaced in phase space around a closed path; the phases accumulated are proportional to the enclosed area in phase space. The gates can be viewed in a common framework, the main difference being whether the forces act on the spins in the z -basis (eigenstates $|\downarrow\rangle, |\uparrow\rangle$) or in the x - y basis (eigenstates of the form $\frac{1}{\sqrt{2}}(|\downarrow\rangle + e^{i\xi}|\uparrow\rangle), \frac{1}{\sqrt{2}}(|\downarrow\rangle - e^{-i\xi}|\uparrow\rangle)$) [13]. The state-dependent forces required for the displacements are optical-dipole forces, which arise from spatial gradients of AC Stark shifts. Since the AC Stark shifts are usually different for the two qubit states, geometric phases lead to entangling gates. Two-qubit phase gates have been implemented in the z -basis [55, 56] and in the x - y basis [47, 57–59]. In the Innsbruck experiment of [58] a state with fidelity of 0.993 with respect to a Bell state was produced, setting a standard for all QIP experiments.

Qubits are typically composed of states separated by optical energies or two states in the electronic ground state hyperfine/Zeeaman manifold [6, 10]. Optical qubits have the advantage that single-photon transitions can implement gate operations, but have the complication that radiative lifetimes (e.g. ~ 1 s in Ca^+) limit long-term memory, and very good laser spectral purity is required. Hyperfine/Zeeaman qubits have extremely long radiative lifetimes, implying potentially very good memory. Gates can be performed with coherent two-photon stimulated-Raman transitions, which requires two laser beams having a difference frequency near the qubit frequency. These beams can typically be

generated with a single beam of modest spectral purity and a RF/microwave frequency modulator to provide the stable difference frequency between the beams. However, hyperfine/Zeeaman transitions driven by stimulated-Raman transitions have the disadvantage of spontaneous emission decoherence, which can be reduced only by the use of high laser power and large detuning from allowed transitions [60, 61].

The use of single- and multiqubit gates has enabled the demonstration of several (deterministic) QIP algorithms. These include the Deutsch–Jozsa algorithm [62], dense coding [63], qubit teleportation [64, 65], quantum error correction [66], entanglement-assisted detection [67], the quantum Fourier transform [68], Grover’s search algorithm [69], entangled state purification [70], entanglement swapping [71], the Toffoli gate [72] and the production of an 8-qubit W-state [73] and a 6-qubit ‘spin-cat’ state [74]. A technique to generate arbitrary motional state superpositions, proposed by Law and Eberly [75], was demonstrated in [76]. (this method has recently been applied with great success to the fields in a strip-line cavity coupled to a Josephson junction phase qubit [77]). Although the basic features of these algorithms were observed, in all cases, considerable technical effort will be required to reach fault-tolerant levels.

3. Beginning applications

Although primary drivers for the field of QIP have been a factoring machine [1] and a device for performing unstructured searches [78, 79], it is very likely that other applications will emerge first. Some of these are described briefly here.

3.1. Spectroscopy and metrology

Some potential applications are motivated by the idea of using entangled states to improve spectroscopic sensitivity [21, 24, 25, 29, 80], and simple demonstrations of this increased sensitivity have been made [28, 29, 74, 80]. The improvements in sensitivity derived from these techniques assume that noise is dominated by ‘projection noise,’ the fundamental noise arising from the fluctuations in which state the system is projected into upon measurement [81, 82]. However, if significant additional phase noise is present in either the atoms themselves [83], or the interrogating radiation [10, 84], the gain from entanglement can be lost. This puts a premium on finding probe oscillators that are stable enough that the projection noise dominates for the desired probe duration.

Another interesting use of entanglement in spectroscopy was demonstrated in [80, 85]. Here, a precise measurement of the quadrupole moment of $^{40}\text{Ca}^+$ was made by performing spectroscopy on an entangled state of two ions in which the spectroscopy yielded the quadrupole moment, but was immune to perturbations from ambient fluctuating magnetic fields.

Some ions of spectroscopic interest may be difficult to detect because they do not have a cycling transition, or lack a cycling transition at a convenient wavelength. In some cases, this limitation can be overcome by simultaneously

storing the ion(s) of spectroscopic interest with another ‘logic’ ion whose states can be more easily detected. Using the state-transfer process of the Cirac/Zoller gate described above, it is possible to transfer the states of interest in the spectroscopy ion to a mode of the ions’ coupled motion and then transfer this information to the logic ion, which is subsequently measured [86, 87]. This technique has been used to detect optical transitions in $^{27}\text{Al}^+$ ions by transferring the relevant state amplitudes to a $^9\text{Be}^+$ logic ion, which is then measured [87]. It is now used routinely in an accurate optical clock based on $^{27}\text{Al}^+$ [88] and might also be extended to molecular ions [89].

The information transfer and readout process employed in [87, 88] typically had a fidelity of about 0.85, limited by errors caused by the ions’ motion. However, this detection process is a quantum-nondemolition (QND) type of measurement in that it does not disturb the detected populations of the $^{27}\text{Al}^+$ ion. It can therefore be repeated to gain better information on the $^{27}\text{Al}^+$ ion’s state. By use of real-time Bayesian analysis on successive detection cycles, the readout fidelity was improved from 0.85 to 0.9994 [90]. This experiment shares similarities with those of Serge Haroche’s group, where successive atoms are used to perform QND measurements of the field in a cavity [91]. In [90], the same atom ($^9\text{Be}^+$) is reset after each detection cycle and used again. Also, because the detection was accomplished in real time, the procedure was adaptive, requiring on each run a variable number of detection cycles to reach a certain measurement fidelity.

3.2. Tests of quantum correlations and entanglement

With the continued interest in Einstein–Podolsky–Rosen correlations [92] and Bell-type inequalities [93], trapped ions have been able to probe these fundamental aspects of quantum mechanics in some new regimes. By producing entangled pairs of ions, Bell’s inequalities of the CHSH form [94] have been measured. Since the pairs are produced deterministically, correlation measurements can be performed in every experiment, as opposed to, for example, photon-based experiments where not every pair produced results in a measurement. This feature enabled experiments that showed a violation of Bell’s inequalities and overcame the ‘detection loophole’ [95, 96]. These experiments also provided the first such tests on massive particles that employed a complete set of correlation measurements. Some experiments have explored other aspects of entanglement such as size, by production of an 8-qubit W-state [73] and a 6-qubit ‘spin-cat’ state [74]. Longevity has been explored with the production of decoherence-free subspace Bell states [96, 97] with coherence lifetimes of 7 s in [98] and 34 s in [73]. In [99], Bell’s inequalities were violated for entangled states between an ion and a photon. Distance scales for entanglement have been explored by producing entangled spins over length scales from sub-millimeter [6] to ~ 1 m [100]. The highest fidelity (0.993) of deterministic entanglement in any system has been reported in [58]. A recent verification of (state-independent) contextuality in quantum mechanics has been made in [48]. By transferring entanglement from spin states to the motion of ion pairs held in different locations, entanglement has been created between separated mechanical oscillators [101].

3.3. Towards quantum simulation

In the early 1980s, Richard Feynman proposed that one quantum system might be used to efficiently simulate the dynamics of other quantum systems of interest [102, 103]. This is now a highly anticipated application of QIP, and will likely occur well before useful factorization is performed. Of course the universality of a large-scale quantum computer will allow it to simulate any quantum system of interest. However, before such a device is built, it may be possible to use the readily available interactions in a quantum processor to simulate certain classes of physical problems. For trapped ions, it might be possible to use the interactions employed in the various gates to simulate other systems of interest such as nonlinear optical systems [10, 104]. Along these lines, it has been possible to simulate the phase sensitivity of a Mach–Zehnder interferometer comprised of two n th-order nonlinear optical beam splitters, showing the increased gain in sensitivity from the nonlinearity [105]. Currently, there is considerable interest in, and efforts are underway in several laboratories to use, QIP interactions to simulate condensed-matter systems. Some of the basic ideas for how this might work with ions have been outlined in [106–114]. Here, one can take advantage of the fact that phase gates between ions i and j invoke a spin–spin-like interaction of the form $\sigma_{\hat{u}i}\sigma_{\hat{u}j}$, where $\hat{u} \in \{\hat{x}, \hat{y}, \hat{z}\}$. Spin rotations about a direction \hat{u} act like magnetic fields along \hat{u} . In [59, 115], these basic interactions have been implemented on a few ions and efforts are underway to scale to much larger numbers. One interesting application is to study quantum phase transitions by varying the relative strengths of the (simulated) spin–spin and magnetic field interactions, from small to large numbers.

4. Future

4.1. Scaling to many ion qubits

Very large crystalline arrays of ions have been observed in ion traps [116–118]. These arrays have recently been used to demonstrate strong coupling with photons [119] and as a model quantum memory for studying the error-suppressing capabilities of dynamical-decoupling pulse sequences [120, 121]. In these large crystals, the ions are relatively well separated, implying that individual qubit addressing can be accomplished with appropriately focused and steered laser beams. However, for almost all of the gates that have been demonstrated, information is transferred through one mode of motion. The modes generally have different frequencies and therefore can be spectrally isolated, but as the numbers of ions increase, the mode spectrum becomes more dense and it becomes difficult to isolate the mode of interest, or the gate speeds must become very slow to maintain this isolation.

To mitigate this problem (and to ease the problem of single-qubit addressing by laser beam focusing) ions could be distributed over arrays of individual trap zones so that in each zone the number of ions is relatively small and single-mode addressing is not problematic. One way to transfer information throughout the array would be to physically move the ions between zones [10, 122–124]. Another way this could be accomplished is to first create entangled pairs

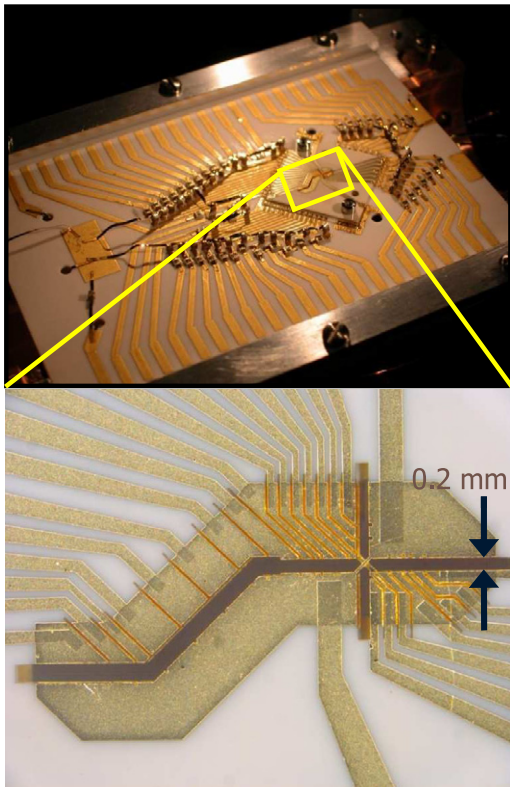


Figure 3. Photographs of a multizone trap that incorporates an ‘X’ junction [135]. The trap is formed from two gold-coated alumina wafers similar to that described in [124]. An array of interconnected trap zones that includes junctions will enable two ions selected from arbitrary locations in the array to be transported to a common zone with time-dependent potentials applied to the segmented electrodes, in order to perform two-qubit logic gates.

where the qubits in each pair are strategically distributed to different locations in the array. Subsequently, information is transferred and gates are performed between separated qubits by teleporting [8, 125–129]. The initial entangled-pair distribution could occur by physically transporting the ions or through projective entanglement, as demonstrated in [100]. With ions, teleportation has been demonstrated deterministically in [64, 65] and probabilistically in [130].

Multizone arrays are currently being explored in several laboratories, as summarized in [39]. Transport of ions in linear arrays has been studied in [65, 124, 131, 132] and methods to deterministically order ions in [101, 133]. For efficient computation, we want to perform multiqubit gates between ions selected from arbitrary locations in the processor. This can be accomplished with 2D arrays with junctions [134, 135] as shown in figure 3, or the ability to swap ion positions in a linear array [133]. It will be important to separate and move ions with minimal increase in kinetic energy while preserving coherence [65, 124, 135].

4.2. Ion heating

A rather ubiquitous problem that plagues trapped-ion QIP is ion heating. In contrast to the Cirac/Zoller gate, some types of gate operations are relatively insensitive to this heating (as long as the Lamb–Dicke limit is maintained) [50–53, 136], but at some point, it compromises all types of gates. Heating

can come from thermal electronic noise. In free space, this noise is manifested as blackbody radiation, but for typical trapped-ion conditions, it can be described as coming from Johnson noise in any resistive elements associated with the trap electrodes [10, 137, 138]. It has typically been small enough to not cause significant problems. However, as noted above, to increase gate speeds in QIP we want high motional frequencies, which can be obtained, in part, by using very small trap structures. At smaller trap dimensions (< 1 mm) additional electric field noise has been observed. Representative data, expressed as electric-field noise spectral density at the position of the ions, compiled from several groups are contained in [39, 139, 140]. Unfortunately, observed heating rates are typically orders of magnitude higher than predicted for thermal noise heating. A model that seems to approximately represent the observations assumes that electric field noise is due to randomly fluctuating potentials on patches located on the electrode surfaces, where the size of the patches is small compared with the electrode–ion spacing [141]. There is evidence from some groups that the noise may be due to surface contamination, since its strength depends, in part, on electrode cleaning and vacuum processing. Interestingly, it has been observed that the heating drops more quickly with ambient temperature than would be expected for thermal electronic noise [142, 143]. It appears that it is a thermally activated process and is dramatically reduced near liquid helium temperatures [143]. Of course, ion trappers hope to identify and eliminate the cause, but in the meantime, it may be possible to mitigate the heating by operating at low temperature.

Even for very low heating rates, it will likely be necessary to provide some sort of qubit cooling, particularly for lengthy computations. Since cooling implies dissipation, the qubits themselves cannot be used as the cooling elements. However, cooling can be performed ‘sympathetically’ [144–147], where one ion or group of ions is used as a refrigerator to cool the qubit ions. In QIP, the refrigerator ions might be identical to the qubit ions [148], a different isotope [149, 150], or an entirely different species [101, 151]. During a multiqubit gate, the cooling must be interrupted, so that significant heating while the cooling is off will still compromise fidelity. This requirement might be relaxed in certain circumstances where the change in ion temperature is slow compared with the gate dynamics [152].

4.3. Gates

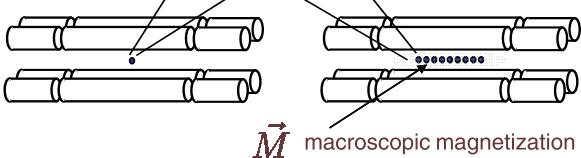
Nearly all gates that have been demonstrated rely on addressing a single mode of motion. Their speed is limited in part by the requirement that the pulse be not so short as to couple to other modes. Generally, this is aided by use of very high mode frequencies, implying very small traps and putting a premium on suppressing the anomalous heating (above). However, as discussed in [8, 153, 154], it should not be required to use single motional modes to perform multiqubit gates. For example, ‘push gates’ [155–158], which could be implemented with quasi-static dipole forces, can be viewed as geometric phase gates in the limit of large detuning where all modes participate. Here, the same basic ideas for the gates apply, but by utilizing multiple modes the gate speeds can be

considerably increased and the number of ions in a given zone could be made substantially larger than when single-mode addressing is required [152]. In the future, it may be that a combination of strategies will be used for efficient, large-scale processing.

Magnetic field gradients can be used to implement gates. If an array of ions is placed in a static magnetic field gradient, the individual ion qubit frequencies are position-dependent; therefore, the qubits can be addressed for single-qubit gates without the need for the qubit excitation fields (microwave or optical) to be focussed onto the ions [10, 106, 159–161]. In addition, sideband transitions and multiqubit gates can be implemented with uniform RF fields. If the two-qubit states have different magnetic moments, their confining wells are displaced spatially in a static gradient field; therefore, Franck–Condon-type overlap factors allow simultaneous change of spin and motional states [106, 159, 160]. On the other hand, when the qubit transition frequencies are independent of position, it should be possible to use inhomogeneous ac magnetic fields for sideband transitions and multiqubit gates on hyperfine qubits [112, 162, 163], similar to Dehmelt’s AC Stern–Gerlach effect for electrons. The potential advantages of RF fields for gates are significant. Since the Lamb–Dicke limit for the RF transitions would likely be easy to reach with Doppler cooling, and because stimulated-Raman transitions would be replaced by RF transitions, it would reduce laser power requirements by orders of magnitude. Spontaneous emission decoherence would be absent. Intensity and phase stability would likely be much easier to control with RF versus optical fields. However, the advantage of qubit addressing with focused laser beams would be lost, and crosstalk between zones, particularly for single qubit rotations, could be problematical. These effects can in principle be mitigated by use of composite pulses [14] and proper shielding between zones.

4.4. Lasers

A significant source of decoherence in many experiments stems from laser-driven transitions. For qubits based on optical transitions, the fundamental limit is caused by the radiative lifetime of the excited state. For hyperfine qubits driven by two-photon stimulated-Raman transitions, the fundamental limit is caused by spontaneous emission from the weakly coupled optically excited states [60, 61]. However, in practice, decoherence is often dominated by classical noise. The effects are easy to state in general terms, but can be technically challenging to correct. One effect is phase noise in the laser beams caused by phase noise in the lasers themselves (particularly important for optical qubits) or phase noise caused by fluctuations in beam path length. Intensity noise might be caused by power fluctuations or fluctuations between the relative positions of the beams and the ions due to air currents or mechanical vibrations. Operation at shorter wavelengths can be especially troublesome because of poorer quality optics, degradation of beam qualities when they are produced by nonlinear conversion, and inability to purify beam quality with mode cleaners such as single-mode fibers. Laser beam switching is typically accomplished with RF-driven acousto-optic modulators, but

$$\begin{aligned}\Psi &= \frac{1}{\sqrt{2}} \left[|\downarrow\rangle_1 |\downarrow\rangle_2 \cdots |\downarrow\rangle_N + |\uparrow\rangle_1 |\uparrow\rangle_2 \cdots |\uparrow\rangle_N \right] \\ &= \frac{1}{\sqrt{2}} \left[|\downarrow\rangle_k \prod_{j \neq k} |\downarrow\rangle_j + |\uparrow\rangle_k \prod_{j \neq k} |\uparrow\rangle_j \right] \\ &= \frac{1}{\sqrt{2}} \left[|\downarrow\rangle_k \vec{M}_\downarrow + |\uparrow\rangle_k \vec{M}_\uparrow \right] \quad (\text{for large } N)\end{aligned}$$


\vec{M} macroscopic magnetization

Figure 4. Experimental configuration that mimics the conditions of Schrödinger’s cat [164]. From a large number of ions in the state Ψ , we select the k th ion and transport it to a separate location (on the left) while the remaining ions are located in the right-hand trap zone. For large N , these remaining ions comprise a large spin and associated macroscopic magnetic moment \vec{M} . We associate the k th ion with the single radioactive particle in Schrödinger’s example; we associate the mesoscopic magnetic moment pointing up or down with Schrödinger’s cat being dead or alive.

in many cases the on–off ratio is not sufficient, due to scattering in the modulators. Moreover, the RF drive can lead to temperature-dependent index effects in the modulators that cause time-dependent beam steering and mode quality. This becomes a particular problem in longer algorithms with intermediate measurements and branching, where the duty cycle of switching is generally not constant. These classical fluctuations in principle have straightforward solutions, such as better passive control and active feedback using power and position sensors, but may in practice be difficult to implement. Therefore, significant effort on the engineering side is already needed to help overcome these problems.

4.5. Fundamental tests

As trapped-ion experiments become more refined, we should be able to provide more stringent tests of certain quantum phenomena. A general long-sought goal is to perform a ‘loophole-free’ test of Bell’s inequalities. Following the recent successes of producing remote entanglement in ions with good memory qualities via photons in fibers [100], it may be possible to perform such a test. With anticipated technical improvements, QIP systems will become larger, more complex and more entangled. This will press issues such as the measurement problem and fundamental sources of decoherence, and may enable the possibility of generating situations like Schrödinger’s cat [164] (figure 4). As an example of this latter possibility, trapped-ion experiments [29, 74, 165] have been able to make small- N approximations to the state

$$\Psi = \frac{1}{\sqrt{2}} [|\downarrow\rangle_1 |\downarrow\rangle_2 \cdots |\downarrow\rangle_N + |\uparrow\rangle_1 |\uparrow\rangle_2 \cdots |\uparrow\rangle_N]. \quad (4)$$

If these states can eventually be produced for very large N , it should be possible to realize a situation like that in figure 4. Unless there might be some as of yet undetected mechanism that prevents the formation of large entangled superpositions [166], and if we can overcome the (admittedly

formidable) technical challenges, we should therefore be able to realize analogues of Schrödinger's cat. Of course, as is often the case, as the field progresses, new unanticipated fundamental phenomena will hopefully emerge.

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